9.2. In supergravity

Dp world sheet ✓ $\longleftrightarrow$ target space
(p+1) - dim. 10 - dim

Dp - brane is a $1/2$ BPS solution
remember only half of the Poincaré supercharges Q_α contribute

Symmetries: $P(p,1) = \mathbb{R}^{p,1} \rtimes SO(p,1)$

$$\underset{\text{without } Dp}{P(9,1)} \rightarrow \underset{\text{with } Dp}{P(p,1)} \times SO(9-p)$$ translations $\perp$ to Dp are broken

solution: $ds^2 = H_p(r)^{-1/2} \eta_{\mu\nu} dx^\mu dx^\nu + H_p(r)^{1/2} dy^i dy^i$

$$e^\phi = g_s H_p(r)^{(3-p)/4} \quad \text{and} \quad \Box H_p(r) = 0$$

$$C_{(p+1)} = \left(H_p(r)^{-1} - 1 \right) dx^0 \wedge \ldots \wedge dx^p$$

$\rightsquigarrow$ harmonic function $H_p(r) = 1 + \left(\frac{L_p}{r}\right)^{7-p}$
choosen such that $r \to \infty \mathrel{\hat{=}} 10$ dim. Minkowski space

10. The AdS/CFT correspondence

$\mathcal{N}=4$ SYM theory with gauge group $SU(N)$ and g_{YM}	is dynamically equivalent to	type IIB superstring theory with $l_s = \sqrt{\alpha'}$ and g_s on $AdS_5 \times S^5$ *)

*) $AdS_5 \times S^5$ with curvature radius L and N units of $F^{(5)}$ flux on the $S^{(5)}$

$$\boxed{g_{YM}^2 = 2\pi g_s \quad \text{and} \quad \underbrace{2 g_{YM}^2 N}_{\lambda \text{ ('t Hooft coupling)}} = \frac{L^4}{\alpha'^2}}$$

dynamically equivalent = all physics of one discription is mapped onto all of the other

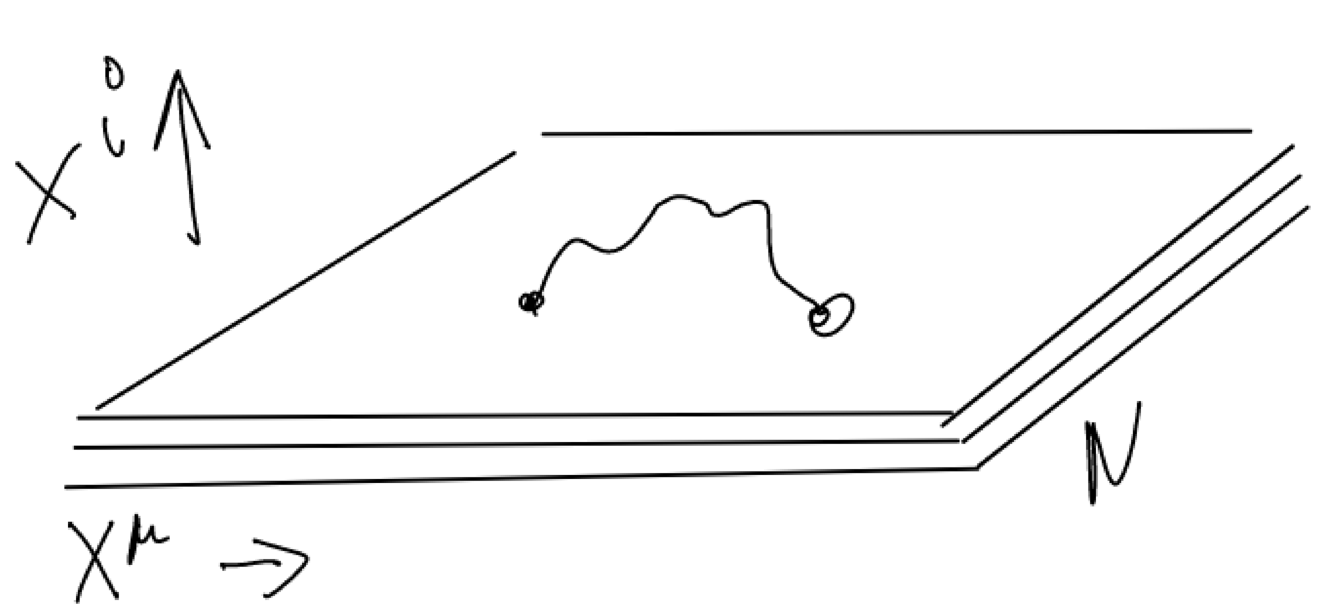
limits: $\mathcal{N}=4$ SYM — IIB on $AdS_5 \times S^5$

- $N \to \infty$, λ fixed but arbitrary — classical string theory, $g_s \to 0$, $\frac{\alpha'}{L^2} \neq 0$
- $N \to \infty$, $\lambda \to \infty$ — classical SUGRA, $g_s \to 0$, $\frac{\alpha'}{L^2} \to 0$

strong coupling $\longleftrightarrow$ weak coupling

10.1. Near horizon limit of N D3 branes

remember: a) open string perspective



$g_s \ll 1$ and $E \ll \alpha'^{-1/2}$ (only massless modes)

$\to$ 4d SYM with gauge group $U(N)$

bosons are A_μ = longitudinal excitations

ϕ^i = transversal deformation of brane ($i = 1, \dots, 6$)

