Gauge / Gravity Duality

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office: 448

lectures: Fri. 16:15 – 18:00 } 442
tutorials: Fri. 18:15 – 20:00 }

exercises & handwritten notes at
https://www.fhassler.de/teaching/ws_24/ads-cft
- online ~ 1 week before tutorial
- assigned at Sunday 21:00 –"– ⇝ email

WILL BE GRADED

exam
- in written at end of semster
- at least 50% of points of assigned problems to qualify

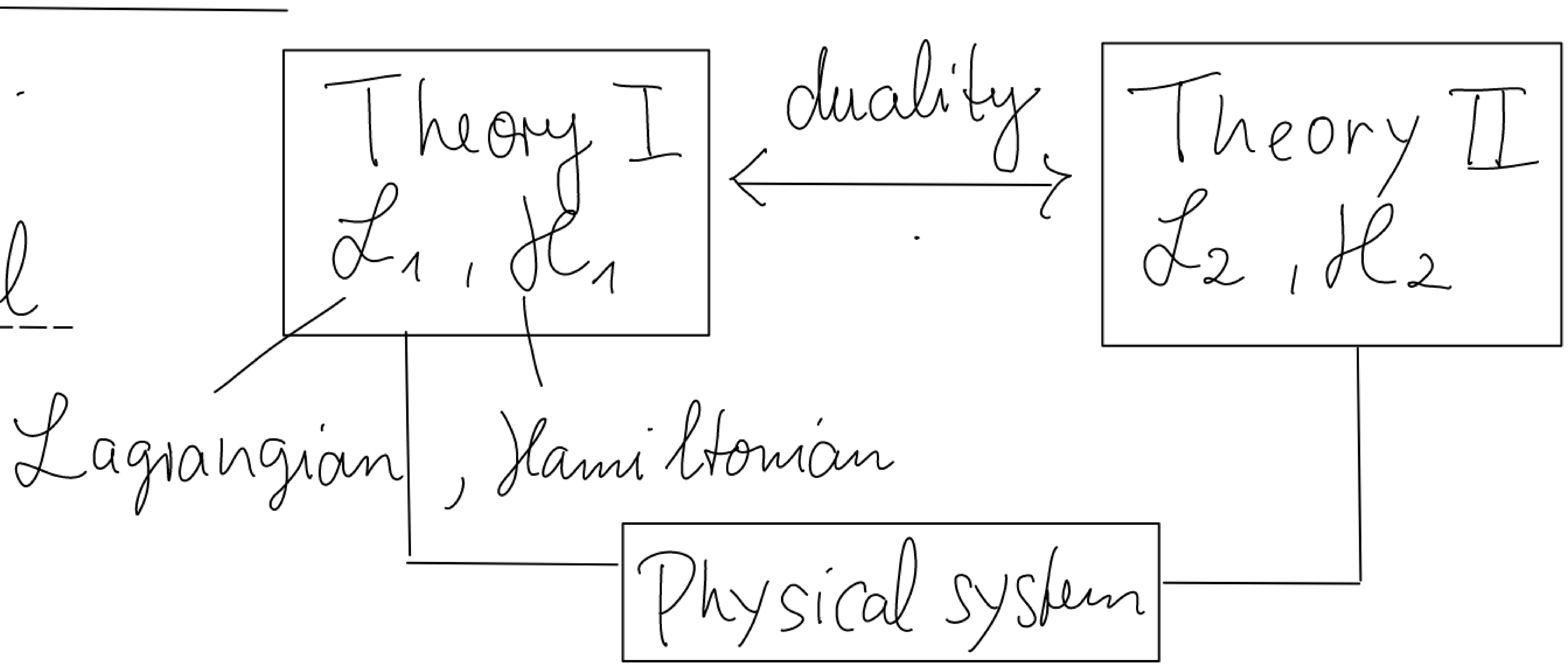
Problems? Contact me or Luca Scala
(luca.scala@uwr.edu.pl)

office hours: Tue. 15:00 – 17:00

1. Introduction

Duality:

classical



$\mathcal{H}_1$ and $\mathcal{H}_2$ are related by a canonical transformation $\rightsquigarrow$ equation of motion are preserved

Phase space $(\vec{q}_1, \vec{p}_1) \longleftrightarrow (\vec{q}_2, \vec{p}_2)$

degrees of freedom (dof)

duality

Quantum: $\mathcal{L} \longrightarrow Z[J]$ ← Partition function which generates all correlation functions

now the duality implies $\boxed{Z_1 \sim Z_2}$

dictionary for observables

Why do we care?

