last time AdS spacetime in global coordinates

tutorial ⇝ metric

$$\boxed{ds^2 = L^2\left(-\cosh^2\rho\, d\tau + d\rho^2 + \sinh^2\rho\, d\Omega^2_{d-1}\right)}$$

coordinate transformation $\rho \to \theta$: $\tan\theta = \sinh\rho$

$$\rightsquigarrow ds^2 = \cancel{\frac{L^2}{\cos^2\theta}}\Big(\underbrace{-d\tau^2}_{S^1} + \underbrace{d\theta^2 + \sin^2\theta\, d\Omega^2_{d-1}}_{\frac{1}{2}S^d}\Big)$$

because $0 \le \theta < \frac{\pi}{2}$, $0 \le \tau < 2\pi$

causal structure remains unchanged

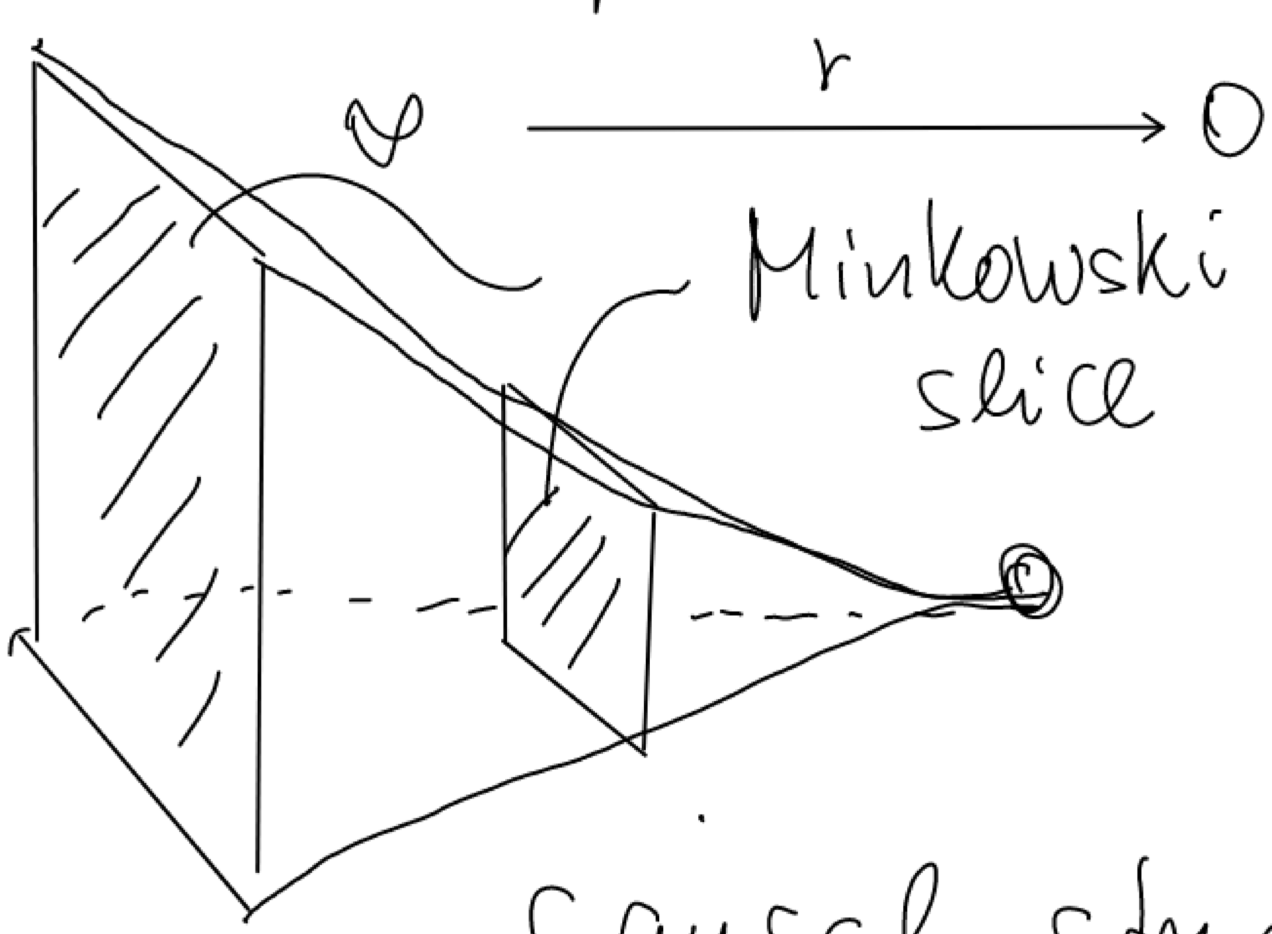
⇝ conformal equivalent to $\boxed{d\tilde{s}^2 = -d\tau^2 + d\theta^2 + \sin^2\theta\, d\Omega^2_{d-1}}$

