Gauge/Gravity Duality, Winter 2024/25

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5. Renormalisation (8 points)

To be discussed on Wednesday, 20th November, 2024 in the tutorial.
Please indicate your preferences until Friday, 15/11/2024, 21:00:00 on the website.

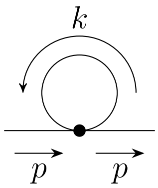
Exercise 5.1: Renormalisation of ϕ^4 theory

We will now, step-by-step, apply the renormalisation procedure to the following Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4$$

at 1-loop.

a) (1 point) Starting from the Feynman rules we found in the previous exercise sheet, write the 2-points 1-loop contribution $\Gamma^{(2)}_{1L}$ coming from the following divergent diagram:



$$= \frac{ig}{2}\int\frac{d^dk}{(2\pi)^d}\frac{1}{k^2+m^2-i\epsilon}.$$

b) (2 points) Performing a Wick rotation ($k_0 \to ik_0$), and then sending $\epsilon \to 0$, show that, integrating, one obtains

$$\Gamma^{(2)}_{1L} = -\frac{g}{2}\frac{\Gamma(1-\frac{d}{2})}{(4\pi)^{d/2}}m^{\frac{(d-2)}{2}},$$

where Γ is the Euler's Gamma function.
(Hint: $\int\frac{d^dk}{(2\pi)^d}\frac{1}{(k^2+b)^a} = \frac{1}{(4\pi)^{d/2}}\frac{\Gamma(a-d/2)}{\Gamma(a)}\frac{1}{b^{a-d/2}}$ *in Euclidean space.)*
Show that in $d = 4$ this quantity diverges. Expanding the previous expression, keeping $d = 4-\epsilon$, show that it reduces to

$$\Gamma^{(2)}_{1L} \propto \frac{1}{2}\frac{g}{16\pi^2}m^2\left(\frac{2}{\epsilon}+1-\ln m^2\right)(e^{-\gamma}4\pi)^{\epsilon/2},$$

where γ is the Euler-Mascheroni constant.

c) (2 points) Show that, in order to reabsorb this divergent 1-loop contribution, one is led to add a counter-term Lagrangian

$$\mathcal{L}^{(2)CT}_{1L} = -\frac{A}{2}\partial^\mu\phi\partial_\mu\phi - \frac{B}{2}\phi^2 - \frac{C}{4!}\phi^4,$$

with $A = 0, B = \frac{gm^2}{16\pi^2\epsilon}, C = 0$.

d) (1 point) Considering the 4-points 1-loop corrections, one has to add to the previous expressions a $C = \frac{3g^2}{16\pi^2\epsilon}$ term, leading to a new counter-term Lagrangian with a non-vanishing C term. knowing that $\phi_0 = Z_\phi^{1/2}\phi$, show that the bare quantities are defined as

$$Z_\phi = 1 + A, \qquad m_0^2 = \frac{m^2 + B}{Z_\phi}, \qquad g_0 = \frac{g + C}{Z_\phi^2},$$

and compute them for our case.

e) (2 points) In dimensional regularisation we are computing quantities in $d = 4 - \epsilon$. This means that we have to send our g (the coupling in d=4) into $g\mu^\epsilon$ (where μ is an arbitrary mass scale). Show how this reflects in m_0^2 and compute the following quantities that appear in the renormalisation group equation:

$$\beta = \mu\frac{\partial g}{\partial \mu}, \qquad \gamma_m = \frac{\mu}{m}\frac{\partial m}{\partial \mu}, \qquad \gamma = \frac{\mu}{2Z_\phi}\frac{\partial Z_\phi}{\partial \mu}.$$

Explain what happens when β is larger, smaller or equal than 0.