Gauge/Gravity Duality, Winter 2024/25

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6. Large N expansion (8 points)

To be discussed on Friday, 22nd November, 2024 in the tutorial.
Please indicate your preferences until Sunday, 17/11/2024, 21:00:00 on the website.

Exercise 6.1: Fun with diagrams.

Consider a $U(N)$ gauge theory, in the $N \to \infty$ (remember that we are keeping $\lambda = g^2 N$ fixed and finite). We can rewrite its Lagrangian (as we did during the lecture) as

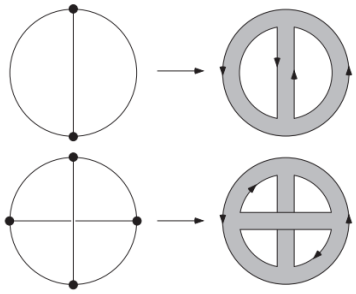
$$\mathcal{L} = \frac{1}{g^2}\left[-\frac{1}{2}\,\mathrm{Tr}(\partial_\mu \tilde{\phi}\partial^\mu \tilde{\phi}) + \mathrm{Tr}(\tilde{\phi}^3) + \mathrm{Tr}(\tilde{\phi}^4)\right].$$

a) (2 points) Write the diagrams for the propagator of this theory, the cubic vertex and the quartic vertex. Explain with which power of the coupling they come and use this information to obtain the following formula:

$$\text{diagram} \sim N^{V-E+F}\lambda^{E-V},$$

where V is the number of vertices, E is the number of propagators, F is the number of loops, $\chi = V - E + F = 2 - 2g$, where χ is the Euler characteristic and g is the genus.

b) (2 points) Consider the following diagrams for vacuum amplitudes:



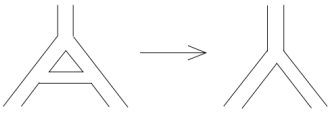
Evaluate V, E, F, the Euler characteristic χ and the genus g for the two diagrams. How do they scale with N?

c) (2 points) From the expansion of the generating functional W one sees that dominant diagrams are those for which the genus vanishes. These are called planar diagrams. Consider the first diagram of task b) along with the following one:

Evaluate the λ-scaling of the two diagrams and argue which one is dominant in the large N expansion. Comparing the two, what do you expect to be the topological condition for

a general diagram to be dominant in the large N expansion? (In other words: why are dominant diagrams called planar?)

d) (2 points) The Euler characteristic is a topological invariant. To understand the meaning of this statement consider the following diagram deformation:



Compute χ for the two diagrams. Has this deformation changed the characteristic? Why? Argue from these two diagrams what is the genus of a sphere. Is the genus the same when we consider a non-planar diagram? Why?