↯ closed time like curves ⇝ universal cover with $\tau \in \mathbb{R}$

∂AdS_{d+1} for $\theta = \frac{\pi}{2}$

alternatively Poincaré coordinates with metric

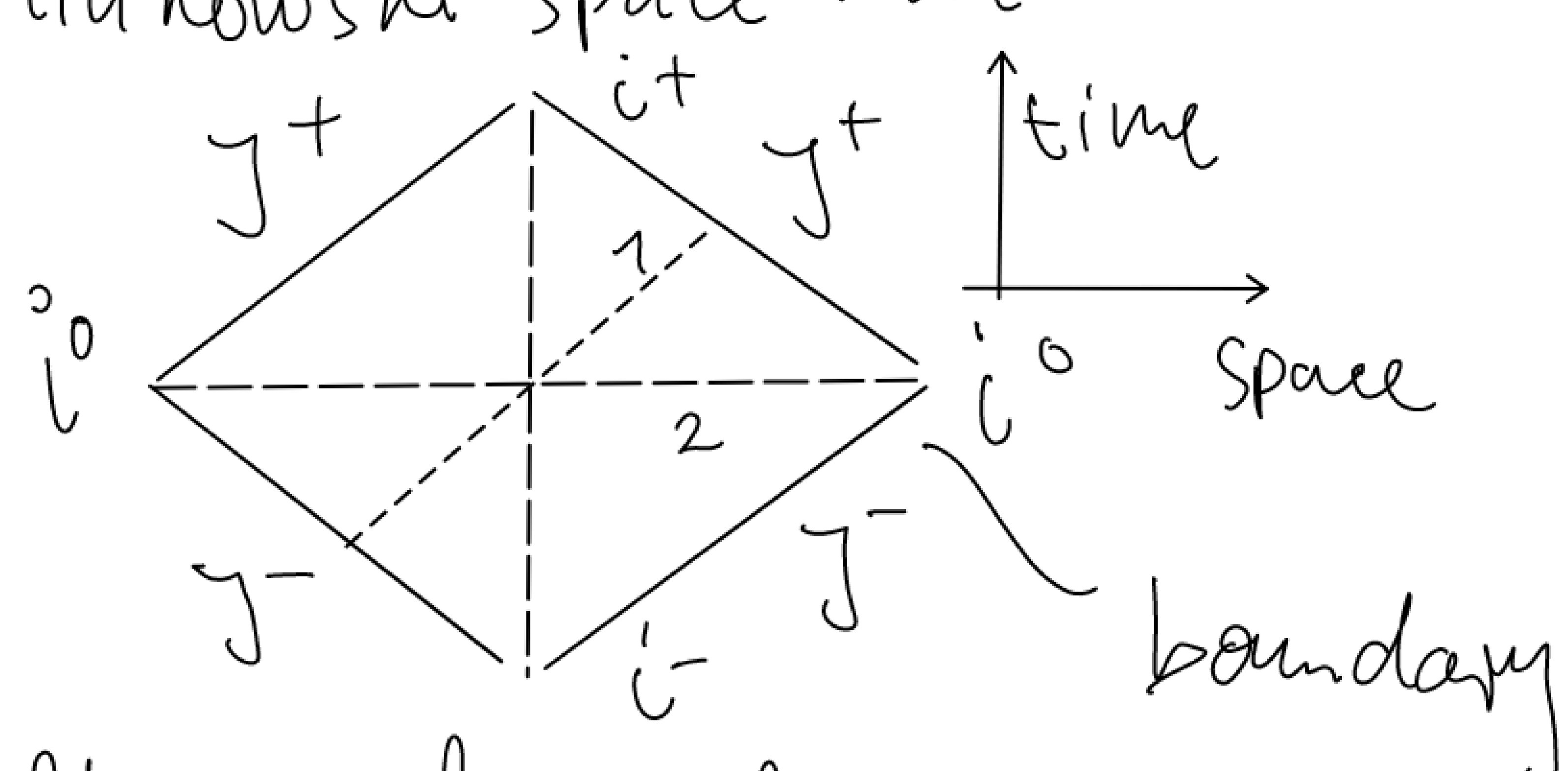
$$ds^2 = \frac{L^2}{r^2}dr^2 + \frac{r^2}{L^2}\big(\underbrace{\eta_{\mu\nu}dx^\mu dx^\nu}_{\mathbb{R}^{d-1,1}}\big)$$



