Extended Space for (half) Maximally Supersymmetric Theories

Falk Hassler

bases on

arXiv: 1611.07978 and 1705.09304

in collaboration with

Pascal du Bosque and Dieter Lüst

University of North Carolina at Chapel Hill

June 7, 2017

Typical conversation with non-DFT/EFT hep-th people

- Why an extended space?

Typical conversation with non-DFT/EFT hep-th people

- Why an extended space?
    - additional coordinates are winding modes on T^d
    - they make U/T-duality manifest

Typical conversation with non-DFT/EFT hep-th people

- Why an extended space?
    - additional coordinates are winding modes on T^d
    - they make U/T-duality manifest
- Tori are boring. Can you do more?

Typical conversation with non-DFT/EFT hep-th people

- Why an extended space?
  - additional coordinates are winding modes on T^d
  - they make U/T-duality manifest
- Tori are boring. Can you do more?
  - yes, after imposing the SC (section condition)

Typical conversation with non-DFT/EFT hep-th people

- Why an extended space?
    - additional coordinates are winding modes on T^d
    - they make U/T-duality manifest
- Tori are boring. Can you do more?
    - yes, after imposing the SC (section condition)
- But then you have GG (generalized geometry).

Typical conversation with non-DFT/EFT hep-th people

- Why an extended space?
  - additional coordinates are winding modes on T^d
  - they make U/T-duality manifest
- Tori are boring. Can you do more?
  - yes, after imposing the SC (section condition)
- But then you have GG (generalized geometry).
  - yes, but extended space is great bookkeeping device

Typical conversation with non-DFT/EFT hep-th people

- Why an extended space?
  - additional coordinates are winding modes on T^d
  - they make U/T-duality manifest
- Tori are boring. Can you do more?
  - yes, after imposing the SC (section condition)
- But then you have GG (generalized geometry).
  - yes, but extended space is great bookkeeping device
- Examples please!

Typical conversation with non-DFT/EFT hep-th people

- Why an extended space?
  - additional coordinates are winding modes on T^d
  - they make U/T-duality manifest
- Tori are boring. Can you do more?
  - yes, after imposing the SC (section condition)
- But then you have GG (generalized geometry).
  - yes, but extended space is great bookkeeping device
- Examples please!
  - generalized Scherk-Schwarz reductions ...

Typical conversation with non-DFT/EFT hep-th people

- Why an extended space?
  - additional coordinates are winding modes on T^d
  - they make U/T-duality manifest
- Tori are boring. Can you do more?
  - yes, after imposing the SC (section condition)
- But then you have GG (generalized geometry).
  - yes, but extended space is great bookkeeping device
- Examples please!
  - generalized Scherk-Schwarz reductions …
- You classify all consistent reductions with maximal SUSY?
- By the way, what happens to the manifest dualities?

Scherk-Schwarz reductions at a glance

- compactification on group manifold M=G
- frame field $e^a{}_i$ (left-invariant Maurer-Cartan form)

  $t_a e^a{}_i dx^i = g^{-1} dg$ with $t_a \in \mathfrak{g}$ and $g \in G$

Scherk-Schwarz reductions at a glance

- compactification on group manifold M=G
- frame field $e^a{}_i$ (left-invariant Maurer-Cartan form)

  $t_a e^a{}_i dx^i = g^{-1} dg$ with $t_a \in \mathfrak{g}$ and $g \in G$
- frame algebra with const. $f_{ab}{}^c$ generated by Lie derivative

  $L_{e_a} e_b{}^i e^c{}_i = f_{ab}{}^c e_c{}^i$

Scherk-Schwarz reductions at a glance

- compactification on group manifold $M{=}G$
- frame field $e^a{}_i$ (left-invariant Maurer-Cartan form)

  $t_a e^a{}_i dx^i = g^{-1} dg \quad \text{with} \quad t_a \in \mathfrak{g} \quad \text{and} \quad g \in G$
- frame algebra with const. $f_{ab}{}^c$ generated by Lie derivative

  $L_{e_a} e_b{}^i e^c{}_i = f_{ab}{}^c e_c{}^i$
- invariant, non-degenerate, symmetric two-form $\delta_{ab} \rightarrow$ metric

  $ds^2 = \delta_{ab} e^a{}_i e^b{}_j dx^i dx^j$

Scherk-Schwarz reductions at a glance

- compactification on group manifold M=G
- frame field $e^a{}_i$ (left-invariant Maurer-Cartan form)

  $t_a e^a{}_i dx^i = g^{-1} dg$ with $t_a \in \mathfrak{g}$ and $g \in G$
- frame algebra with const. $f_{ab}{}^c$ generated by Lie derivative

  $L_{e_a} e_b{}^i e^c{}_i = f_{ab}{}^c e_c{}^i$
- invariant, non-degenerate, symmetric two-form $\delta_{ab} \rightarrow$ metric

  $ds^2 = \delta_{ab} e^a{}_i e^b{}_j dx^i dx^j$
- no fluxes
- M is parallelizable space
- tangent bundle trivial, maximal SUSY
- consistent reduction to gauged SUGRA

