with this ansatz we find ↗ see EX 11.2

$$R(G) = R(g) - \frac{1}{4}k^2 F^{ij} F_{ij} - \frac{2}{k}\nabla^i \nabla_i k$$

$F_{ij} = 2\,\partial_{[i} A_{j]}$ ← electro-magnetic field strength

↳ $$S = \int d^{D-1}x \sqrt{-g}\left(R - (\partial \ln k)^2 - \frac{k^2}{4} F^{ij} F_{ij}\right)$$

GR + ED unified

↯ massless scalar

$\longrightarrow$ not observed ☹

Same in more sophisticated compactifications

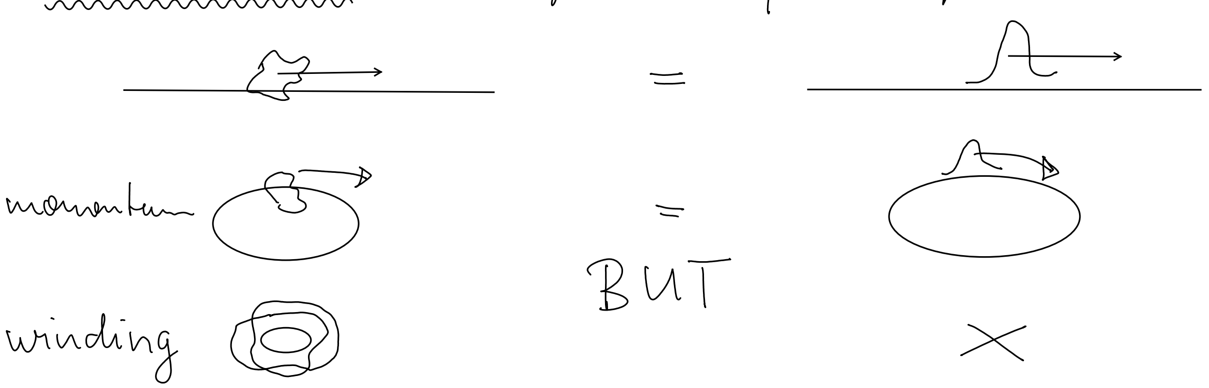
≙ moduli stabilisation problem

⇝ use H-flux ≙ flux compactifications

⇝ research

12. Closed string on torus & T-duality

Motivation: strings ≠ point particles

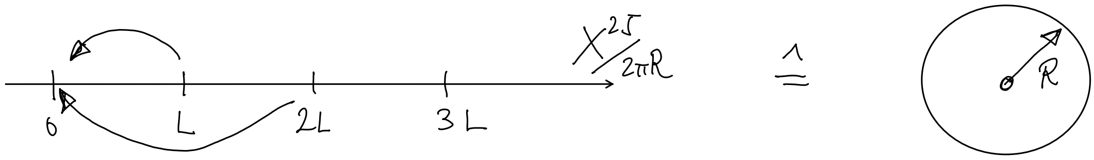


Important for i.e. Kaluza-Klein. Implications?

12.1. Closed string on a circle

remember: $D=26$ ⇝ compactify X^{25}

$X^{25} \sim X^{25} + 2\pi R \cdot L \qquad L \in \mathbb{Z}$



$S^1 = \mathbb{R}/(2\pi R\,\mathbb{Z})$ and therefore

$$\boxed{X^{25}(\sigma+2\pi,\tau) = X^{25}(\sigma) + 2\pi R L}$$

⇝ Into mode expansion

$$X^{25}(\sigma,\tau) = \underbrace{x^{25} + \alpha' p^{25}\cdot\tau + L R\sigma}_{} + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0} \text{ oscillatory}$$

zero modes with $[x^{25}, p^{25}] = i$

wave function on S^1 single-valued

$$\longrightarrow \quad \boxed{p^{25} = \frac{M}{R} \quad M \in \mathbb{Z}}$$

momentum quantum number

$$= X_L^{25}(\tau+\sigma) + X_R^{25}(\tau-\sigma)$$

(know from KK)

↳ Left- and Right moving modes

$$X_L^{25}(\tau+\sigma) = \frac{1}{2}(x^{25}+c) + \underbrace{\frac{\alpha'}{2}\left(\frac{M}{R} + \frac{LR}{\alpha'}\right)}_{p_L^{25}}(\tau+\sigma) + \text{osc.}$$

$$X_R^{25}(\tau-\sigma) = \frac{1}{2}(x^{25}-c) + \underbrace{\frac{\alpha'}{2}\left(\frac{M}{R} - \frac{LR}{\alpha'}\right)}_{p_R^{25}}(\tau-\sigma) + \text{osc.}$$

with mass

$$\alpha' m_L^2 = 2(\bar{L}_0 - 1) = \frac{\alpha'}{2}\left(\frac{M}{R} + \frac{LR}{\alpha'}\right)^2 + 2(N_L - 1)$$