~ conformal boundary at $r = \infty$

Causal structure is captured by Penrose diagram

Minkowski space time



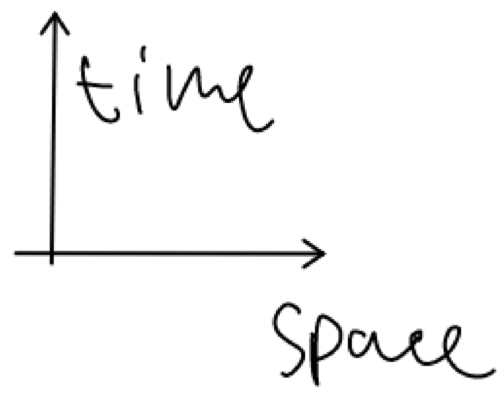
after conformal map

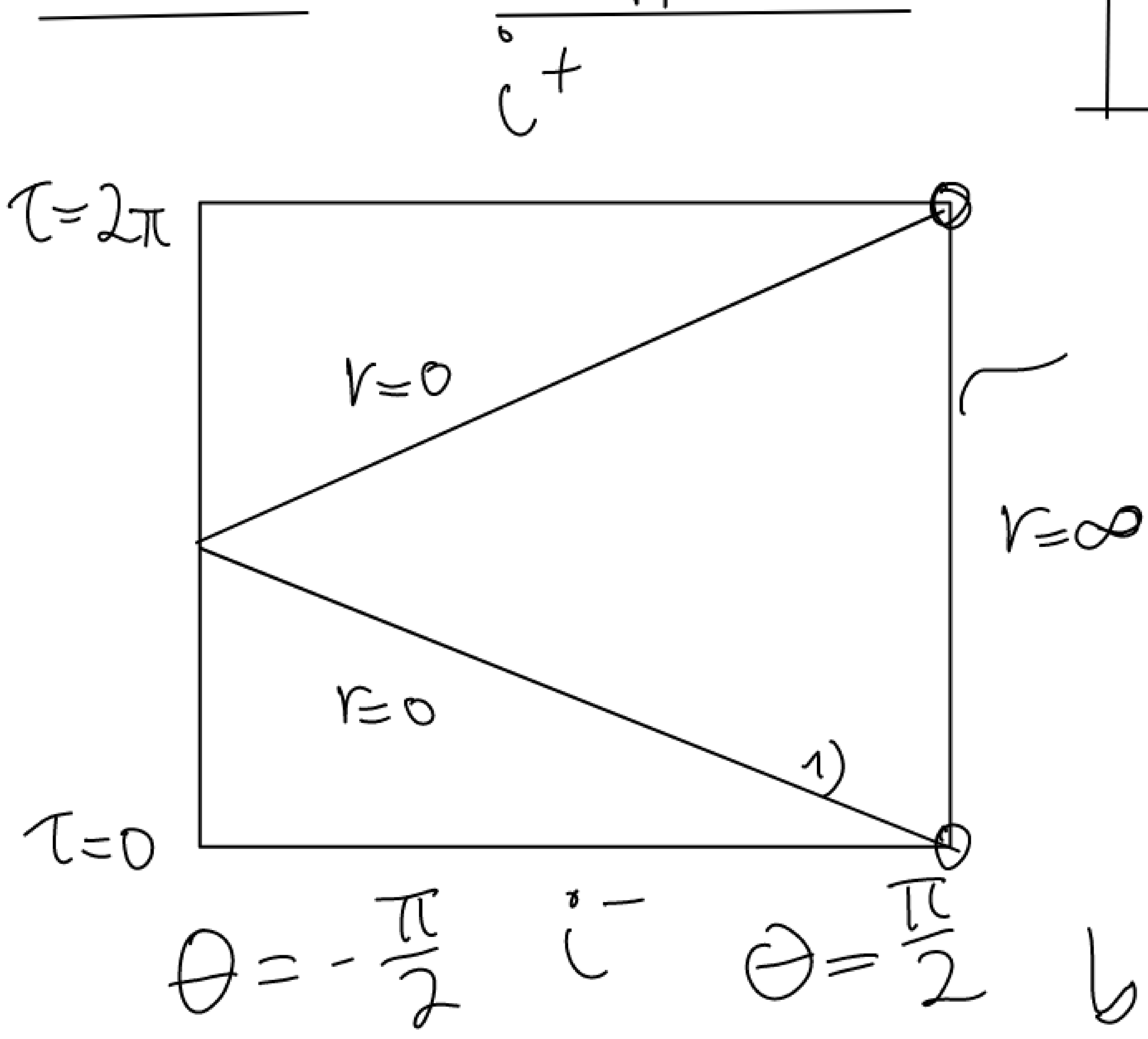
geodesics:

1) light rays $\mathcal{J}^- \longrightarrow \mathcal{J}^+$

2) matter (time like) $i^- \longrightarrow i^+$

AdS is different





boundary ∂

Geodesics again:

$$\boxed{\begin{array}{ll} \text{1) light rays} & \partial \to \partial \\ \text{2) time-like} & i^- \to i^+ \end{array}}$$

bulk can be probed from the boundary
$\rightsquigarrow$ Central for the AdS/CFT corresp.

4. Quantization

2 main approches $\to$ canonical (Hamiltonian); Path integral (Lagrangian)

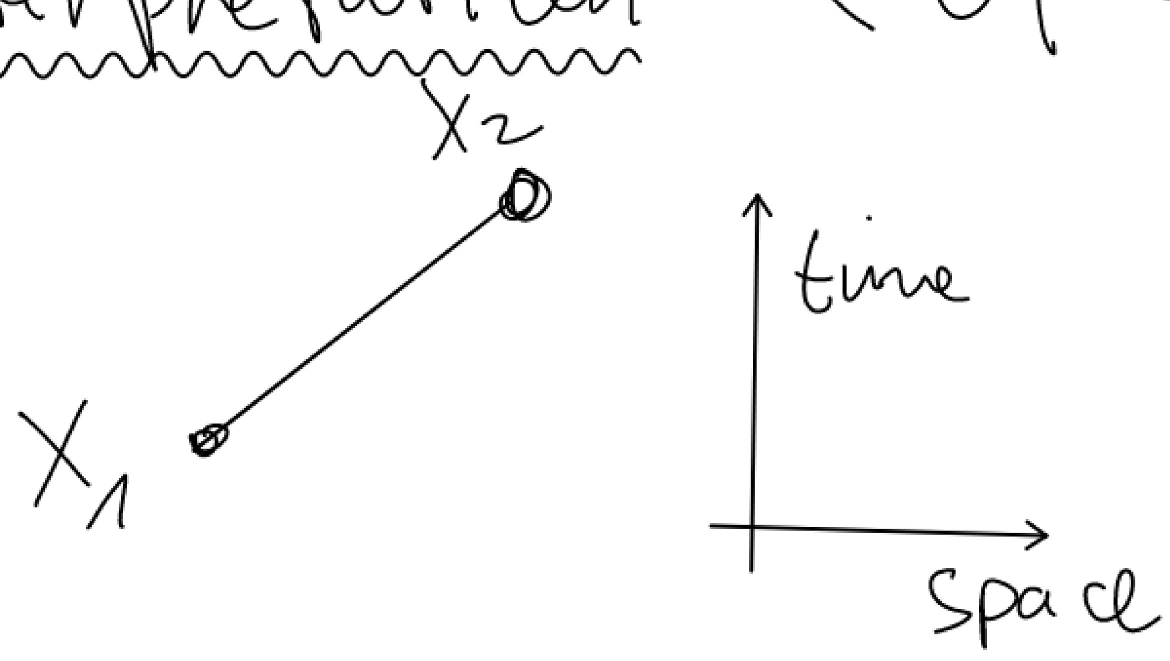
4.1. The path integral and the partition function