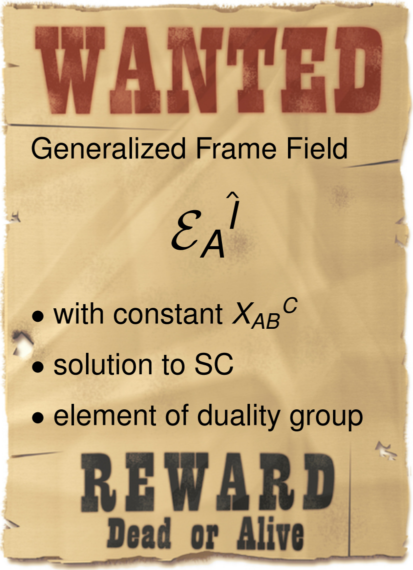
Generalized Scherk-Schwarz reductions at a glance

- compactification on ~~group~~ manifold M
- generalized frame field $\mathcal{E}^A{}_{\hat{I}}$ ~~(left-invariant Maurer-Cartan form)~~ ???
- frame algebra with const. $X_{AB}{}^C$ generated by gen. Lie derivative $\widehat{\mathcal{L}}_{\mathcal{E}_A}\mathcal{E}_B{}^{\hat{I}}\mathcal{E}^C{}_{\hat{I}} = X_{AB}{}^C$
- invariant, non-degenerate, symmetric two-form $\delta_{AB} \rightarrow$ gen. metric $\mathcal{H}_{\hat{I}\hat{J}} = \delta_{AB}\mathcal{E}^A{}_{\hat{I}}\mathcal{E}^B{}_{\hat{J}}$
- ~~no~~ fluxes
- M is generalized parallelizable space
- generalized tangent bundle trivial, maximal SUSY
- consistent reduction to gauged SUGRA

Generalized Frame Field

$$\mathcal{E}_A{}^{\hat{I}}$$

- with constant $X_{AB}{}^C$
- solution to SC
- element of duality group



- examples, no construction yet

today

1. revisit extended space
2. solve SC
3. construct $\mathcal{E}_A{}^{\hat{I}}$

Alternative proposal for extended space

- extended space = group manifold G

Alternative proposal for extended space

- extended space = group manifold G
- relevant structure

1. left-invariant Maurer-Cartan form

   $$t_A E^A{}_I dx^I = g^{-1} dg \quad \text{with} \quad t_A \in \mathfrak{g} \quad \text{and} \quad g \in G$$

Alternative proposal for extended space

- extended space = group manifold G
- relevant structure

1. left-invariant Maurer-Cartan form

   $t_A E^A{}_I dx^I = g^{-1} dg \quad \text{with} \quad t_A \in \mathfrak{g} \quad \text{and} \quad g \in G$

2. flat derivative

   $D_A = E_A{}^I \partial_I$

Alternative proposal for extended space

- extended space = group manifold G
- relevant structure

1. left-invariant Maurer-Cartan form

   $$t_A E^A{}_I dx^I = g^{-1} dg \quad \text{with} \quad t_A \in \mathfrak{g} \quad \text{and} \quad g \in G$$

2. flat derivative

   $$D_A = E_A{}^I \partial_I$$

3. covariant derivative

   $$\nabla_A V^B = D_A V^B + \Gamma_{AC}{}^B V^C$$

Alternative proposal for extended space

- extended space = group manifold G
- relevant structure

1. left-invariant Maurer-Cartan form

   $$t_A E^A{}_I dx^I = g^{-1} dg \quad \text{with} \quad t_A \in \mathfrak{g} \quad \text{and} \quad g \in G$$

2. flat derivative

   $$D_A = E_A{}^I \partial_I$$

3. covariant derivative

   $$\nabla_A V^B = D_A V^B + \Gamma_{AC}{}^B V^C$$

- why?
  - suggested by CSFT (Closed String Field Theory) on WZW-model
  - for G abelian $\rightarrow$ standard formulation
  - it works

Action of the duality group $E_{d(d)}$/O(d-1,d-1)

- indices A, B, ... transform in coordiante irrep of duality group e.g. **10** of SL(5)=$E_{4(4)}$ or vector irrep of O(d-1,d-1)

Action of the duality group $E_{d(d)}$/O(d-1,d-1)

- indices A, B, … transform in coordiante irrep of duality group
  e.g. **10** of SL(5)=$E_{4(4)}$ or vector irrep of O(d-1,d-1)
- introduce generalized Lie derivative
  $\mathcal{L}_\xi V^A = \xi^B \nabla_B V^A - V^B \nabla_B \xi^A + Y^{AB}{}_{CD} \nabla_B \xi^C V^D$
- whose closure requires SC constraint
  $Y^{CD}{}_{AB} D_C \cdot D_D \cdot = 0$

Action of the duality group $E_{d(d)}$/O(d-1,d-1)

- indices A, B, ... transform in coordiante irrep of duality group
  e.g. **10** of SL(5)=$E_{4(4)}$ or vector irrep of O(d-1,d-1)
- introduce generalized Lie derivative
  $\mathcal{L}_\xi V^A = \xi^B \nabla_B V^A - V^B \nabla_B \xi^A + Y^{AB}{}_{CD} \nabla_B \xi^C V^D$
- whose closure requires SC constraint
  $Y^{CD}{}_{AB} D_C \cdot D_D \cdot = 0$

fix connection Γ

by requiring
- Y-tensor is covariant constant
- closure of generalized Lie derivative

