Supergeneralized geometry, dualities and integrable deformations

Falk Hassler

Based on 2307.05665 with

D. Butter, C. Pope and H. Zhang

Uniwersytet Wrocławski

NARODOWE CENTRUM NAUKI

String dualities

- Historical development of T-dualites

abelian $\subset$ non-abelian $\subset$ Poisson-Lie $\subset$ WZW-Poisson

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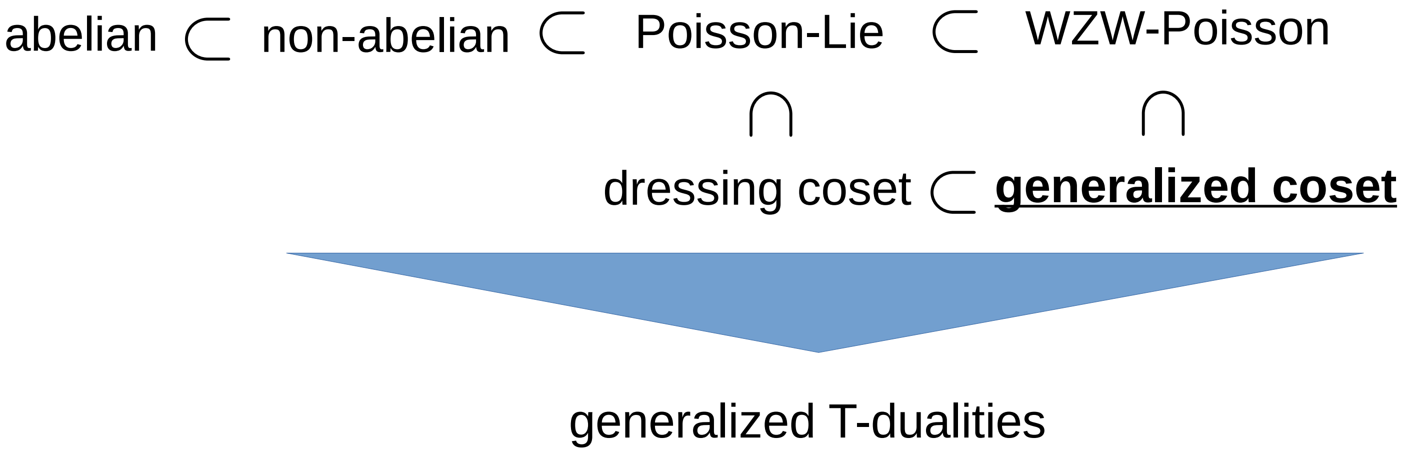


abelian $\subset$ non-abelian $\subset$ Poisson-Lie $\subset$ WZW-Poisson

$\cap$ $\cap$

dressing coset $\subset$ **generalized coset**

generalized T-dualities

String dualities

- Historical development of T-dualites

abelian ⊂ non-abelian ⊂ Poisson-Lie ⊂ WZW-Poisson

∩ ∩

dressing coset ⊂ **generalized coset**

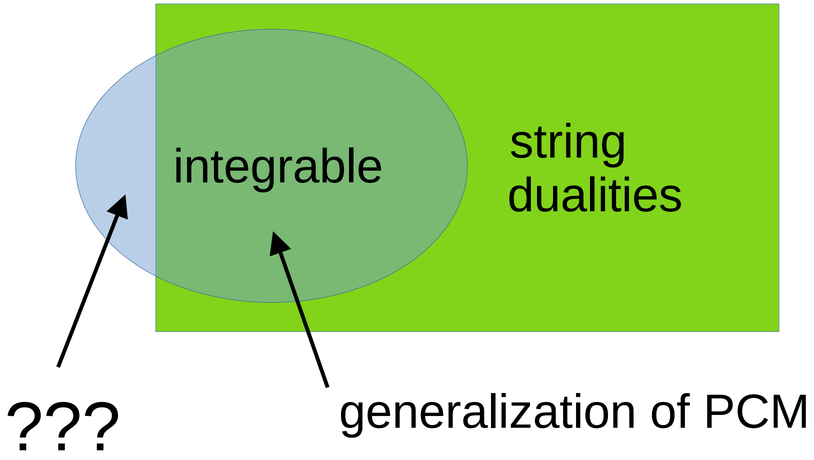
generalized T-dualities

Applications:

- solution generating techniques
- consistent truncations
- integrable strings

Examples

- principal chiral model
- Yang-Baxter deformations
- η- and λ-deformations
- affine Gaudin models [Lacroix, Vicedo 21; Liniado, Vicedo 23]



integrable

string dualities

???

