Gauge/Gravity Duality, Winter 2024/25

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4. Quantisation of field theories (14 points)

To be discussed on Wednesday, 20th November, 2024 in the tutorial.
Please indicate your preferences until Friday, 01/11/2024, 21:00:00 on the website.

Exercise 4.1: Path integral quantisation

Starting from the relation

$$Z[J] = \exp\left[i\int d^dx \mathcal{L}_{\text{int}}\left(\frac{\delta}{i\delta J(x)}\right)\right] Z_0[J],$$

a) (4 points) derive the Feynman rules for a ϕ^4 theory whose Lagrangian is given by

$$\mathcal{L} = -\frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{g}{4!}\phi^4.$$

b) (4 points) Transforming the n-point correlation functions into momentum space,

$$G^{(n)}(p_1,\dots,p_n) = \int d^dx_1 \cdots \int d^dx_n G^{(n)}(x_1,\dots,x_n)e^{-i(p_1x_1+\cdots+p_nx_n)},$$

where all the momenta p_i are taken to be ingoing, derive the momentum space Feynman rules.

c) (2 points) Consider the *retarded Green's function* G_R given by

$$G_R(x-y) = \int \frac{d^dk}{(2\pi)^d}\frac{e^{ik(x-y)}}{-(k^0+i\epsilon)^2+\vec{k}^2+m^2}.$$

Show that

$$(-\Box + m^2)G_R(x-y) = \delta^d(x-y)$$

by explicitly computing the integral on the left-hand side.

d) (2 points) Show that

$$G_R(x-y) = -i\Theta(x^0-y^0)\langle 0|[\hat{\phi}(x),\hat{\phi}(y)]|0\rangle,$$

with Θ the *Heaviside step function* defined as $\Theta(x) = \begin{cases} 1 & \text{if} \quad x>0 \\ 0 & \text{if} \quad x<0 \end{cases}$.

e) (1 point) During the lecture we introduced the functionals $\Gamma[\phi]$ and $W[J]$. Show that

$$\frac{\delta\Gamma[\phi]}{\delta\phi(x)} = J(x).$$

f) (1 point) Using chain rule show that

$$\frac{\delta}{\delta J(x)} = \int d^dy \frac{\delta^2 W[J]}{\delta J(x)\delta J(y)}\frac{\delta}{\delta\phi(y)}.$$