Action of the duality group $E_{d(d)}/O(d\text{-}1,d\text{-}1)$

- indices A, B, ... transform in coordiante irrep of duality group
  e.g. **10** of $SL(5)=E_{4(4)}$ or vector irrep of $O(d\text{-}1,d\text{-}1)$
- introduce generalized Lie derivative
  $\mathcal{L}_\xi V^A = \xi^B \nabla_B V^A - V^B \nabla_B \xi^A + Y^{AB}{}_{CD} \nabla_B \xi^C V^D$
- whose closure requires SC constraint
  $Y^{CD}{}_{AB} D_C \cdot D_D \cdot = 0$

fix connection Γ

by requiring
- Y-tensor is covariant constant
- closure of generalized Lie derivative

results in
- two linear constraints
- one quadratic constraint

Solutions

- linear constraints restrict G to embedding tensor solutions

e.g. SO(3,3) $\cong$ SL(4): $\mathbf{6} \times \mathbf{15} \to \mathbf{10} + \overline{\mathbf{10}}$

or SL(5): $\mathbf{10} \times \mathbf{24} \to \mathbf{15} + \overline{\mathbf{40}}$

Solutions

- linear constraints restrict G to embedding tensor solutions

  e.g. SO(3,3) $\cong$ SL(4): $\mathbf{6} \times \mathbf{15} \to \mathbf{10} + \overline{\mathbf{10}}$

  or SL(5): $\mathbf{10} \times \mathbf{24} \to \mathbf{15} + \overline{\mathbf{40}}$

- if embedding tensor $X_{AB}{}^{C} \neq X_{[AB]}{}^{C}$

  dim $G <$ dim coordinate irrep $\to$ break duality group

- happens in **40** of SL(5)

  e.g. dim G=9, branch to SL(3)$\times$SL(2)

  coordinate irrep: $\mathbf{10} \to (\mathbf{3}, \mathbf{2}) + (\overline{\mathbf{3}}, \mathbf{1}) + \cancel{(\mathbf{1}, \mathbf{1})}$

  embedding tensor: $(\mathbf{1}, \mathbf{3}) + (\mathbf{3}, \mathbf{2}) + (\mathbf{6}, \mathbf{1}) + (\mathbf{1}, \mathbf{2})$

Solutions

- linear constraints restrict G to embedding tensor solutions

  e.g. SO(3,3) $\cong$ SL(4): $\mathbf{6} \times \mathbf{15} \to \mathbf{10} + \overline{\mathbf{10}}$

  or SL(5): $\mathbf{10} \times \mathbf{24} \to \mathbf{15} + \overline{\mathbf{40}}$

- if embedding tensor $X_{AB}{}^{C} \neq X_{[AB]}{}^{C}$

  dim $G <$ dim coordinate irrep $\to$ break duality group

- happens in **40** of SL(5)

  e.g. dim G=9, branch to SL(3)$\times$SL(2)

  coordinate irrep: $\mathbf{10} \to (\mathbf{3},\mathbf{2}) + (\overline{\mathbf{3}},\mathbf{1}) + \cancel{(\mathbf{1},\mathbf{1})}$

  embedding tensor: $(\mathbf{1},\mathbf{3}) + (\mathbf{3},\mathbf{2}) + (\mathbf{6},\mathbf{1}) + (\mathbf{1},\mathbf{2})$

- also for $E_{6(6)}$ and G=SO(6) where $E_{6(6)} \to$SL(6)$\times$SL(2)

  coordinate irrep: $\mathbf{27} \to (\mathbf{15},1) + \cancel{(\mathbf{6},\mathbf{2})}$

Solutions

- linear constraints restrict G to embedding tensor solutions

  e.g. SO(3,3) $\cong$ SL(4): $\mathbf{6} \times \mathbf{15} \rightarrow \mathbf{10} + \overline{\mathbf{10}}$

  or SL(5): $\mathbf{10} \times \mathbf{24} \rightarrow \mathbf{15} + \overline{\mathbf{40}}$

- if embedding tensor $X_{AB}{}^{C} \neq X_{[AB]}{}^{C}$

  dim $G <$ dim coordinate irrep $\rightarrow$ break duality group

- happens in **40** of SL(5)

  e.g. dim G=9, branch to SL(3)$\times$SL(2)

  coordinate irrep: $\mathbf{10} \rightarrow (\mathbf{3},\mathbf{2}) + (\overline{\mathbf{3}},\mathbf{1}) + \cancel{(\mathbf{1},\mathbf{1})}$

  embedding tensor: $(\mathbf{1},\mathbf{3}) + (\mathbf{3},\mathbf{2}) + (\mathbf{6},\mathbf{1}) + (\mathbf{1},\mathbf{2})$

- also for $E_{6(6)}$ and G=SO(6) where $E_{6(6)}$ $\rightarrow$SL(6)$\times$SL(2)

  coordinate irrep: $\mathbf{27} \rightarrow (\mathbf{15},1) + \cancel{(\mathbf{6},\mathbf{2})}$

- quadratic constraint = Jacobi identity

SC and H-principal bundle

- SC selects d physical directions in extended space
- split TG into $\begin{cases} \text{d} & \text{physical} \\ \dim G - d & \text{SC violating} \end{cases}$ directions