generalization of PCM

Examples

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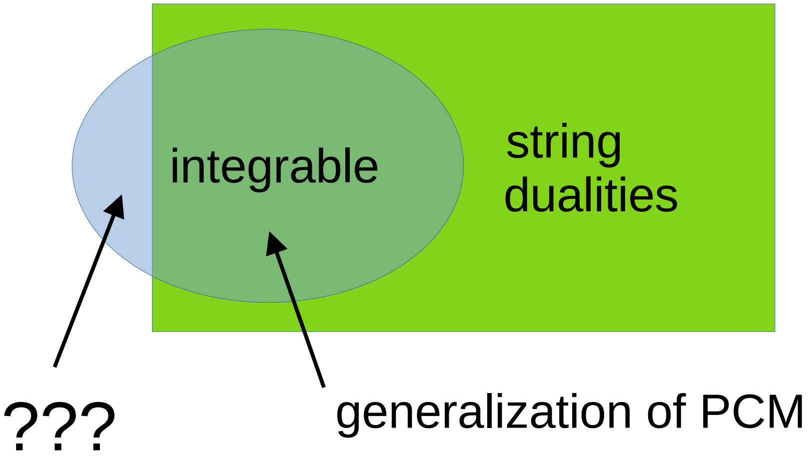


<u>Benefits:</u>

- structures are the same as in the Hamiltonian analysis
- very useful for computation of RG flows
- relation to consistent truncations and gauged supergravity
- hints towards non-commutative geometry

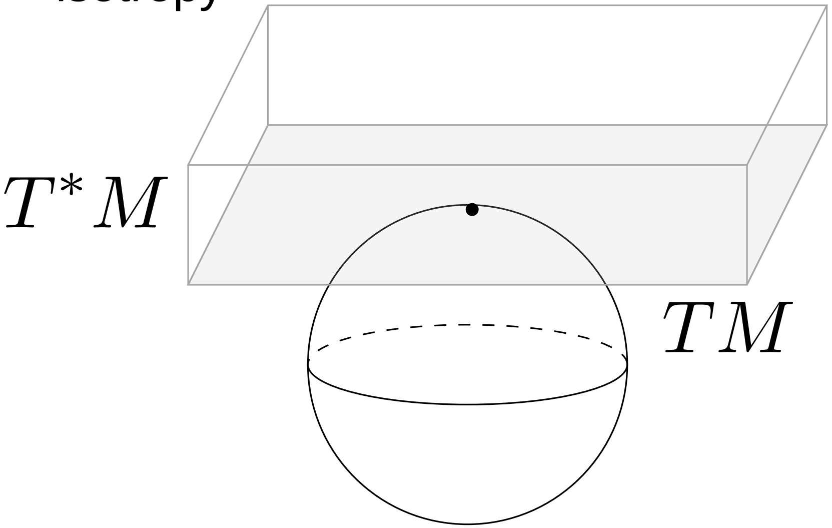

Felix Klein

Underlying structure

Homogenious space: A space that looks everywhere the same as you move through it.

isometry → G/F ← isotropy

but in Generalized Geometry



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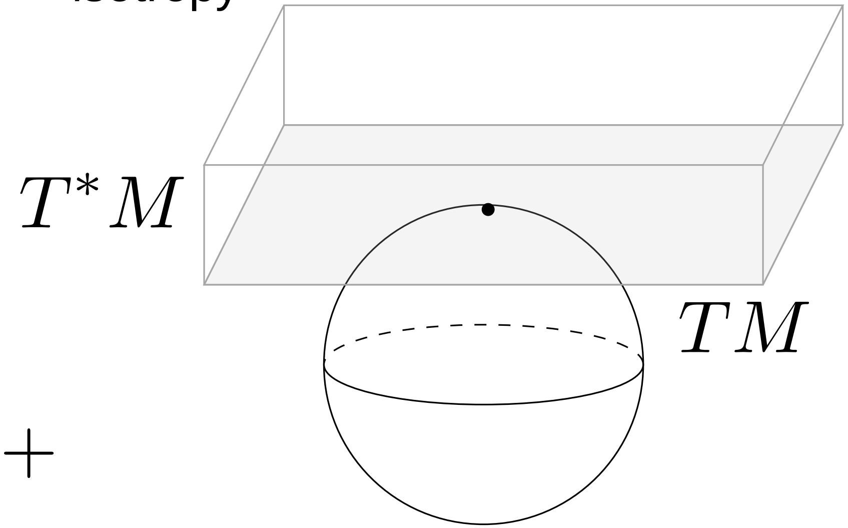
<u>generalized Lie derivative:</u>

$$\mathbb{L}_U V^M = U^N \partial_N V^M - \alpha P_{(\mathrm{adj})}{}^M{}_{N,}{}^P{}_Q \partial_P U^Q V^N + \beta \partial_N U^N V^M$$