$\boxed{\text{Theory } 1} \longleftrightarrow \boxed{\text{Theory } 2}$

- cool (strongly coupled)
- very hard

- easier (weakly coupled)

results $\xleftarrow{\text{dictionary}}$ Compute observables

In this lecture:

$\boxed{\text{Theory } 1} \longleftrightarrow \boxed{\text{Theory } 2}$

- gauge theory without gravity

- General relativity with gravity

$\Downarrow$ more precisely $\Downarrow$

4d $\mathcal{N}=4$ Super-Yang-Mills
= 4d maximal SYM

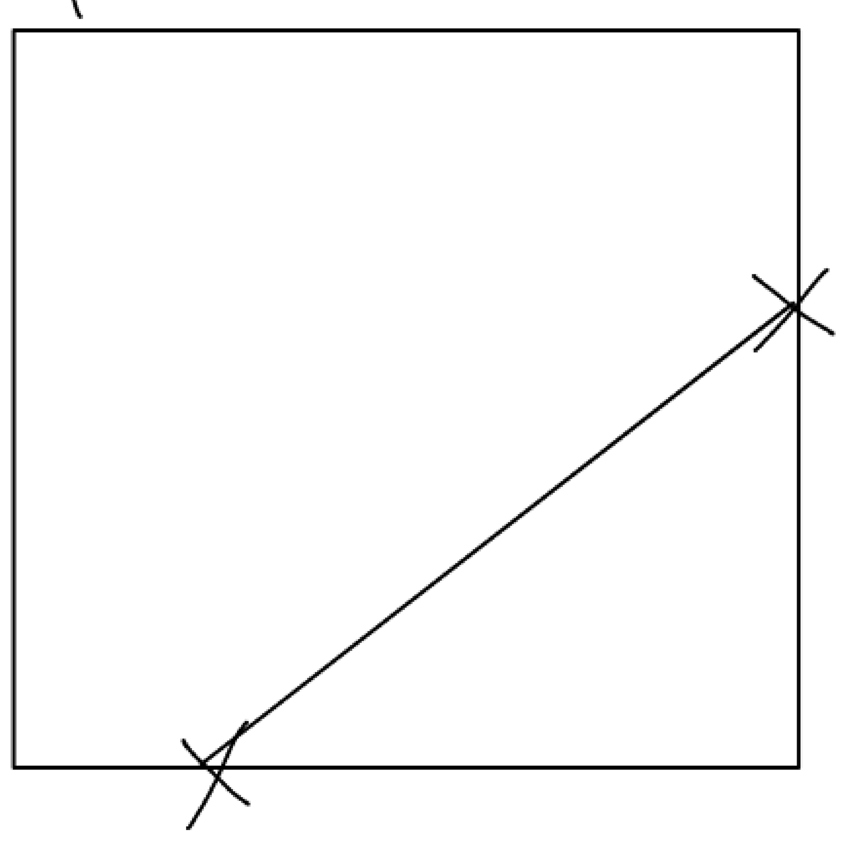
5d $\mathcal{N}=8$ gauged Super-Gravity
= 5d max gSUGRA