SC and H-principal bundle

- SC selects d physical directions in extended space
- split TG into $\begin{cases} \text{d} & \text{physical} \\ \dim G - d & \text{SC violating} \end{cases}$ directions
- convenient tool is a H-principal bundle G

  splits TG into $\begin{cases} HG & \text{horizontal=physical} \\ VG & \text{vertical=auxiliary} \end{cases}$ subbundles
- physical manifold $M = G/H$ is coset space

SC and H-principal bundle

- SC selects d physical directions in extended space
- split TG into $\begin{cases} \text{d} & \text{physical} \\ \dim G - d & \text{SC violating} \end{cases}$ directions
- convenient tool is a H-principal bundle G

  splits TG into $\begin{cases} HG & \text{horizontal=physical} \\ VG & \text{vertical=auxiliary} \end{cases}$ subbundles
- physical manifold $M = G/H$ is coset space
- H is a maximally isotropic subgroup of G for O(d-1,d-1)
- for SL(5) EFT linear SC $v_a^0 \epsilon^{aBC} \partial_C \cdot = 0$ is a map **10** $\rightarrow$ **10**

  $\mathfrak{m}$ is its kernel and $\mathfrak{h}$ the complement
- in total $\mathfrak{g} = \mathfrak{m} + \mathfrak{h}$

Sections and connections

- the bundle $\pi : G \to M$ has sections $\sigma_i : M \supset U_i \to G$
- and a $\mathfrak{h}$-valued connection one-form ω

Sections and connections

- the bundle $\pi : G \to M$ has sections $\sigma_i : M \supset U_i \to G$
- and a $\mathfrak{h}$-valued connection one-form ω
- at each $g \in G$, $\omega_g(X) = 0$ for all $X \in H_g G$
- solving SC in patch U_i requires $\omega \sigma_{i\,*} = \sigma_i^* \omega = A_i = 0$

Sections and connections

- the bundle $\pi : G \to M$ has sections $\sigma_i : M \supset U_i \to G$
- and a $\mathfrak{h}$-valued connection one-form ω
- at each $g \in G$, $\omega_g(X) = 0$ for all $X \in H_g G$
- solving SC in patch U_i requires $\omega \sigma_{i\,*} = \sigma_i^* \omega = A_i = 0$
- A is gauge connection on M
- corresponding field strength has to vanish (flat connection)

Sections and connections

- the bundle $\pi : G \to M$ has sections $\sigma_i : M \supset U_i \to G$
- and a $\mathfrak{h}$-valued connection one-form ω
- at each $g \in G$, $\omega_g(X) = 0$ for all $X \in H_g G$
- solving SC in patch U_i requires $\omega \sigma_{i\,*} = \sigma_i^* \omega = A_i = 0$
- A is gauge connection on M
- corresponding field strength has to vanish (flat connection)
- A severely constraint by linear version of SC

  e.g. O(d-1,d-1): $A = t^a(-B_{ab}E^b{}_i + \delta^b{}_a E_{bi})$

  SL(5): $A = t_{\tilde\alpha}(\eta^{\gamma\delta,\tilde\alpha} C_{\beta\gamma\delta} E^\beta{}_i + \delta^{\tilde\alpha}_{\tilde\beta} E^{\tilde\beta}{}_i) dx^i$
- B_{ab}, $C_{\alpha\beta\gamma}$ are totally anti-symmetric

Two ways to find A=0 (and solve the SC)

- the hard way

  1. take arbitrary coset representative $m(x^i)$
  2. choose B/C in connection A that F=DA=0 (in general very hard)
  3. A is (at least locally) pure gauge
  4. set A=0 by gauge transformation

Two ways to find A=0 (and solve the SC)

- the hard way
  1. take arbitrary coset representative $m(x^i)$
  2. choose B/C in connection A that F=DA=0 (in general very hard)
  3. A is (at least locally) pure gauge
  4. set A=0 by gauge transformation

- much simpler if
  1. $\mathfrak{m}$ and $\mathfrak{h}$ form symmetric pair

     $$[\mathfrak{h},\mathfrak{h}] \subset \mathfrak{h} \qquad [\mathfrak{h},\mathfrak{m}] \subset \mathfrak{m} \qquad [\mathfrak{m},\mathfrak{m}] \subset \mathfrak{h}$$
  2. $\mathfrak{m}$ is a subgroup

     e.g. for O(d-1,d-1) Drinfeld double

then coset representative $m = \exp(f(x^i))$ results in $A = 0$

Connection to generalized geometry

- identify $\begin{cases} HG & \text{with tangent bundle } TM \text{ of } M \\ VG & \text{the reset of the generalized tangent bundle} \end{cases}$

Connection to generalized geometry

- identify $\begin{cases} HG & \text{with tangent bundle } TM \text{ of } M \\ VG & \text{the reset of the generalized tangent bundle} \end{cases}$
- for all $g \in G$, $V_g G \cong \mathfrak{h}$; introduce map

  $\eta_g : \mathfrak{h} \to \Lambda^n T_g^* M$ $\quad$ n=1 for O(d-1,d-1) / n=2 for SL(5)