+ section condition for closure

Generalized group manifold

$\cancel{F}$

$$\mathbb{L}_{E_A} E_B{}^M = F_{AB}{}^C E_C{}^M$$

gen. frame

structure constants

Generalized group manifold $\xcancel{F}$

$$\mathbb{L}_{E_A} E_B{}^M = F_{AB}{}^C E_C{}^M$$

gen. frame

structure constants

O(D,D) recipe to contruct gen. frame:

1) Lie algebra with generators T_A

$$[T_A, T_B] = F_{AB}{}^C T_C$$

2) with ad-invariant, O(D,D)-pairing

$$\langle T_A, T_B \rangle = \eta_{AB}$$

3) maximally isotropic subgroup

Generalized group manifold $\cancel{F}$

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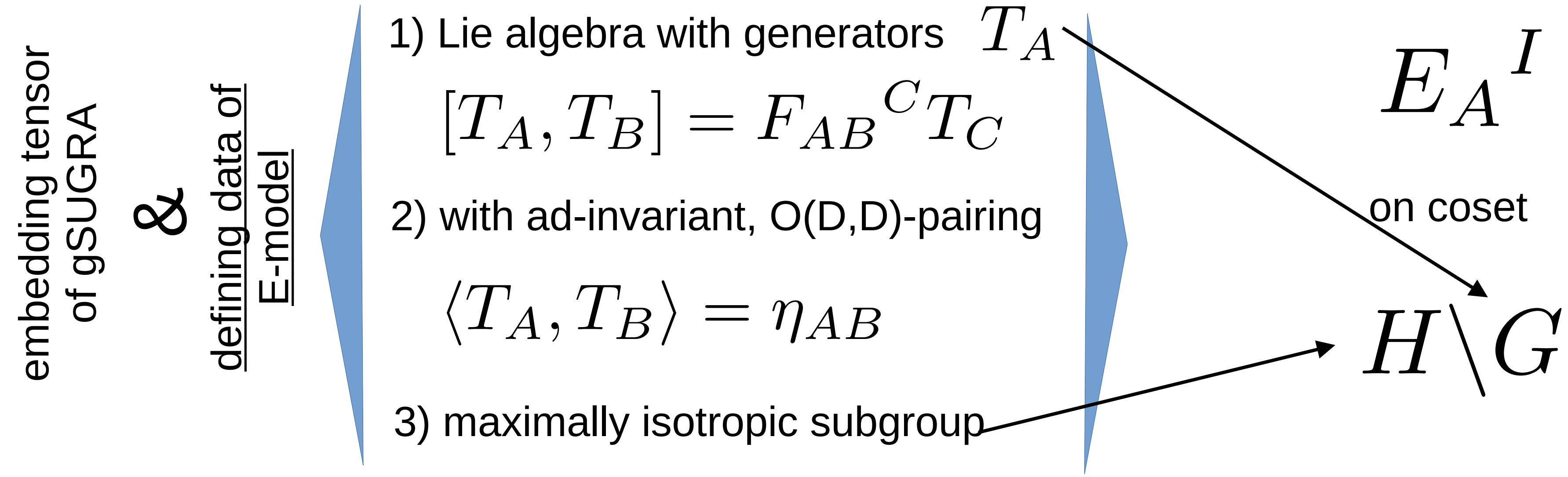

embedding tensor of gSUGRA

&

defining data of E-model

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3) maximally isotropic subgroup

$$E_A{}^I$$

on coset

$$H \backslash G$$

Homogeneous space

<u>Theorem:</u>

[Ambrose, Singer 1958]

Let (M,g) be a connected and simply-connected complete Riemannian manifold. Then, the following statements are equivalent:

1) The manifold M is Riemannian homogenous

2) M admits a linear connection ∇ satisfying

$$\nabla R = 0\,, \quad \nabla S = 0\,, \quad \nabla g = 0$$

Riemann tensor

$$S = \nabla^{\mathrm{LC}} - \nabla$$

metric

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➡ frame and <u>connection</u> required

$$\nabla_i {e_a}^j = \partial_i {e_a}^j - {\omega_{ia}}^b {e_b}^j + {\Gamma_{ik}}^j {e_a}^k = 0$$