symmetry relating bosons and fermions

on the boundary of bulk Anti-de Sitter space

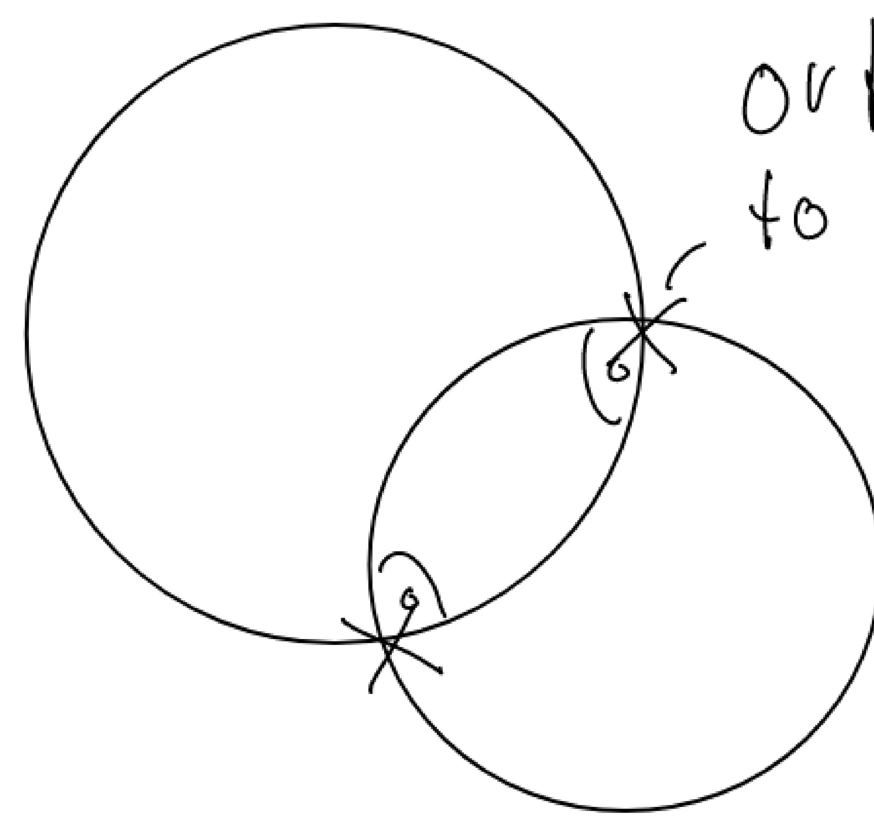
space with const. neg. curvature

flat euclidian



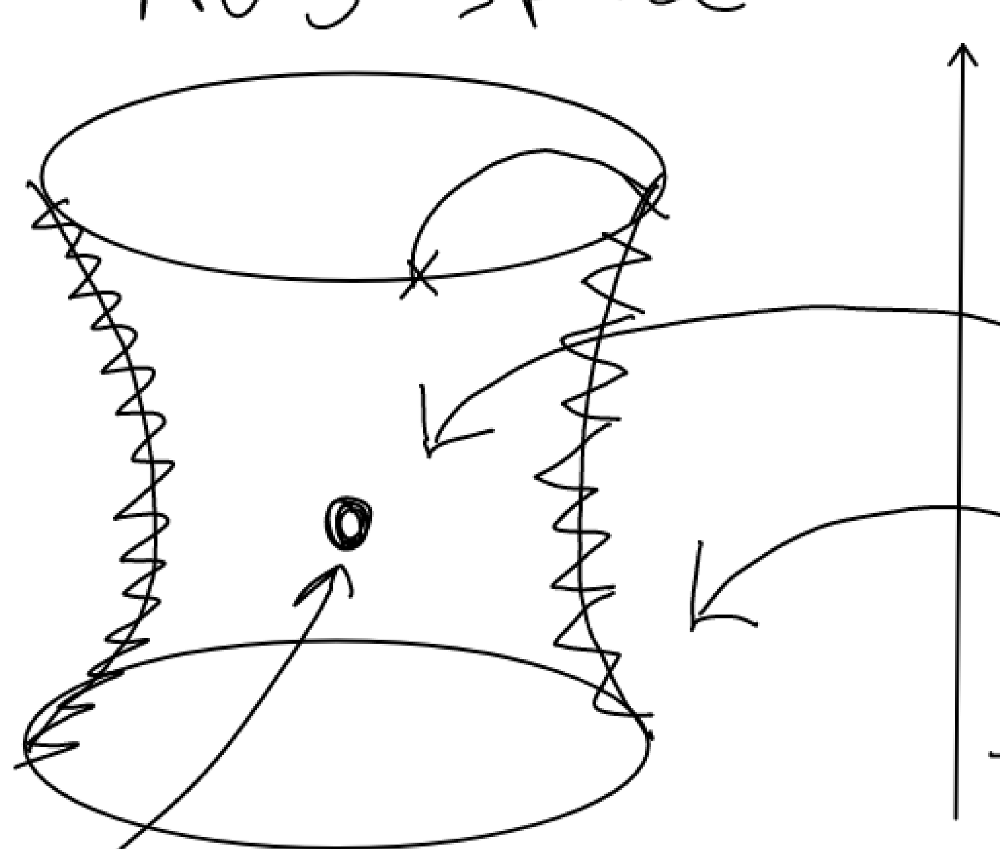
geodesic = shortest connection between two points