Connection to generalized geometry

- identify $\begin{cases} HG & \text{with tangent bundle } TM \text{ of } M \\ VG & \text{the reset of the generalized tangent bundle} \end{cases}$
- for all $g \in G$, $V_g G \cong \mathfrak{h}$; introduce map

  $\eta_g : \mathfrak{h} \to \Lambda^n T_g^* M$ n=1 for O(d-1,d-1) / n=2 for SL(5)
- in combination with
  1. component $E^a{}_i$ of left-invariant Maurer-Cartan form $E^A{}_I$
  2. B_{ab} / $C_{\alpha\beta\gamma}$ from A=0

  a canoical generalized frame field $\widehat{E}_A{}^{\hat{I}}$ arises

Connection to generalized geometry

- identify $\begin{cases} HG & \text{with tangent bundle } TM \text{ of } M \\ VG & \text{the reset of the generalized tangent bundle} \end{cases}$
- for all $g \in G$, $V_g G \cong \mathfrak{h}$; introduce map

  $\eta_g : \mathfrak{h} \to \Lambda^n T^*_g M$ $\quad$ n=1 for O(d-1,d-1) / n=2 for SL(5)
- in combination with
  1. component $E^a{}_i$ of left-invariant Maurer-Cartan form $E^A{}_I$
  2. B_{ab} / $C_{\alpha\beta\gamma}$ from A=0

  a canoical generalized frame field $\widehat{E}_A{}^{\hat{I}}$ arises
- apply to vectors $V^{\hat{I}} = \begin{pmatrix} V^i & V_i \end{pmatrix} = \widehat{E}_A{}^{\hat{I}} V^A$ and gen. Lie derivative

  $\mathcal{L}_\xi V^{\hat{I}} = \widehat{\mathcal{L}}_\xi V^{\hat{I}} + \mathcal{F}_{\hat{J}\hat{K}}{}^{\hat{I}} \xi^{\hat{J}} V^{\hat{K}}$

Connection to generalized geometry

- $\widehat{\mathcal{L}}$ = untwisted gen. Lie derivative of GG (generalized geometry) on M
- $\mathcal{F}_{\hat{I}\hat{J}}{}^{\hat{K}}$ = additional twist

$$\mathcal{L}_{\widehat{E}_A} \widehat{E}_B{}^{\hat{I}} \widehat{E}^C{}_{\hat{I}} = X_{AB}{}^C$$

Connection to generalized geometry

- $\widehat{\mathcal{L}}$ = untwisted gen. Lie derivative of GG (generalized geometry) on M
- $\mathcal{F}_{\hat{I}\hat{J}}{}^{\hat{K}}$ = additional twist

$$\mathcal{L}_{\widehat{E}_A} \widehat{E}_B{}^{\hat{I}} \widehat{E}^C{}_{\hat{I}} = X_{AB}{}^C$$

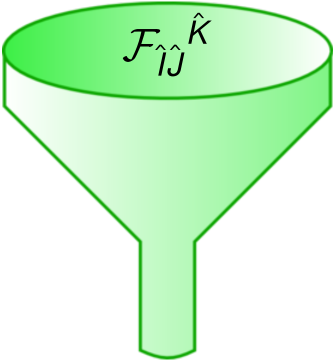
- $\widehat{E}_A \neq \mathcal{E}_A$, because $\mathcal{L} \neq \widehat{\mathcal{L}}$

Connection to generalized geometry

- $\widehat{\mathcal{L}}$ = untwisted gen. Lie derivative of GG (generalized geometry) on M
- $\mathcal{F}_{\hat{I}\hat{J}}{}^{\hat{K}}$ = additional twist

$$\mathcal{L}_{\widehat{E}_A} \widehat{E}_B{}^{\hat{I}} \widehat{E}^C{}_{\hat{I}} = X_{AB}{}^C$$

- $\widehat{E}_A \neq \mathcal{E}_A$, because $\mathcal{L} \neq \widehat{\mathcal{L}}$



- TODO: push twist completely in gen. frame field $\widehat{E}_B{}^{\hat{I}}$

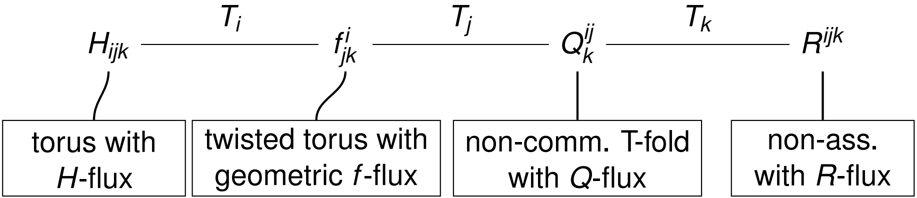
Different solutions and dualities

- in general more than one tuple $(M, \mathcal{E}_A)$ with same $X_{AB}{}^C$

Different solutions and dualities

- in general more than one tuple $(M, \mathcal{E}_A)$ with same $X_{AB}{}^C$
- related by duality transformations

