The Generalized Geometry of Integrable σ-models

2d σ-models $\longleftrightarrow$ (super) geometry

Question: What distinguishes geometry of integrable models?

gen. Isometries: PCM, WZW ✓

YB-, η-, λ-def. affine Gaudin, ... ✓

1. Gen. Geometry of σ-models

$$S = \frac{1}{4\pi\alpha'} \int_\Sigma \left(g_{ij}\, dx^i \wedge * dx^j + B_{ij}\, dx^i \wedge dx^j \right)$$

with Hamiltonian

$$H = \frac{1}{4\pi\alpha'} \int d\sigma\, J_M \mathcal{H}^{MN} J_N \qquad J_N = \begin{pmatrix} \partial_\sigma x^n \\ p_n \end{pmatrix}$$

$$p_n = g_{nm} \partial_\tau X^m + B_{nm} \partial_\sigma X^m$$

and $\{ J_M(\sigma), J_N(\sigma') \} = 2\pi\alpha' \delta'(\sigma - \sigma')\, \eta_{MN}$ $\quad \eta_{MN} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Field redefinition: $\quad J_A = \frac{1}{\sqrt{2\pi\alpha'}} E_A{}^M(x^i) J_M$ results in

$$\{ J_A(\sigma), J_B(\sigma') \} = F_{AB}{}^C J_C(\sigma)\, \delta(\sigma - \sigma') + \underbrace{\eta_{AB}}_{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}} \delta'(\sigma - \sigma') \quad \text{with}$$

$$H = \frac{1}{2} \int d\sigma\, J_A \underbrace{\mathcal{H}^{AB}}_{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}} J_B$$

$$\boxed{F_{ABC} = E_{[A}{}^I \partial_I E_B{}^J E_{C]J}} \quad \text{gen. fluxes}$$

$$\partial_I = (\partial_i \;\; 0)$$

- diff's
- B-field tr.
- T-duality

$$\begin{array}{ccc} g_{ij}, B_{ij} & \rightleftarrows & \\ \downarrow \uparrow & & F_{ABC} \\ g'_{ij}, B'_{ij} & \rightleftarrows & \end{array}$$

Remarks:

- $F_{ABC} = \text{const.}$

  $\mathcal{E}$-model with $\mathcal{E} = \eta \mathcal{H}$

2. Gen. Group manifold

similar to $f_{abc} = 2 e_{[a}{}^i \partial_i e_{b]}{}^j e_{cj} = \text{const.}$

with $t_a e^a{}_i dx^i = g\, dg^{-1}$ $\quad g \in G$

Ingredients for $E_A{}^I$:

1. Lie group/algebra: $[t_A, t_B] = F_{AB}{}^C t_C$
2. Invariant pairing: $\langle t_A, t_B \rangle = \eta_{AB}$
3. max. isotropic subgroup H with
   $\langle t^a, t^b \rangle = 0$

↳ unique $E_A{}^I$ on G/H permits dualities

3. Gen. Cosets

$$m^{-1} dm = t_a e^a{}_i dx^i + t_\alpha A^\alpha{}_i dx^i \qquad m \in G/F$$

Same in gen. geom.? $e^a{}_i \to E^A{}_I$

$A^\alpha{}_i \to \Omega^\alpha{}_I$ and $\rho^{\alpha\beta}$

new connection required

all from double coset $\boxed{F\backslash G/H}$ = gen. coset or gen. homogeneous space

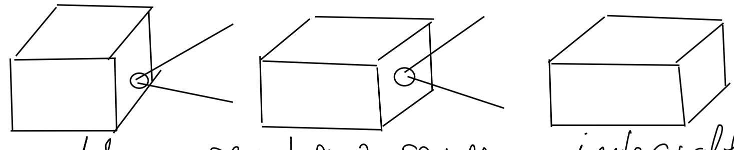
F = isotropic ⇝ degenerate $\mathcal{E}$-model

4. β-functions

$$\boxed{\frac{d\mathcal{H}_{AB}}{d\log\mu} = R_{AB} = \text{const.}}$$

@ 1-loop
@ 2-loops for $F = 1$
Vision to all orders

Suggests:



all σ-models | gen. homo. spaces | integrable models