[Demulder, FH, Piccinini, Thompson 19; Butter, FH, Pope, Zhang 22]

Generalized coset

$$\nabla_I E_A{}^J = \partial_I E_A{}^J - \Omega_{IA}{}^B E_B{}^J + \Gamma_{IK}{}^J E_A{}^K = 0$$

O(D,D) recipe to contruct gen. frame and spin connection:

1) gen. frame on H\G = mega-space

2) another isotropic subgroup F

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$E_A{}^I$ and $\Omega_{IA}{}^B$

on double coset

$$H\backslash G/F$$

gen. structure group

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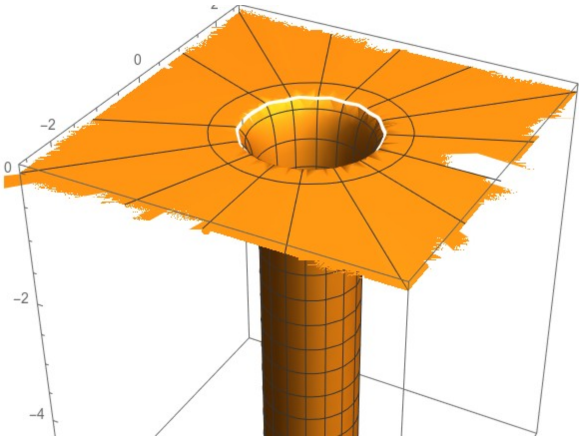

New

- *higher derivative* connections from *tensor hierarchy*
- *singularities* @ fixed points of F action

Why is this structure useful?

worldsheet

current algebra

$$\{J_A(\sigma), J_B(\sigma')\} = F_{AB}{}^{C} J_C(\sigma)\delta(\sigma-\sigma') + \eta_{AB}\delta'(\sigma-\sigma')$$

⇕

generalized Lie derivative

Hamiltonian

$$H = \int \mathrm{d}\sigma\, J_A \mathcal{H}^{AB} J_B$$

generalized metric = E-operator

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$$\Updownarrow$$

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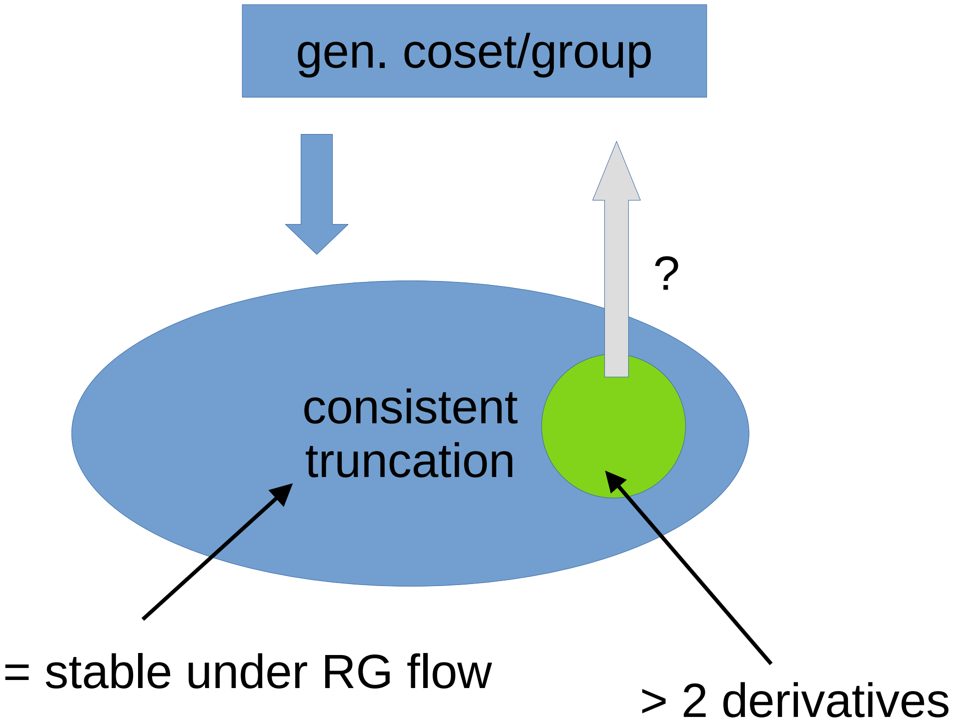
<u>target space</u>



gen. coset/group

?