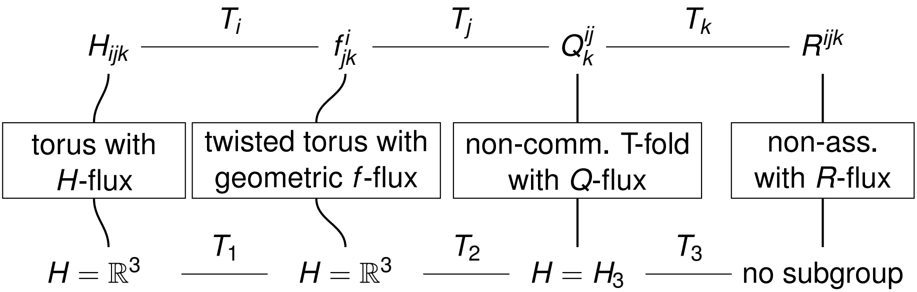
  e.g. G=CSO(1,0,3) $\subset$ SO(3,3)



Different solutions and dualities

- in general more than one tuple $(M, \mathcal{E}_A)$ with same ${X_{AB}}^C$
- related by duality transformations

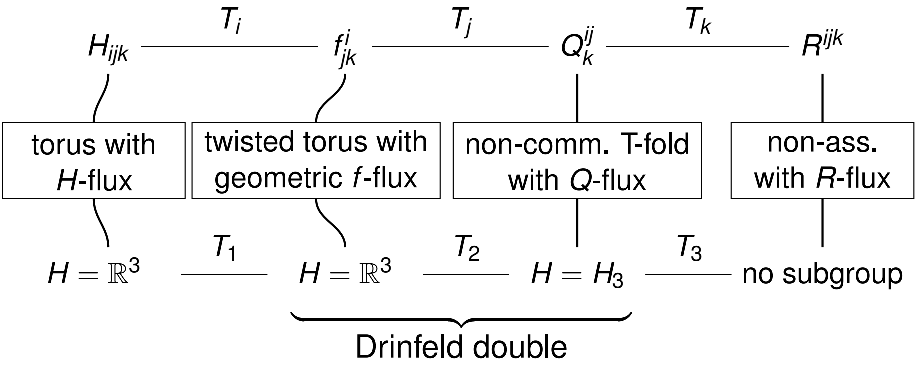
  e.g. G=CSO(1,0,3) $\subset$ SO(3,3)



Different solutions and dualities

- in general more than one tuple $(M, \mathcal{E}_A)$ with same $X_{AB}{}^C$
- related by duality transformations

  e.g. G=CSO(1,0,3) $\subset$ SO(3,3)



Construction of the generalized frame field $\mathcal{E}_A{}^{\hat{I}}$

- ansatz $\mathcal{E}_A{}^{\hat{I}} = -M_A{}^B \widehat{E}'_B{}^{\hat{I}}$ with

$$M_A{}^B t_B = m^{-1} t_A m$$

$$\widehat{E}'_A{}^{\hat{I}} = \text{similar as } \widehat{E}_A{}^{\hat{I}} \text{ but with } \mathcal{B}/\mathcal{C} \text{ instead of } B/C$$

$$G/H \ni m = \text{from splitting } g = mh \text{ induced by SC solution}$$

Construction of the generalized frame field $\mathcal{E}_A{}^{\hat{I}}$

- ansatz $\mathcal{E}_A{}^{\hat{I}} = -M_A{}^B \widehat{E}'_B{}^{\hat{I}}$ with

$$M_A{}^B t_B = m^{-1} t_A m$$

$$\widehat{E}'_A{}^{\hat{I}} = \text{similar as } \widehat{E}_A{}^{\hat{I}} \text{ but with } \mathcal{B}/\mathcal{C} \text{ instead of } B/C$$

$$G/H \ni m = \text{from splitting } g = mh \text{ induced by SC solution}$$

- requires
  1. additional linear constraint
  2. appropriate choice of $\mathcal{B}/\mathcal{C}$

     e.g. SL(5): $d\mathcal{C} = -\dfrac{3}{4} Y_{11}\,\mathrm{vol}$

Construction of the generalized frame field $\mathcal{E}_A{}^{\hat{I}}$

- ansatz $\mathcal{E}_A{}^{\hat{I}} = -M_A{}^B \widehat{E}'_B{}^{\hat{I}}$ with

$$M_A{}^B t_B = m^{-1} t_A m$$

$$\widehat{E}'_A{}^{\hat{I}} = \text{similar as } \widehat{E}_A{}^{\hat{I}} \text{ but with } \mathcal{B}/\mathcal{C} \text{ instead of } B/C$$

$$G/H \ni m = \text{from splitting } g = mh \text{ induced by SC solution}$$

- requires
  1. additional linear constraint
  2. appropriate choice of $\mathcal{B}/\mathcal{C}$

     e.g. SL(5): $d\mathcal{C} = -\frac{3}{4} Y_{11} \,\mathrm{vol}$

- by construction element of duality group & SC solution

Solutions of the additional constraint for SL(5)

- SC solution requires choosing a v_a^0
- breaks SL(5) to SL(4), ...