hyperbolic space



orthogonal to boundary

AdS-space



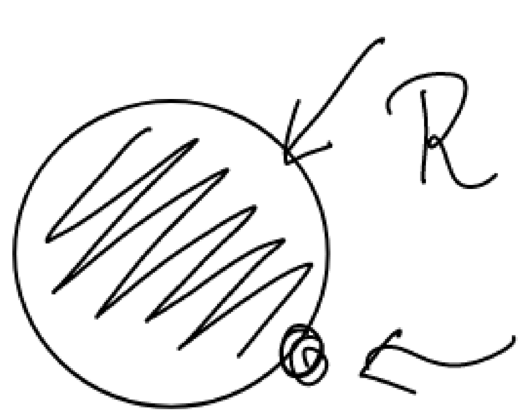
time

5d max gSUGRA
||
4d max SYM = conformal

AdS / CFT correspondence

Holographic principle or why can this possibly work?

black hole characterized by mass M or Schwarzschild-radius R



R

surface gravity κ

Thermodynamics

temperature T	κ
energy E	M

First law $\quad dE = T\,dS \qquad dM = \frac{\kappa}{8\pi G}\,dA$

$\hookrightarrow \quad S \sim A \qquad \boxed{S = \frac{k_B}{4 l_p^2} A}$ Bekenstein - Hawking formula

entropy $\hat{=}$ count of degrees of freedom

expectation from statistic grows with volume but for gravity only with area $\rightsquigarrow$ holographic principle

All "information" can be encode in a lower dim. surface like on a hologram.