consistent truncation

= stable under RG flow

> 2 derivatives

(Super)generalized geometries

	$\mathrm{O}(D,D)$	$\mathrm{E}_{n(n)}\,,\ n \leq 8$	$\mathrm{OSp}(D,D\vert 2s)$
theory	bosonic	M & type IIB	type II D=10, s=32
gen. group	✓	✓	✓
gen. coset	✓	🚧	✓
full spacetime	✓	✗	✓
fermions	✗	✗	✓
gen. duality	bosonic-T	U	super-T

Supergeneralized Lie deriviate

$$\mathbb{L}_\xi V^{\mathcal{M}} = \xi^{\mathcal{N}} \partial_{\mathcal{N}} V^{\mathcal{M}} - V^{\mathcal{N}} \Big(\partial_{\mathcal{N}} \xi^{\mathcal{M}} - \partial^{\mathcal{M}} \xi_{\mathcal{N}} (-)^{mn} \Big)$$

leaves the OSp(D,D|2s) metric

$$\eta_{\mathcal{M}\mathcal{N}} = \begin{pmatrix} 0 & \delta_M{}^N \\ \delta^M{}_N (-)^{mn} & 0 \end{pmatrix}$$

invariant

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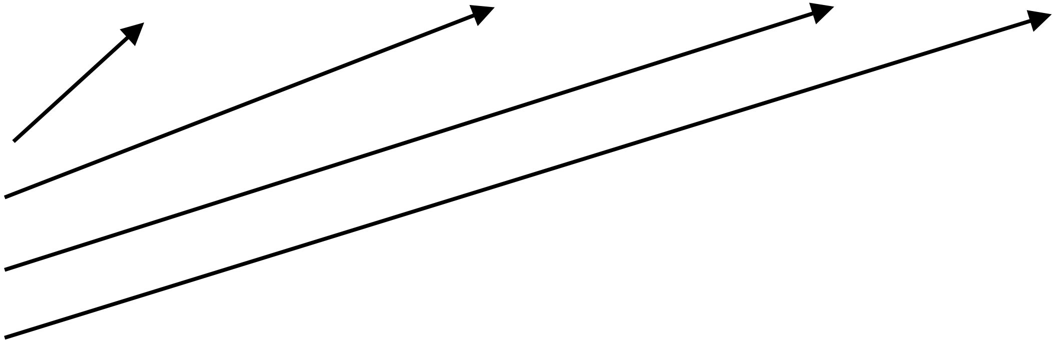
closes under the section condition $\eta^{\mathcal{M}\mathcal{N}} \partial_{\mathcal{N}} \otimes \partial_{\mathcal{M}} = 0$ solved by

$\partial_{\mathcal{M}} = \begin{pmatrix} \partial_M & 0 \end{pmatrix}$ with $\partial_M = \dfrac{\partial}{\partial z^M}$ and $\mathrm{z}^M = \begin{pmatrix} x^m & \theta^\mu \end{pmatrix}$

Supergeneralized frame

$$\mathrm{OSp}(D,D|2s) \ni \mathcal{E}_{\mathcal{M}}{}^{\mathcal{A}} = (\mathcal{E}_B)_{\mathcal{M}}{}^{\mathcal{N}} \times (\mathcal{E}_E)_{\mathcal{N}}{}^{\mathcal{C}} \times (\mathcal{E}_S)_{\mathcal{C}}{}^{\mathcal{B}} \Lambda_{\mathcal{B}}{}^{\mathcal{A}}$$



contributions:

1) super B-field
2) super frame
3) R/R field strengths & dilatini
4) super double Lorentz $\mathrm{H}_L \times \mathrm{H}_R$

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New

$$\mathrm{V}_{\mathcal{A}} = \left(V_{\mathrm{a}} \quad V_{\alpha} \quad V^{\alpha} \quad \middle| \quad V_{\bar{\mathrm{a}}} \quad V_{\bar{\alpha}} \quad V^{\bar{\alpha}} \right)$$

left right

generators of H_L , $\lambda_{\mathcal{B}\mathcal{A}} = -\lambda_{\mathcal{B}\mathcal{A}}(-)^{ab}$ are further constraint by

$$\lambda_{\beta\alpha} = \lambda_{\mathrm{b}\alpha} = 0\,, \quad \lambda_{\beta}{}^{\alpha} = \frac{1}{4}\lambda_{\mathrm{ba}}(\gamma^{\mathrm{ba}})_{\beta}{}^{\alpha}\,, \quad (\gamma^{\mathrm{b}})_{\beta\alpha}\lambda_{\mathrm{b}}{}^{\alpha} = 0$$