$$\alpha' m_R^2 = 2(L_0 - 1) = \frac{\alpha'}{2}\left(\frac{M}{R} - \frac{LR}{\alpha'}\right)^2 + 2(N_R - 1)$$

rest frame in $\mathbb{R}^{1,24}$ uncompactified space

remember Level matching $L_0|\psi\rangle = \bar{L}_0|\psi\rangle$

physical state

$\Rightarrow \quad m_L^2 = m_R^2$ or

$$\boxed{N_R - N_L = M \cdot L}$$

with mass

$$\alpha' m^2 = \alpha'(m_L^2 + m_R^2) = \alpha' \frac{M^2}{R^2} + \frac{1}{\alpha'} L^2 R^2 + 2(N_L + N_R - 2)$$

momentum   energy of and   winding modes

Spectrum:

1. $|0\rangle \quad \alpha' m^2 = -4$ ⇝ tachyon
2. $|G^{\mu\nu}\rangle = \alpha^\mu_{-1} \bar{\alpha}^\nu_{-1} |0\rangle$ ⇝ massless spin 2

   metric, B-field & dilaton in $\mathbb{R}^{1,24}$
3. $|V_1^\mu\rangle = \alpha^\mu_{-1} \bar{\alpha}^{25}_{-1} |0\rangle$ ⇝ massless spin 1

   $|V_2^\mu\rangle = \alpha^{25}_{-1} \bar{\alpha}^\mu_{-1} |0\rangle$ two vectors
4. $|\phi\rangle = \alpha^{25}_{-1} \bar{\alpha}^{25}_{-1} |0\rangle$ ⇝ massless scalar

for $M = L = 0$ we recover fields from KK

- BUT much more excited states (in KK only $L = 0$)
- $\mathbb{Z}_2$ symmetry of spectrum

$$\boxed{R \rightleftarrows \frac{\alpha'}{R}, \quad L \rightleftarrows M \quad \text{called T-duality}}$$

T= Target space or Torus

- not only spectrum but also modes with

$(p_L^{25}, p_R^{25}) \longrightarrow (p_L^{25}, -p_R^{25})$ and osc. such that

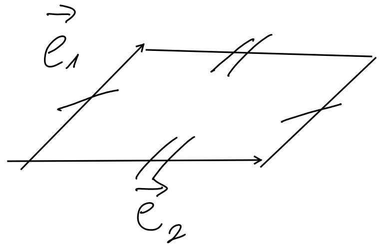
$(X_L^{25}, X_R^{25}) \longrightarrow (X_L^{25}, -X_R^{25})$

full sym. of closed string!

12.2 Closed string on D-torus

torus directions

$$X^I \sim X^I + 2\pi \boxed{\sum_{i=1}^{D} n^i e_i^I} = L^I$$



$T^D = \mathbb{R}^D / 2\pi \Lambda_D$

Torus

lattice spanned by e^I_i

again $[X^I, P_J] = i\,\delta^I_J$, $P_I = m_i\, e^{*i}_I$ quantised, $m_i \in \mathbb{Z}$

e^{*i}_I = <u>dual lattice</u> with

$e^I_i\, e^{*j}_I = \delta^j_i$ and $e^I_i\, e^{*i}_J = \delta^I_J$

⋮

mass formula $\alpha' m^2 = 2(N_L + N_R - 2) + \sum_{i,j=1}^{D} \left(\alpha' m_i\, g^{ij} m_j + \frac{1}{\alpha'} n^i g_{ij} n^j\right)$

$g_{ij} = e^I_i\, e^J_j\, \delta_{IJ}$

$g^{ij} = e^{*i}_I\, e^{*j}_J\, \delta^{IJ}$

and

level matching $N_R - N_L = p_I L^I = \sum_{i=1}^{D} m_i n^i$
$= \frac{1}{2} N^{\hat{I}}\, \eta_{\hat{I}\hat{J}}\, N^{\hat{J}}$

with $N^{\hat{I}} = \begin{pmatrix} n^i \\ m_i \end{pmatrix}$ and $\eta_{\hat{I}\hat{J}} = \begin{pmatrix} 0 & \delta_i^j \\ \delta^j_i & 0 \end{pmatrix}$

invariant metric of the Lie group $O(D,D)$

similar $\alpha' m^2 = \text{osc} + N^{\hat{I}}\, \mathcal{H}_{\hat{I}\hat{J}}\, N^{\hat{J}}$

with $\mathcal{H}_{\hat{I}\hat{J}} = \begin{pmatrix} \frac{1}{\alpha'} g_{ij} & 0 \\ 0 & \alpha' g^{ij} \end{pmatrix}$ ≙ generalised metric

$\boxed{\text{T-duality} \;\hat{=}\; O(D,D,\mathbb{Z}) \text{ action}}$

non-linearly realised on g_{ij} and B_{ij} ⇝ Buscher rules