Gauge/Gravity Duality, Winter 2024/25

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14. Correlators & holographic renormalisation. (16 points)

To be discussed on Friday, 31st January, 2025 in the tutorial.
Please indicate your preferences until Tuesday, 28/01/2025, 21:00:00 on the website.

Exercise 14.1: Bulk-to-boundary propagator. 5 points

The bulk-to-boundary propagator can be given as

$$K_\Delta(z,x;y) = \lim_{w\to 0} \frac{2\Delta - d}{w^\Delta} G_\Delta(z,x;w,y)$$

where G_Δ is the bulk-to-bulk propagator. Verify this expression using Green's second identity

$$\int_{\mathcal{M}} \mathrm{d}z \mathrm{d}^d x \sqrt{g} \left(\phi(\Box_g - m^2)\psi - \psi(\Box_g - m^2)\phi\right) = \int_{\partial\mathcal{M}} \mathrm{d}^d x \sqrt{\gamma}(\phi\partial_n\psi - \psi\partial_n\phi),$$

where γ is the determinant of the induced metric on $\partial\mathcal{M}$ and ∂_n is the derivative normal to the boundary (in our case $\partial_n = z\partial_z$, check it).

Hint: You will ned the following asymptotic behaviours:

$$\lim_{z\to\epsilon} K(z,x;x') = e^{\Delta_-}\delta(x-x'),$$
$$\lim_{z\to\epsilon} G(z,x;z'',x'') \sim e^{\Delta_+}.$$

Exercise 14.2: Holographic renormalisation.

Consider the $(d+1)$-dimensional AdS metric in Fefferman-Graham coordinates:

$$\mathrm{d}s^2 = L^2\left(\frac{\mathrm{d}\rho^2}{4\rho^2} + \frac{1}{\rho} g_{\mu\nu}(x,\rho)\mathrm{d}x^\mu \mathrm{d}x^\nu\right),$$

where x^μ, $\mu \in \{0,1,2,3\}$ are the coordinates parallel to the boundary of AdS and ρ is the redial direction.

a) (4 points) Consider the following Penrose-Brown-Henneaux diffeomorphism:

$$\rho = \rho'(1-2\sigma(x')),$$
$$x^\mu = (x')^\mu + a^\mu(x',\rho'),$$

with $g'_{\rho\rho} = g_{\rho\rho}$ and $g'_{\mu\rho} = g_{\mu\rho}$ (where ρ is again the radial direction, not a generic 4-index). Show that

$$\partial_\rho a^\mu = \frac{L^2}{2} g^{\mu\nu}\partial_\nu\sigma, \tag{1}$$

$$g_{\mu\nu} \to g_{\mu\nu} + 2\sigma\left(1 - \rho\frac{\partial}{\partial\rho}\right) g_{\mu\nu} + \nabla_\mu a_\nu + \nabla_\nu a_\mu. \tag{2}$$

b) (3 points) Explain why (1) induces a conformal transformation $g_{\mu\nu}(x) \to e^{2\sigma(x)} g_{\mu\nu}(x)$ at the boundary.

c) (4 points) Consider a scalar field with boundary conditions

$$\begin{aligned} \phi(\rho, x) &= \rho^{(d-\Delta)/2} \bar{\phi}(\rho, x), \\ \bar{\phi}(\rho, x) &= \rho_{(0)}(x) + \rho \phi_{(2)}(x) + \rho^2 \phi_{(4)}(x) + \dots \end{aligned}$$

Derive the equation of motions for $\phi(x, \rho)$ and using them show that

$$\phi_{(2)}(x) = \frac{1}{2(2\Delta - d - s)} \Box_{(0)} \phi_{(0)}(x).$$