Osp(D,D|2s) transformations

relate all generalized frames $\mathcal{E}'_{\mathcal{A}}{}^{\mathcal{M}} = \mathcal{E}_{\mathcal{A}}{}^{\mathcal{N}} \mathcal{U}_{\mathcal{N}}{}^{\mathcal{M}}$ ← linear (simple)

After fixing $\Lambda_{\mathcal{A}}{}^{\mathcal{B}}$ transformations of super frame, B-field, R/R field strengths and dilatini ← non-linear (complicated)

↖ central for higher loop RG flows in s=0

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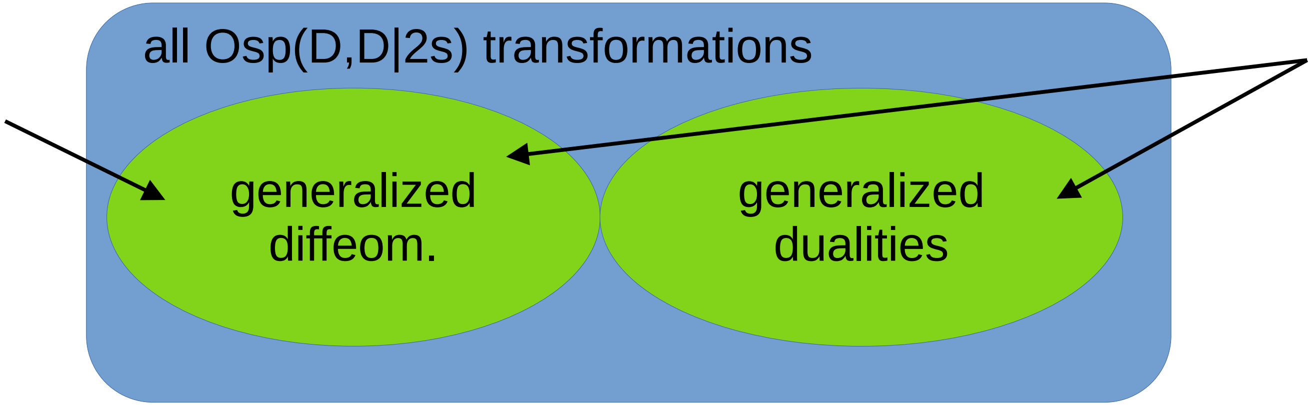

all Osp(D,D|2s) transformations

generated by generalized Lie derivative

generalized diffeom.

generalized dualities

preserve equations of motion and (one-loop) β-functions

Supergeneralized group manifold

on the super coset $\mathrm{H}\backslash\mathrm{G} \ni m$

1) one-forms $\mathrm{d}mm^{-1} = V^A T_A + A_A T^A (-)^a$

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$$\mathrm{d}B_{\mathrm{WZW}} = -\frac{1}{12}\left\langle \mathrm{d}mm^{-1}, [\mathrm{d}mm^{-1}, \mathrm{d}mm^{-1}]\right\rangle$$

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$$\mathcal{E}_{\mathcal{A}}{}^{\mathcal{M}} = \mathrm{M}_{\mathcal{A}}{}^{\mathcal{B}} \begin{pmatrix} V_B{}^M & -V_B{}^N B_{NM} (-)^m \\ 0 & V^B{}_M (-)^m \end{pmatrix}$$

Supergeneralized coset

split G’s generators into $\mathrm{T}_{\widehat{\mathcal{A}}} = \begin{pmatrix} T_{\underline{A}} & T_{\mathcal{A}} & T^{\underline{A}} \end{pmatrix}$ with

subalgebra generators and their dual

$$\eta_{\widehat{\mathcal{A}}\widehat{\mathcal{B}}} = \begin{pmatrix} 0 & 0 & \delta_{\underline{A}}{}^{\underline{B}} \\ 0 & \eta_{\mathcal{A}\mathcal{B}} & 0 \\ \delta^{\underline{A}}{}_{\underline{B}}(-)^b & 0 & 0 \end{pmatrix}$$

and double coset elements

$$\mathrm{n} \in H\backslash G/F\,, \quad m = nf\,, \quad f \in F$$

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frame is of the form:

spin connection

$$\widehat{\mathcal{E}}_{\widehat{\mathcal{A}}}{}^{\widehat{\mathcal{M}}} = (\mathrm{Adj}\, f)_{\widehat{\mathcal{A}}}{}^{\widehat{\mathcal{B}}} \begin{pmatrix} \delta_{\underline{B}}{}^{\underline{C}} & 0 & 0 \\ -\Omega_{\mathcal{B}}{}^{\underline{C}} & \mathcal{E}_{\mathcal{B}}{}^{\mathcal{N}} & 0 \\ \rho^{\underline{BC}} - \frac{1}{2}\Omega^{\underline{B}}\Omega^{\underline{C}} & \Omega^{\underline{B}\mathcal{N}} & \delta^{\underline{B}}{}_{\underline{C}} \end{pmatrix} \begin{pmatrix} \widetilde{v}_{\underline{C}}{}^{\underline{I}} & 0 & 0 \\ 0 & \delta_{\mathcal{N}}{}^{\mathcal{M}} & 0 \\ 0 & 0 & \widetilde{v}^{\underline{C}}{}_{\underline{I}}(-)^i \end{pmatrix}$$