Solutions of the additional constraint for SL(5)

- SC solution requires choosing a v_a^0
- breaks SL(5) to SL(4), ...
- linear constraints remove certain irreps from embedding tensor

$$\dim G = 10 : \quad \mathrm{SL}(5) \to \mathrm{SL}(4)$$
$$\mathbf{15} \to \mathbf{1} + \cancel{\mathbf{4}} + \mathbf{10}$$

Solutions of the additional constraint for SL(5)

- SC solution requires choosing a v_a^0
- breaks SL(5) to SL(4), ...
- linear constraints remove certain irreps from embedding tensor

$$\dim G = 10: \quad \mathrm{SL}(5) \to \mathrm{SL}(4)$$

$$\mathbf{15} \to \mathbf{1} + \cancel{\mathbf{4}} + \mathbf{10}$$

$$\dim G = 9: \quad \mathrm{SL}(3) \times \mathrm{SL}(2) \to \mathrm{SL}(2) \times \mathrm{SL}(2)$$

$$(\mathbf{1},\mathbf{3}) + (\mathbf{3},\mathbf{2}) + (\mathbf{6},\mathbf{1}) + (\mathbf{1},\mathbf{2}) \to$$

$$(\mathbf{1},\mathbf{3}) + \cancel{(\mathbf{1},\mathbf{2})} + (\mathbf{2},\mathbf{2}) + (\mathbf{1},\mathbf{1}) + \cancel{(\mathbf{2},\mathbf{1})} + (\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{2})$$

Solutions of the additional constraint for SL(5)

- SC solution requires choosing a v_a^0
- breaks SL(5) to SL(4), ...
- linear constraints remove certain irreps from embedding tensor

$$\dim G = 10: \quad \mathrm{SL}(5) \to \mathrm{SL}(4)$$
$$\mathbf{15} \to \mathbf{1} + \cancel{\mathbf{4}} + \mathbf{10}$$

$$\dim G = 9: \quad \mathrm{SL}(3) \times \mathrm{SL}(2) \to \mathrm{SL}(2) \times \mathrm{SL}(2)$$
$$(\mathbf{1},\mathbf{3}) + (\mathbf{3},\mathbf{2}) + (\mathbf{6},\mathbf{1}) + (\mathbf{1},\mathbf{2}) \to$$
$$(\mathbf{1},\mathbf{3}) + \cancel{(\mathbf{1},\mathbf{2})} + (\mathbf{2},\mathbf{2}) + (\mathbf{1},\mathbf{1}) + \cancel{(\mathbf{2},\mathbf{1})} + (\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{2})$$

$$\dim G = 7: \quad \ldots$$

Solutions of the additional constraint for SL(5)

- SC solution requires choosing a v_a^0
- breaks SL(5) to SL(4), ...
- linear constraints remove certain irreps from embedding tensor

$$\dim G = 10: \quad \mathrm{SL}(5) \to \mathrm{SL}(4)$$
$$\mathbf{15} \to \mathbf{1} + \cancel{\mathbf{4}} + \mathbf{10}$$

$$\dim G = 9: \quad \mathrm{SL}(3) \times \mathrm{SL}(2) \to \mathrm{SL}(2) \times \mathrm{SL}(2)$$
$$(\mathbf{1},\mathbf{3}) + (\mathbf{3},\mathbf{2}) + (\mathbf{6},\mathbf{1}) + (\mathbf{1},\mathbf{2}) \to$$
$$(\mathbf{1},\mathbf{3}) + \cancel{(\mathbf{1},\mathbf{2})} + (\mathbf{2},\mathbf{2}) + (\mathbf{1},\mathbf{1}) + \cancel{(\mathbf{2},\mathbf{1})} + (\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{2})$$

$$\dim G = 7: \quad \dots$$

- all allowed gaugings in the **15** only result in symmetric spaces

Example S^4 with radius R

- group G=SO(5), only one subgroup H=SO(4)
- M=SO(5)/SO(4), symmetric space

Example S^4 with radius R

- group G=SO(5), only one subgroup H=SO(4)
- M=SO(5)/SO(4), symmetric space
- A vanishes globally for

  $$C = R^3 \tan\left(\frac{\phi^1}{2}\right) \sin^3(\phi^1) \sin^2(\phi^2) \sin(\phi^3)\, d\phi^2 \wedge d\phi^3 \wedge d\phi^4$$

Example S^4 with radius R

- group G=SO(5), only one subgroup H=SO(4)
- M=SO(5)/SO(4), symmetric space
- A vanishes globally for

  $$C = R^3 \tan\left(\frac{\phi^1}{2}\right) \sin^3(\phi^1) \sin^2(\phi^2) \sin(\phi^3)\, d\phi^2 \wedge d\phi^3 \wedge d\phi^4$$

- choose $\mathcal{C}$ in $\widehat{E}'_A{}^{\hat{I}}$ such that

  $$d\mathcal{C} = 3R^3 \sin^3(\phi^1) \sin^2(\phi^2) \sin(\phi^3)\, d\phi^1 \wedge d\phi^2 \wedge d\phi^3 \wedge d\phi^4 = \frac{3}{R}\mathrm{vol}$$

  and we construct $\mathcal{E}_A{}^{\hat{I}}$

Example S^4 with radius R

- group G=SO(5), only one subgroup H=SO(4)
- M=SO(5)/SO(4), symmetric space
- A vanishes globally for

  $$C = R^3 \tan\left(\frac{\phi^1}{2}\right) \sin^3(\phi^1) \sin^2(\phi^2) \sin(\phi^3)\, d\phi^2 \wedge d\phi^3 \wedge d\phi^4$$

- choose $\mathcal{C}$ in $\widehat{E}'_A{}^{\hat{I}}$ such that

  $$d\mathcal{C} = 3R^3 \sin^3(\phi^1) \sin^2(\phi^2) \sin(\phi^3)\, d\phi^1 \wedge d\phi^2 \wedge d\phi^3 \wedge d\phi^4 = \frac{3}{R}\mathrm{vol}$$

  and we construct $\mathcal{E}_A{}^{\hat{I}}$
- alternative parameterization, embedding in $\mathbb{R}^5$
- reproduces known results

Summary

DFT/EFT gives us an explicit construction of generalized parallelizable spaces, if we change the perspective on the extended space.