Applications

strong - weak duality

i.e. Strongly coupled QFT
- hard, BUT important for

weakly coupled gravity
- can be treated with perturbation theory

- quark-gluon plasma
- condense matter physics i.e. Super conductor

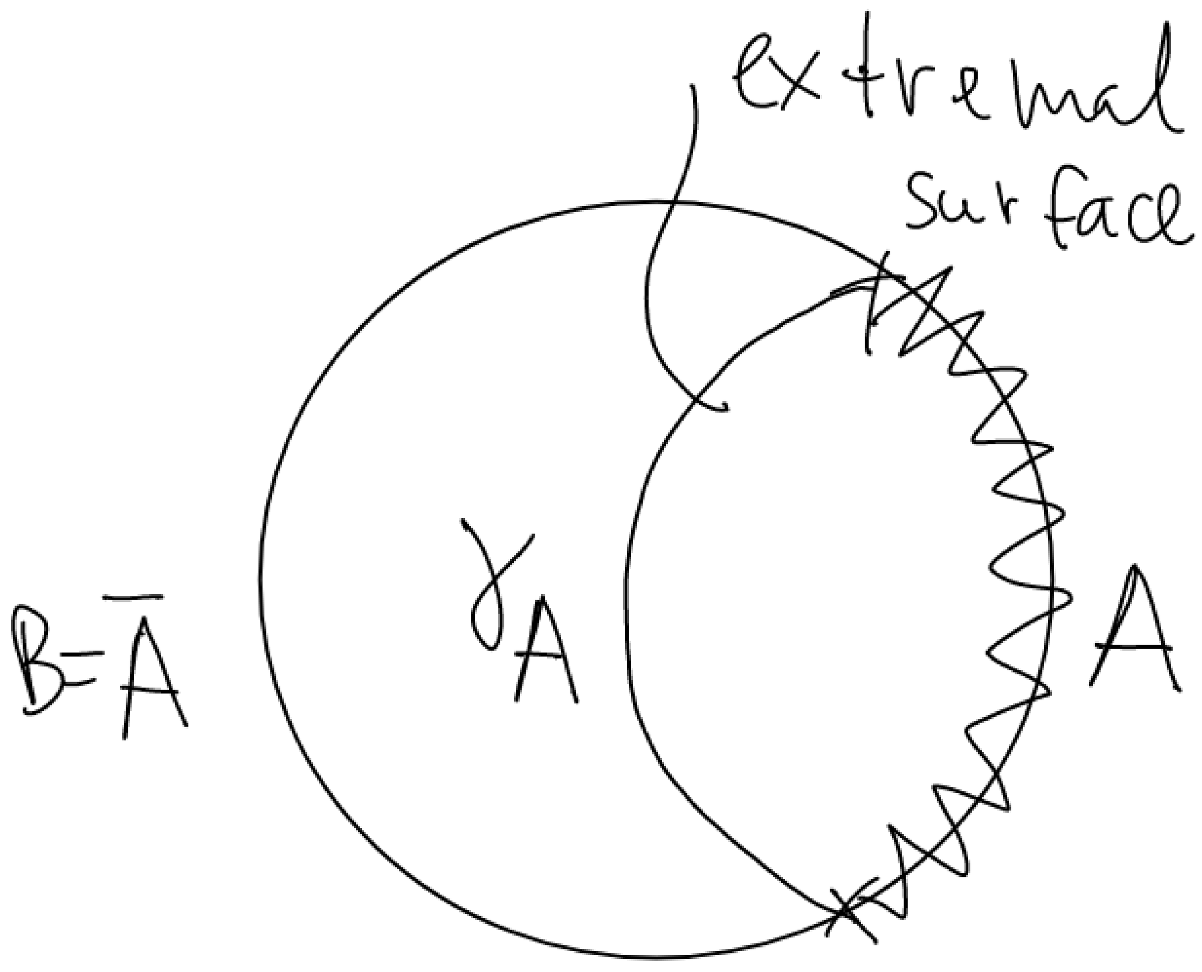
new insights

$\lightning$ Problem dictionary only known for very symmetric theories, with SUSY but not observed in nature

a way to study / define quantum gravity

Central question: What is the relation between between dof in the bulk and boundary

Tool: entanglement entropy



density matrix for boundary state $|\psi\rangle$: $\rho = |\psi\rangle\langle\psi|$

reduced density matrix

$\rho_A = \mathrm{Tr}_B |\psi\rangle\langle\psi|$

entanglement entropy $S_A = -\mathrm{Tr}(\rho_A \log \rho_A)$

$S_A = 0$ for seperable states $|\psi\rangle = |\phi_A\rangle|\phi_B\rangle$

$\hookrightarrow$ measures the entanglement of the regions A and B

conjecture: $\boxed{S_A = \frac{\text{Area of } \gamma_A}{4G}}$ Ryu-Takayangi formula

Goal for course: understand this formula

2. Gauge theory

Symmetries in physical system

global $\swarrow$ $\searrow$ local

- Captured by a (Lie)-group

abelian $\swarrow$ $\searrow$ non-abelian

2.1. Global symmetries

example complex scalar field ϕ with

$$\mathcal{L}(\phi) = \partial_\mu \phi \, \partial_\mu \bar{\phi} - m^2 \phi \bar{\phi}$$

transformation $\phi' = e^{-i\Lambda} \phi$ leaves $\mathcal{L}$ invariant

i.e. $\mathcal{L}(\phi) = \mathcal{L}(\phi')$ no coord. dependence same for the infinitesimal transformation

$$\delta_\Lambda \phi = -i\Lambda\phi \quad \text{and} \quad \delta\bar{\phi} = i\Lambda\bar{\phi}$$

$$\delta_\Lambda S = 0 = \int d^4x \left[\text{eom}(\bar{\phi})\, \delta_\Lambda \phi + \text{eom}(\phi)\, \delta_\Lambda \bar{\phi} + \partial_\mu J^\mu \Lambda \right]$$

equation(s) of motion

with conserved current $\boxed{J^\mu \Lambda = \frac{\delta \mathcal{L}}{\delta(\partial_\mu \phi)} \delta_\Lambda \phi + \frac{\delta \mathcal{L}}{\delta(\partial_\mu \bar{\phi})} \delta_\Lambda \bar{\phi}}$

satisfying under the eoms $\partial_\mu J^\mu = 0$

for our example: $J^\mu = i\left(\bar{\phi}\,\partial^\mu \phi - \phi\,\partial^\mu \bar{\phi}\right)$

Why do we care?