In QFT we want to compute correlation functions

$$\langle 0 | \mathcal{T} \hat{\phi}(x_1)\, \hat{\phi}(x_2) \ldots \hat{\phi}(x_n) | 0 \rangle = \langle \phi(x_1) \ldots \phi(x_n) \rangle$$

Green's function $\Rightarrow G^{(n)}(x_1, \ldots, x_n)$

Interpretation $\langle 0 | \mathcal{T} \phi(x_1)\, \phi(x_2) | 0 \rangle$



create particle at x_1, probability to detect it at x_2

$x_1^0 < x_2^0$ because of causality

$$\boxed{\langle \phi(x_1) \ldots \phi(x_n) \rangle = N \cdot \int \mathcal{D}\phi\, \phi(x_1) \ldots \phi(x_n) \exp\left[i \int d^d x\, \mathcal{L}(\phi, \partial\phi) \right]}$$

normalisation; integrate over all different field configs

such that $\langle 0 | 0 \rangle = 1 = N \cdot \int \mathcal{D}\phi \exp\left[i \int d^d x\, \mathcal{L}(\phi, \partial\phi) \right]$

How to compute? 1. Hard way for free theory:
discretize ⇝ finite number of Gaussian integrals
→ solve them, take the cont' limit

2. use generating function

$$\boxed{\begin{aligned} &Z_0[J] = \left\langle \exp\left[i \int d^d x\, J(x)\, \phi(x)\right] \right\rangle \\ &\text{with } \langle \phi(x_1) \dots \phi(x_n) \rangle = (-i)^n \frac{\delta^n Z_0[J]}{\delta J(x_1) \dots \delta J(x_n)} \bigg|_{J=0} \end{aligned}}$$

For the free Scalar:

$$Z_0[J] = N \int \mathcal{D}\phi \exp\Big[i \int d^d x \Big(-\tfrac{1}{2} \phi \underbrace{(-\Box + m^2 - i\overset{\varepsilon \to 0,\ \text{for convergence}}{\varepsilon})}_{\text{after integration by parts}} \phi + J \phi \Big) \Big]$$

Here we need 1. :-)

look at $\int_{-\infty}^{\infty} dy \exp\left(-\tfrac{1}{2} a y^2 + by\right) = \sqrt{\frac{2\pi}{a}}\, e^{\frac{b^2}{2a}}$

with normalization $1 = N \cdot \int_{-\infty}^{\infty} \exp\left(-\tfrac{1}{2} a y^2\right) = N \cdot \sqrt{\frac{2\pi}{a}}$

we have
$$N \cdot \int_{-\infty}^{\infty} dy \exp\left(-\tfrac{1}{2} a y^2 + by\right) = e^{\frac{b^2}{2a}}$$

$$\rightsquigarrow \boxed{Z_0[J] = \exp\Big[\frac{i}{2} \int d^d x\, d^d y\, \overbrace{J(x)\, \underbrace{\Delta_F(x-y)}_{\hat{=}\, \frac{1}{a}}\, J(y)}^{\hat{=}\, b^2} \Big]}$$

with the Feynman Propagator

$$\Delta_F(x-y) = \int \frac{d^d k}{(2\pi)^d} \frac{e^{ik(x-y)}}{k^2 + m^2 - i\varepsilon} \sim \mathcal{F}(-\Box + m^2 - i\varepsilon)$$

Satisfying $(-\Box + m^2)\, \Delta_F(x-y) = \delta^d(x-y)$

which is the Green's function for the field eqs.

4.2. Interacting theories

now $\mathcal{L} = \mathcal{L}_{\text{free}} + \mathcal{L}_{\text{int}}$ quadratic

and $\quad Z[J] = N \int \mathcal{D}\phi \, \exp\left[i \int d^d x \left(\mathcal{L}_{\text{free}} + \mathcal{L}_{\text{int}} + J\phi \right) \right]$

↯ we cannot compute the path integral directly, but if $\mathcal{L}_{\text{int}} \ll 1$ (a perturbation) we can expand

$$Z[J] = N \exp\left[i \int d^d x \, \mathcal{L}_{\text{int}}\left(-i \frac{\delta}{\delta J(x)} \right) \right] Z_0[J]$$

$\rightsquigarrow$ Feynman diagrams (see EX 4)