last lecture: motivation of correspondence from stack of D3 branes, today

10.2. Dictionary

10.2.1. Symmetries

- 4-dim. $\mathcal{N}=4$ SYM is super conformal

$SU(4)$ R-symmetry $SO(4,2)$

bosonic part of super group $PSU(2,2|4)$

fermionic -"- generated by Q^a_α and S^a_α

Poincaré supercharge superconformal superch.

- $AdS_5 \times S^5$ has isometry group $SO(4,2) \times \underbrace{SO(6)}_{SU(4)}$

$\rightsquigarrow$ global symmetries match :-)

10.2.2. Field-operator map

CFT side: 1/2 BPS states of conformal dim. Δ, scalars under gauge symmetry

$$\boxed{\mathcal{O}^I_\Delta(x) = C^I_{i_1 \ldots i_\Delta} \operatorname{Tr}\left(\phi^{i_1}(x) \ldots \phi^{i_\Delta}(x)\right)}$$

totally symmetric tensors of rank Δ

$\operatorname{Tr}(\ldots)$ over $SU(N)$ gauge group

type IIB: spherical harmonics on S^5

$$Y^I := C^I_{i_1 \ldots i_\ell} X^{i_1} \cdot \ldots \cdot X^{i_\ell} \quad \text{with}$$

orthogonal $\quad C^{I}_{i_1 \dots i_l} C^{J\, i_1 \dots i_l} = \delta^{IJ}$

$$\boxed{\Box\, Y^I = -\frac{1}{L^2}\, l(l+4)\, Y^I}$$

a 10 d field can now be written as

$$\varphi(x, z, \Omega_5) = \sum_{I=0}^{\infty} \varphi^I(x,z)\, Y^I(\Omega_5) \qquad \text{K K-expansion}$$

Kaluza-Klein

$x^\mu = 0, 1, 2, 3$ directions along D^3 brane

coordinates on S^5

like fluctuations $g_{MN} = \bar{g}_{MN} + h_{MN}$ &

$$F = \bar{F} + \delta F$$

in the eom $\quad R_{MN} = \frac{1}{3!} F_{M\,ABCD} F_N{}^{ABCD}$

background $\quad \bar{F}_{m_1 \dots m_5} = \frac{4}{L} \sqrt{-g_{AdS^5}}\, \varepsilon_{m_1 \dots m_5}$

$$\bar{F}_{\alpha_1 \dots \alpha_5} = \frac{4}{L} \sqrt{g_{S^5}}\, \varepsilon_{\alpha_1 \dots \alpha_5}$$

$$\bar{g}_{MN} = \begin{pmatrix} g_{mn} & 0 \\ 0 & g_{\alpha\beta} \end{pmatrix} \quad AdS_5 \times S^5$$

example: $h_{\alpha\beta} = \frac{h_2}{5}$, $\quad \delta F_{\alpha\beta\gamma\delta\varepsilon} = \nabla_{[\alpha}\, a_{\beta\gamma\delta\varepsilon]}$

with $\quad h_2(x, z, \Omega_5) = \sum h_2^I(x,z)\, Y^I(\Omega_5)$ and

$$a_{\alpha\beta\gamma\delta}(x, z, \Omega_5) = \sum b^I(x,z)\, \varepsilon_{\alpha\beta\gamma\delta\varepsilon} \nabla^\varepsilon Y^I(\Omega^5)$$

for the mode $\quad s^I = \frac{1}{20(l+2)} \left(h_2^I - 10(l+4)\, b^I \right)$

$$S = -\frac{4N^2}{(2\pi)^5 L^8} \int d^4x\, dz \sqrt{-g}\, \frac{A_I}{2} \left(\bar{g}^{mn} \partial_m s^I \partial_n s^I + l(l-4) (s^I)^2 \right)$$

with $A_I = 32 \frac{\ell(\ell-1)(\ell+2)}{\ell+1} z(\ell)$, $z(\ell)\, \delta^{IJ} = \int_{S^5} d\Omega\, Y^I(\Omega) Y^J(\Omega)$

dictionary: $\ell = \Delta$

$s^I \Leftrightarrow \mathcal{O}^I_\Delta$ and more general

d for general AdS$_{d+1}$/CFT$_d$

$m^2 L^2 = \Delta(\Delta - d)$ scalars, massive spin 2

$h^{\mu\nu} \Longleftrightarrow T_{\mu\nu}$

$m^2 L^2 = 0, \quad \Delta = d$ massless spin 2

$m^2 L^2 = (\Delta - p)(\Delta + p - d)$ p-form field

$\vdots$

10.2.3 Boundary asymptotics

consider $S = -\frac{c}{2}\int dz\, d^dx \sqrt{-g}\,\left(g^{mn}\partial_m\phi\,\partial_n\phi + m^2\phi^2\right)$ (1)

with $ds^2 = g_{mn}\, dx^m dx^n = \frac{L^2}{z^2}\left(dz^2 + \eta_{\mu\nu}\, dx^\mu dx^\nu\right)$

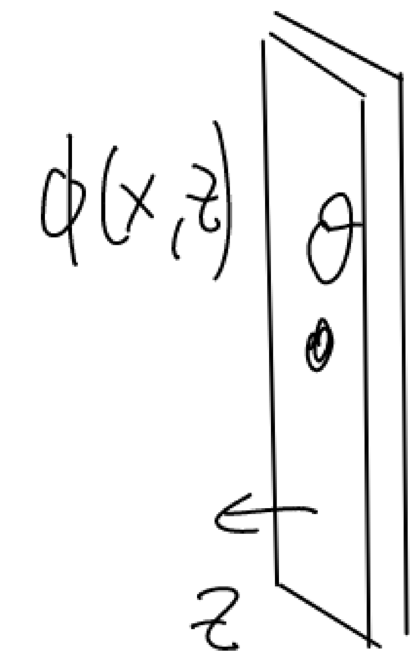
ansatz $\phi(x,z) = e^{i p_\mu x^\mu}\phi_p(z)$ results in Klein-Gordon

eq. $z^2 \partial_z \phi_p(z) - (d-1) z\,\partial_z \phi_p(z) - (m^2 L^2 + p^2 z^2)\phi_p(z) = 0$

$p^2 = p_\mu p^\mu$

for $z \to 0$ $\quad \phi_p(z) \sim z^\Delta \quad \rightsquigarrow m^2 L^2 = \Delta(\Delta - d)$

$$\Delta_\pm = \frac{d}{2} \pm \sqrt{\frac{d^2}{4} + m^2 L^2}$$



note that $\Delta_+ \geq \Delta_-$ and $\Delta_- = d - \Delta_+$

$$\phi(x,z) = \phi_{(0)}(x)\, z^{\Delta_-} + \phi_{(+)}(x)\, z^{\Delta_+} \ldots$$

non-normalizable (action diverges) normalizable (action finite)

source for $\mathcal{O}$ $\qquad \langle \mathcal{O} \rangle$

Question: What is the range of $\Delta_{+/-}(m^2L^2)$?

In flat space $m^2L^2 > 0$ (no tachyons) but

in AdS $\boxed{m^2L^2 \geq -d^2/4}$ Breitenlohner-Freedman bound

$$\rightsquigarrow \quad \Delta_+ \geq \frac{d}{2}$$

⚡: For correlators we cannot use action (1), but need to do integration by parts.

For keeping this "new" action finite $\Delta \geq (d-2)/2$

in $\phi(z) \sim z^{\Delta}$ for $\Delta < d/2$