$$0 = \int_{t_1}^{t_2} d^4x \; \eta^{\mu\nu} \partial_\mu J_\nu \qquad (-1, +1, \ldots, +1)$$

signature

$$= \int_{t_1}^{t_2} dx^0 \Big(-\partial_0 \underbrace{\int d^3\vec{x}\, J^0}_{Q(x^0) = Q(t)} + \underbrace{\int d^3\vec{x}\, \partial_i J^i}_{=0 \text{ as field vanish at } \infty} \Big)$$

$0 = Q(t_2) - Q(t_1)$ or $Q(t) = \text{const.}$

$\boxed{Q(t) = \int d^3\vec{x}\, J^0}$ conserved charge

Noether's theorem: symmetry $\longleftrightarrow$ conserved charges

2.2. Abelian gauge theory

now the parameter $\Lambda = \Lambda(x)$ depends on coordinates

BUT then $\delta \mathcal{L} = \underbrace{(\partial_\mu \Lambda)} \mathcal{J}^\mu \neq 0$

originates from $\partial_\mu \phi$ in $\mathcal{L}$

$\rightsquigarrow$ <u>covariant derivative</u> $\boxed{D_\mu \phi = (\partial_\mu + i A_\mu)\phi}$

with the <u>connection</u> $A_\mu \to A'_\mu = A_\mu + \partial_\mu \Lambda$

such that $D_\mu \phi' = D_\mu (e^{-i\Lambda}\phi) = e^{-i\Lambda} D_\mu \phi$

in $\mathcal{L}$ we replace ∂_μ with D_μ resulting in

$\mathcal{L} = D_\mu \phi D^\mu \bar{\phi} + m^2 \phi \bar{\phi}$ (minimal coupling)

A_μ is not yet dynamic ($\hat{=}$ its eom are algebraic
we can integrate it out)

$$[D_\mu, D_\nu]\phi = \dots = i \underbrace{(\partial_\mu A_\nu - \partial_\nu A_\mu)}_{= F_{\mu\nu}} \phi$$

$\boxed{F_{\mu\nu} = 2\, \partial_{[\mu} A_{\nu]}}$ (electro-magnetic) field strength tensor

both combined give scalar electrodynamics

$\boxed{\mathcal{L} = \frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} + D_\mu \phi D^\mu \bar{\phi} + m^2 \phi \bar{\phi}}$ with the

action $S = -\int d^4x\, \mathcal{L}$

2.3. Non-abelian gauge theory

$$\phi^j \to \phi'^j = \underbrace{\left(e^{i\alpha^a T_a}\right)^j{}_k}_{\text{group element } U \in G \text{ (gauge group)}} \phi^k = U^j{}_k \phi^k$$

matrix

$T_a = (T_a)^j{}_k$ ← row, ← column, form a Lie algebra

$$\boxed{[T_a, T_b] = i f_{ab}{}^c T_c}$$

structure coefficients

infinitesimal, $\delta_\alpha \phi^j = i\alpha^a (T_a)^j{}_k \phi^k$

again, we introduce a covariant derivative

$$(D_\mu)^i{}_j = \delta^i{}_j \partial_\mu + i A^a_\mu (T_a)^i{}_j$$

or with representation (matrix) indices supressed

$$\boxed{D_\mu \phi = \partial_\mu \phi + i A^a_\mu T_a \phi}$$

where the connection $A_\mu := A^a_\mu T_a$ transforms as

$$A_\mu \to A'_\mu = U A_\mu U^{-1} - U \partial_\mu U^{-1}$$

the corresponding field strength arises from

$$[D_\mu, D_\nu]\phi = i F_{\mu\nu} \phi \qquad \text{as}$$

$$\boxed{F_{\mu\nu} = F^a_{\mu\nu} T_a = 2\partial_{[\mu} A_{\nu]} + i[A_\mu, A_\nu]}$$

with the Lagrangian

$$\boxed{\mathcal{L} = \underbrace{\mathrm{Tr}\left(\frac{1}{2g^2} F_{\mu\nu} F^{\mu\nu}\right)}_{\text{Yang-Mills theory}} + \underbrace{D_\mu \phi^\dagger D^\mu \phi + m^2 \phi^\dagger \phi}_{\substack{\text{coupled to scalar fields} \\ \text{in n-dim representation} \\ \text{of } G}}}$$

with $\phi^\dagger = (\bar{\phi}_1 \ldots \bar{\phi}_n)$