2-derivative connection

$$\widetilde{v}^{\underline{A}} T_{\underline{A}} = \mathrm{d}f\, f^{-1}$$

Torsion constraints

O(D,D):

- $\mathrm{F}_{AB}{}^{C}$ only constraint by Bianchi identity
- generalized metric fixes double Lorentz group

OSp(D,D|2s):

- **no** generalized metric, $\mathrm{H}_L \times \mathrm{H}_R$?
- SUSY algebra only closes on-shell

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torsion constraints

κ -symmetry/
gen. SUGRA

$$F_{\alpha\beta\gamma} = F_{\alpha\beta\bar{\gamma}} = F_{\alpha\bar{\beta}\bar{\gamma}} = F_{\bar{\alpha}\bar{\beta}\bar{\gamma}} = 0$$

$$F_{\alpha\beta\mathrm{c}} = -i\sqrt{2}\,(\gamma_{\mathrm{c}})_{\alpha\beta}\ , \qquad F_{\bar{\alpha}\bar{\beta}\mathrm{c}} = -i\sqrt{2}\,(\bar{\gamma}_{\mathrm{c}})_{\bar{\alpha}\bar{\beta}}\ ,$$

$$F_{\alpha\bar{\beta}\mathrm{c}} = F_{\alpha\bar{\beta}\mathrm{c}} = F_{\alpha\beta\mathrm{c}} = F_{\bar{\alpha}\bar{\beta}\mathrm{c}} = 0$$

[Delduc, Magro, Vicedo 14; Hollowood, Miramontes, Schmidtt 14]

η- and λ-deformation

$$\mathrm{AdS}_5 \times S^5$$

$$\mathcal{G} = \mathrm{PSU}(2,2|4)$$

$$F = \mathrm{SO}(4,1) \times \mathrm{SO}(5)$$

- worldsheet theory integrable
- target space max. SUSY

integrability
without
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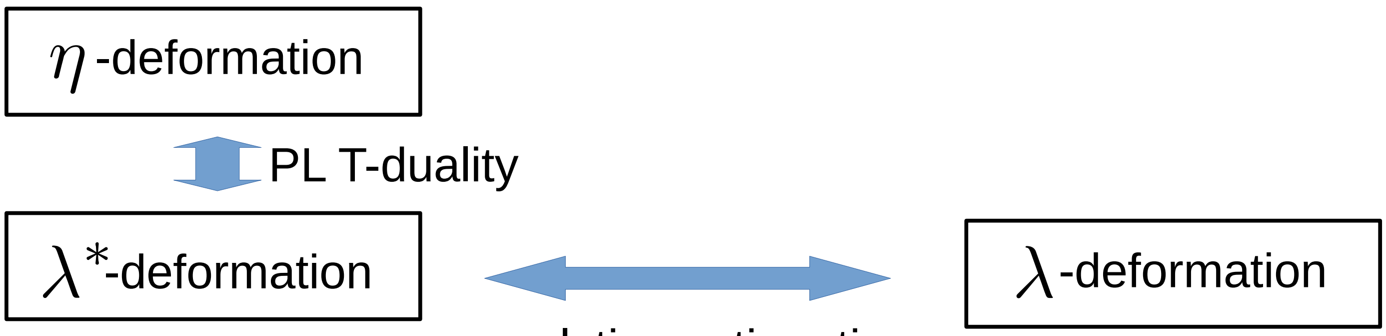


