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11. Superstring theory and branes (21 points)

To be discussed on Friday, 10th January, 2025 in the tutorial.
Please indicate your preferences until Sunday, 05/01/2025, 21:00:00 on the website.

Exercise 11.1: The Fermionic String Spectrum.

a) (2 points) Worldsheet supersymmetry is achieved by adding fermions ψ to the worldsheet. The oscillator expansions of these new operators contain b_m modes, where $m \in \mathbb{Z} + \phi$ and $\phi = 0$ (1/2) for R (NS) boundary condition. The anti- commutation relations are given by:

$$\{b^\mu_r, b^\nu_s\} = \eta^{\mu\nu}\delta_{r+s}.$$

The ground states of the Hilbert space now need to be distinguished depending on whether they lie in the R or NS sector. Define these respective ground states and explain why the Ramond vacuum has a degeneracy.

b) (3 points) For the open fermionic string, obtain the ground and the first excited state and their mass by acting the creation operators on the vacuum in the R and NS sector respectively. You will observe that the tachyonic mode appears again, but this time it can be consistently projected out using the GSO projection. To do so, one introduces the operator $(-1)^F$ requiring its eigenvalues on every state in the Hilbert space be $+1$ in the NS-sector and $+1$ or -1 in the R-sector.

c) (5 points) The closed string spectrum requires both left and right movers. Obtain its spectrum at the massless level by taking a direct product of left and right movers (which could either in the be R or NS sector).

Hint: tensor products of vector (v), spinor (s) and co-spinor (c) representations of $SO(8)$ can be decomposed as

$$\begin{aligned}
8_v \otimes 8_v &= 1 \oplus 28 \oplus 35_v, \\
8_v \otimes 8_s &= 8_c \oplus 56_c, \\
8_v \otimes 8_c &= 8_s \oplus 56_s, \\
8_c \otimes 8_s &= 8_v \oplus 56_v, \\
8_c \otimes 8_c &= 1 \oplus 28 \oplus 35_v.
\end{aligned}$$

Exercise 11.2: Type IIB SUGRA.

a) (3 points) Find the appropriate field redefinition (Weyl rescaling) that turns the action in the string frame

$$S_s = \frac{1}{(2\pi)^7 l_s^8}\int d^{10}x\sqrt{-g_s}e^{-2\phi_s}[R_s + 4(\nabla\phi_s)^2],$$

into the Einstein frame

$$S_E = \frac{1}{(2\pi)^7 l_s^8 g_s^2}\int d^{10}x\sqrt{-g_E}\left[R_E - 4(\nabla\phi_E)^2\right].$$

Exercise 11.3: DBI action for D_p-branes.

The DBI action for a D_p-brane reads

$$S_{D_p} = -\tau_p \int d^{p+1}\zeta e^{-\phi} \sqrt{-\det(\mathcal{P}[g]_{ab} + 2\pi\alpha' F_{ab})},$$

where $\mathcal{P}$ is the pullback on the worldvolume (given by coordinates ζ^a) and F_{ab} a $U(1)$ field strength.

a) (2 points) Show that, for small ϵ, one can write the following expansion:

$$\sqrt{\det(1+\epsilon M)} = 1 + \frac{1}{2}\epsilon \operatorname{Tr} M + \epsilon^2 \left(\frac{1}{8}(\operatorname{Tr} M)^2 - \frac{1}{4}\operatorname{Tr}(M^2) \right) + \mathcal{O}(\epsilon^3).$$

What happens if M is antisymmetric?

b) (2 points) Simplify the DBI action in case of a flat spacetime with vanishing dilaton. Then, take the D_p-brane to be aligned with the coordinate axis and the embedding functions into the target space (given by coordinates X^m, $m = p, \ldots, 9$) to vanish, i.e.,

$$X^a = \begin{cases} \zeta^a, & a = 0, \ldots, p-1, \\ 0, & a = p, \ldots, 9. \end{cases}$$

c) (2 points) Take the expression of the previous exercise and use the expansion of point a) to expand the DBI action up to the first non-trivial order in F.

d) (2 points) Consider an open string whose endpoints lie on a D_p-brane with a background field F_{ab}. Show that the Neumann boundary conditions must be replaced by

$$\partial_\sigma X^a - 2\pi\alpha' F^{ab} \partial_\tau X_b = 0.$$