Quantum Field Theory, Summer 2024

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12. Renormalisation group flow (18 points)

To be discussed on Thursday, 6th June, 2024 in the tutorial.
Please indicate your preferences until Saturday, 01/06/2024, 21:00:00 on the website.

In addition to computing the β-functions of massless Yukawa theory, we further practice how to obtain the renormalisation group flow for the Gross-Neveu model in two dimensions. It was originally introduced by David Gross (Nobel Prize 2004) and André Neveu as a toy model for quantum chromodynamics.

Exercise 12.1: The Gross-Neveu model

We want to compute the one-loop β-function for the coupling g of the Lagrangian

$$\mathcal{L} = \sum_{i=1}^{N} \left[i\overline{\psi}_i \partial\!\!\!/ \psi_i + \frac{1}{2} g^2 (\overline{\psi}_i \psi_i)^2 \right] .$$

It describes N Dirac fermions in two dimensions interacting through a four fermion term.

a) (3 points) Derive the Feynman rules for this model. *Hint: We have done this step already very often and the final result will be not given here. If you are in doubt you should rather try to have a look at the literature. This is in general a good idea when you start to work with a new model.*

b) (3 points) Compute the superficial degree of divergence and identify the divergent diagrams. *Hint: You should find two.*

c) (3 points) Obtain the counterterms for the divergent diagrams and write down their Feynman rules.

d) (3 points) Fix the divergent contribution to the counterterms for the propagator at one-loop in dimensional regularisation.

e) (3 points) Repeat this step for the vertex diagram. *Hint: It has four external legs. Therefore, you already know that there has to be an s-, t- and u-channel. However, you cannot compute them in one shot like we did for scalars, because here the Feynman rules have a more complicated index structure which you have to take into account.*

f) (3 points) Compute the β-function for the coupling g. *Hint: For this computation you need the* $\log M^2$ *divergent term from the counter terms fixed above. But, we just computed the divergent part. Use dimensional analysis to see how the* $\log M^2$ *contribution can be directly obtained from the* ϵ^{-1} *part we already have.*