$$H\backslash\mathcal{G}^{\mathbb{C}}/F$$

$$H\backslash\mathcal{G} \times \mathcal{G}/F$$

$$\eta = \frac{1-\lambda}{1+\lambda}$$

η-deformation

PL T-duality

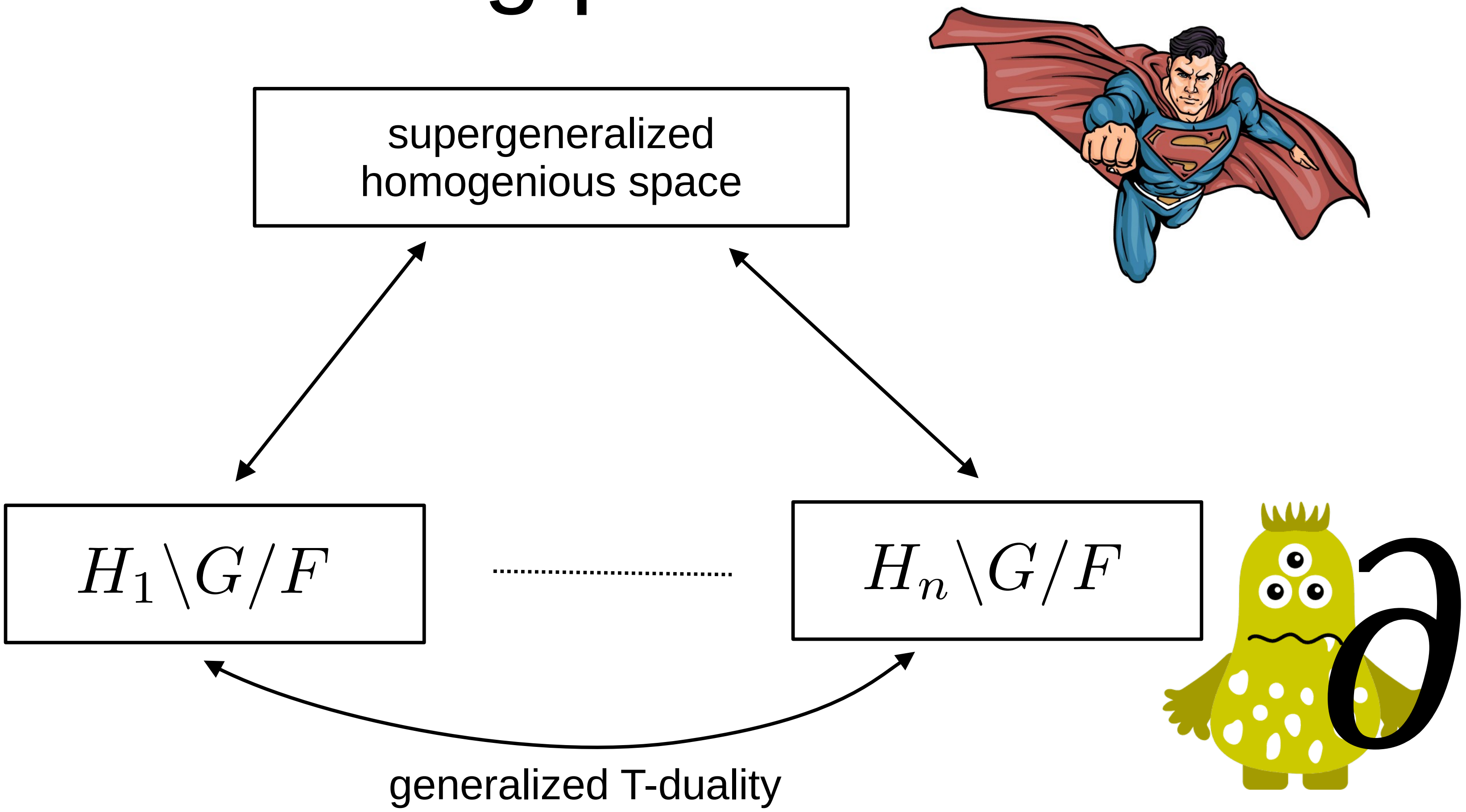
λ^*-deformation

analytic continuation

λ-deformation

only one parameter, η, allowed by torsion constraints

The big picture



supergeneralized homogenious space

Green-Schwarz superstring

$H_1\backslash G/F$

$H_n\backslash G/F$

generalized T-duality

Summary and outlook

- unified all know T-dualities into one framework
- understand both,

  worldsheet: Green-Schwarz string

  target space: generalized supergravity

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- application:
  - lift bosonic integrable strings to superstrings
  - new integrable models from dualities, especially fermionic T-dualities
  - simplify computation of: β-functions, S-matrix, ….

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  target space: generalized supergravity
- application:
  - lift bosonic integrable strings to superstrings
  - new integrable models from dualities, especially fermionic T-dualities
  - simplify computation of: β-functions, S-matrix, ….
- new questions:
  - (How) do branes resolve singularities of generalized cosets?
  - Do loop (=higher derivative) corrections further support

integrable strings ⟷ generalized dualities ⟷ consistent truncations ?