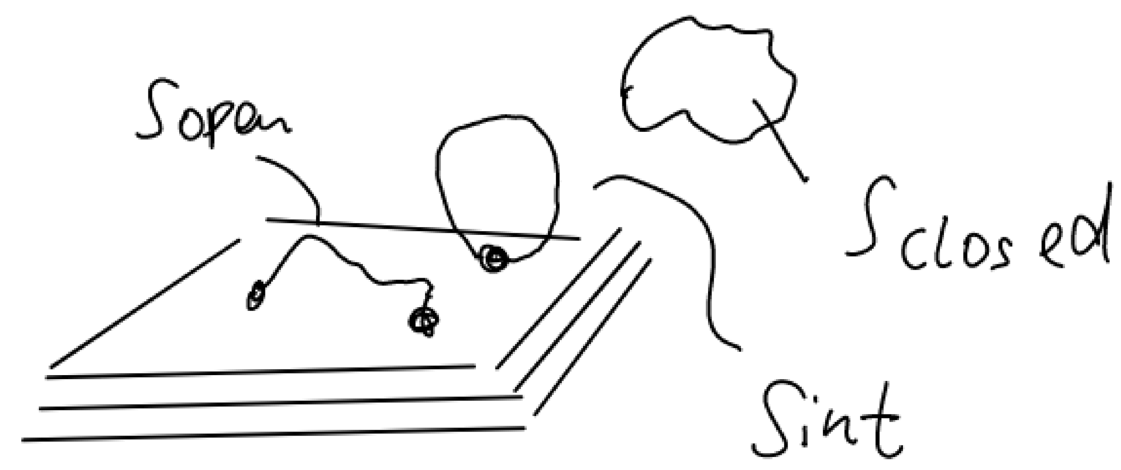
and coupling $\sim g_s N$ ($\ll 1$ to be valid)

b) closed string perspective:

characteristic length scale $L_3^4/\alpha'^2 \gg 1$ with

$$L_3^4 = 4\pi g_s N \alpha'^2 \quad \leadsto \quad \underset{\sim\sim\sim\sim}{g_s \cdot N \gg 1}$$

Decoupling limits



a) $S = S_{closed} + S_{open} + S_{int}$

$$S_{closed} = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-g}\, e^{-2\phi}\left(R + 4|\partial\phi|^2\right) + \underbrace{\ldots}_{\text{R-R fields}}$$

$$2\kappa^2 = (2\pi)^7 \alpha'^4 g_s^2$$

for $g = \eta + \kappa h$ $\quad \sim -\frac{1}{2}\int d^{10}x\, \partial_M h\, \partial^M h + O(\kappa)$

S_{open} and S_{int} from S_{DBI}

single D-brane

$$\begin{cases} S_{open} = -\dfrac{1}{2\pi g_s}\displaystyle\int d^4x \left(\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}\eta^{\mu\nu}\partial_\mu\phi^i\partial_\nu\phi^i + O(\alpha')\right) \\ S_{int} = -\dfrac{1}{8\pi g_s}\displaystyle\int d^4x\, \phi F_{\mu\nu}F^{\mu\nu} + \ldots \end{cases}$$

for N D3's: $\qquad \phi^i = \phi^{ia} T_a, \quad A_\mu = A_\mu^a T_a$

$$F_{\mu\nu}F^{\mu\nu} \to \mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right), \quad \partial \to D$$

and add the potential: $\quad V = \dfrac{1}{2\pi g_s}\sum_{ij}\mathrm{Tr}[\phi^i,\phi^j]^2$

for $\alpha' \to 0$ $\quad S_{open}$ = bosonic part of 4d max SYM with $2\pi g_s = g_{YM}^2$

S_{closed} = SUGRA in 10 dim. Minkowski space

S_{int} = 0 $\quad$ <u>decoupling limit</u>

b) remember 9.2 with $H(r) = 1 + \left(\frac{L}{r}\right)^4$

distance to stack

b1) $r \gg L$

$H(r) \sim 1 \rightsquigarrow$ Minkowski

b2) $r \ll L$

$H(r) \sim \frac{L^4}{r^4}$ near horizon or throat region

for b2) we find

unit S^5

$$ds^2 = \underbrace{\frac{L^2}{z^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2\right)}_{AdS_5} \times \underbrace{+ L^2 \, ds^2_{S^5}}_{S^5}, \quad z = \frac{L^2}{r}$$

In the limit $\boxed{\alpha' \to 0 \quad \text{with} \quad u = \frac{r}{\alpha'} \text{ fixed}}$

Maldacena limit

b$_1$) and b$_2$) decouple !

$$\frac{L^4}{r^4} = 4\pi g_s N \frac{\alpha'^2}{r^4} = 4\pi g_s \cdot N \underbrace{\frac{\alpha'^4}{r^4}}_{\text{const}} \cdot \underbrace{\alpha'^{-2}}_{\to\infty} \to \infty$$

we zoom in to the near horizon region

$\hat{=}$ near horizon limit

decoupling because of gravitational redshift

$$E_\infty = \sqrt{-g_{00}}\, E_r = H(r)^{-1/4} E_r$$

$$\sqrt{\alpha'}\, E_\infty \sim \frac{r}{L} \sqrt{\alpha'}\, E_r \to 0 \qquad \text{because } r \ll L$$

observer @ ∞ sees: 1. String modes of IIB in $AdS_5 \times S^5$

2. supergravity modes propagation in flat 10 dim Minkowski space

a) Maldacena's conjecture b)

- 4 d SYM $U(N)$ $=$ • type IIB superstring theory on $AdS^5 \times S^5$
- 10 d IIB SUGRA in $\mathbb{R}^{9,1}$ $=$ • 10 d IIB SUGRA in $\mathbb{R}^{9,1}$