Quantum Field Theory, Summer 2024

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11. Renormalisation III (21 points)

To be discussed on Wednesday, 29th May, 2024 in the tutorial.
Please indicate your preferences until Friday, 24/05/2024, 21:00:00 on the website.

In exercise 9, we studied divergences in loop integrals of Yukawa theory with the Lagrangian

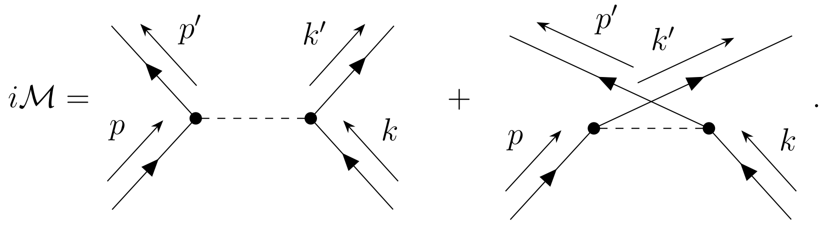
$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m_S^2\phi^2 - \frac{\lambda}{4!}\phi^4 + \overline{\psi}(i\partial\!\!\!/ - m_F)\psi - ig\overline{\psi}\gamma^5\psi\phi\,. \quad (1)$$

Our most important observation was that we have to add the coupling $\lambda\phi^4/4!$ to render the theory renormalisable. In the meanwhile, we practised during the last exercise how to compute renormalised amplitudes by adding appropriate counterterms. We will do this for (1) at one loop in the following.

Exercise 11.1: The Yukawa potential

Before, we start with the more technical part, let us connect Yukawa theory with the nuclear force which stabilises the nucleus of atoms. The latter contain positively charged protons which are repelled by the electromagnetic force. However, at least sufficiently light nuclei are stable. Therefore, there has to be an additional force, the nuclear force, which overcomes the Coulomb force. We will see how this force arises from (1) now.

a) (3 points) Compute the four particle scattering amplitude



We actually need them in the non-relativistic limit, where

$$\overline{u}(p')u(p) = 2m$$

holds. *Hint: Use the Feynman rules we derived in the last exercise. You should find in the non-relativistic limit*

$$i\mathcal{M} = \frac{4ig^2m_F^2}{|\vec{p}' - \vec{p}|^2 + m_S^2}\,.$$

b) (3 points) Compare this result with the Born approximation to scattering amplitudes in non-relativistic quantum mechanics,

$$\frac{i\mathcal{M}}{4m_F^2} = -iV(\vec{q}), \qquad \vec{q} = \vec{p}' - \vec{p}\,.$$

and compute the potential in position space, i.e. $V(r)$. Why is the force it describes attractive? What happens in the limit $m_S \to 0$?

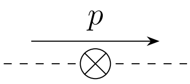
Exercise 11.2: One loop renormalisation

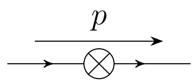
We now continue the discussion from last exercise.

a) (3 points) Figure out the counterterms required to absorb all divergences and derive the corresponding Feynman rules. *Hint: You should use*

$$\phi = \sqrt{Z_\phi}\phi_R\,, \qquad \psi = \sqrt{Z_\psi}\psi_R\,, \qquad m_S^2 = \frac{Z_S}{Z_\phi} m_{S,R}^2\,, \quad m_F^2 = \frac{Z_F}{Z_\psi} m_{F,R}^2\,, \quad g = \frac{Z_g}{Z_\psi\sqrt{Z_\phi}} g_R\,,$$

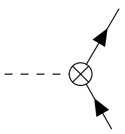
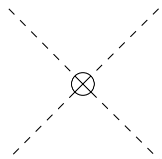
and one more substitution for λ*, which you will figure out yourself, to eventually find*



$= i(\delta Z_\phi p^2 - \delta Z_S m_{S,R}^2)\,,$


$= i(\delta Z_\psi \not{p} - \delta Z_F m_{F,R})\,,$

and


$= \delta Z_g g_R \gamma^5\,,$

$= i\delta Z_\lambda \lambda_R$

with

$$Z_i = 1 + \delta Z_i + \mathcal{O}(g^2)\,.$$

b) (3 points) We now will renormalise the four different types of divergent diagrams that we discovered in the last exercise. Start with the scalar field and write down the two diagrams which contribute at the two loop level to its full propagator. Use dimensional regularisation to compute their divergent contributions and absorb them into δZ_ϕ and δZ_S. *Hint: Work in the minimal subtraction scheme (MS) where only the divergent part enters the counterterm and you only have to compute the divergent part of the two contributing diagrams.*

c) (3 points) Repeat these steps for the fermion propagator to fix δZ_ψ and δZ_F,

d) (3 points) for the Yukawa Coupling (δZ_g) and finally

e) (3 points) for the ϕ^4-vertex (δZ_λ).