Summary

DFT/EFT gives us an explicit construction of generalized parallelizable spaces, if we change the perspective on the extended space.

Advantages

- group G is manifest in extended space
- dualities are manifest as different solutions of the SC
- results depends on having extended space

Summary

DFT/EFT gives us an explicit construction of generalized parallelizable spaces, if we change the perspective on the extended space.

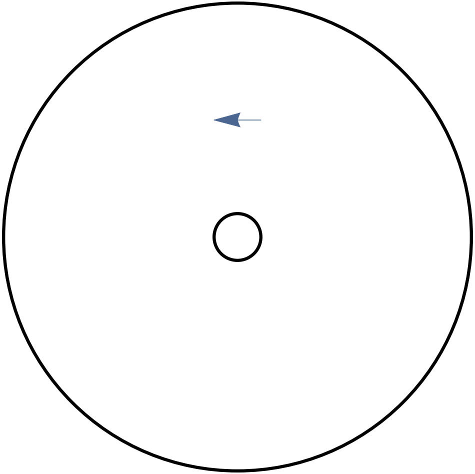
Advantages

- group G is manifest in extended space
- dualities are manifest as different solutions of the SC
- results depends on having extended space

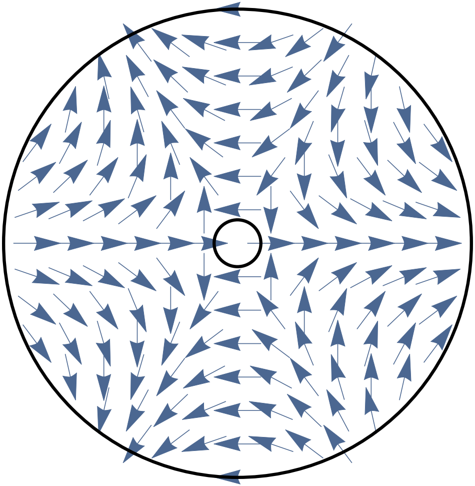
Project proposal: Classification of gen. parall. spaces

- redo analysis for $E_{d(d)}$ with $d > 4$
- solve linear constraints completely
- are there examples not covered by the construction?

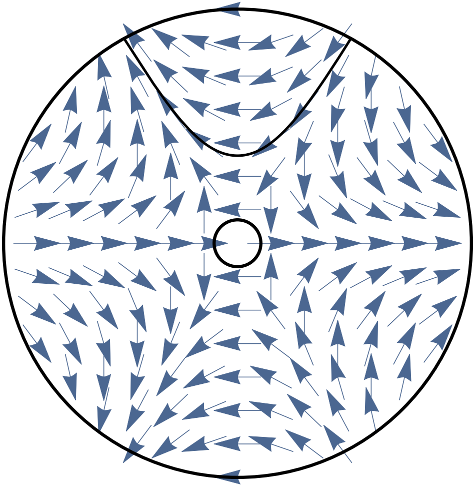
H-principal bundle construction "visualized"



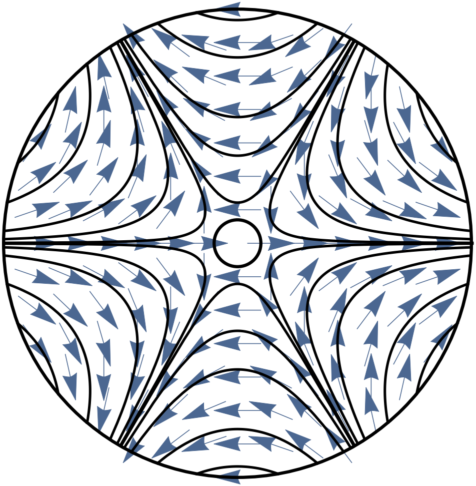
H-principal bundle construction "visualized"



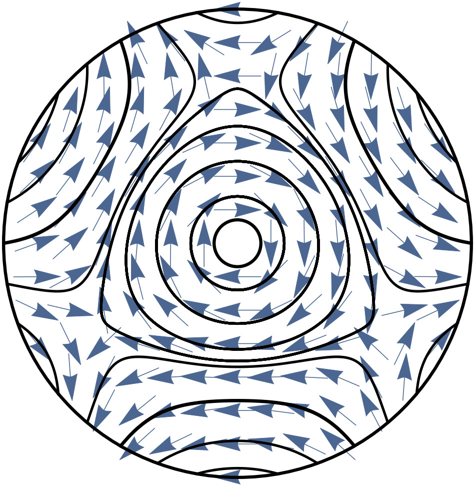
H-principal bundle construction "visualized"



H-principal bundle construction “visualized”



H-principal bundle construction "visualized"



H-principal bundle construction "visualized"