Consistent Truncations and Dualities

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Texas A&M University

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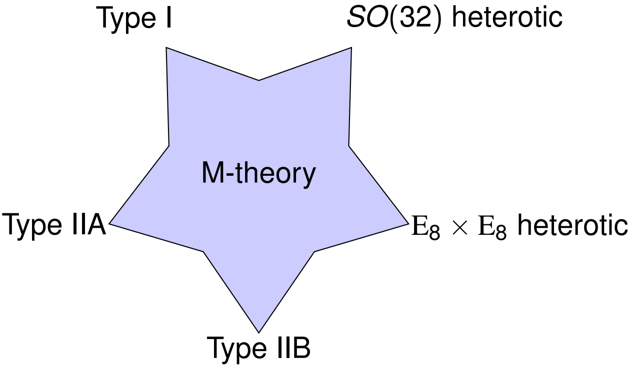
Motivation: Dualities



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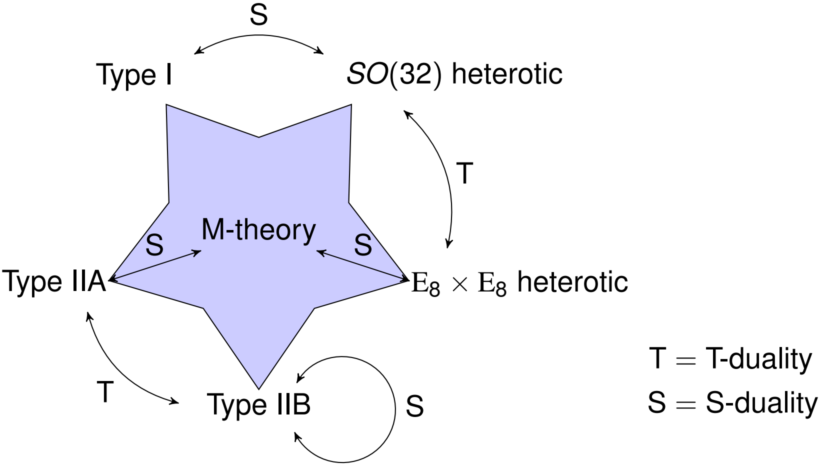

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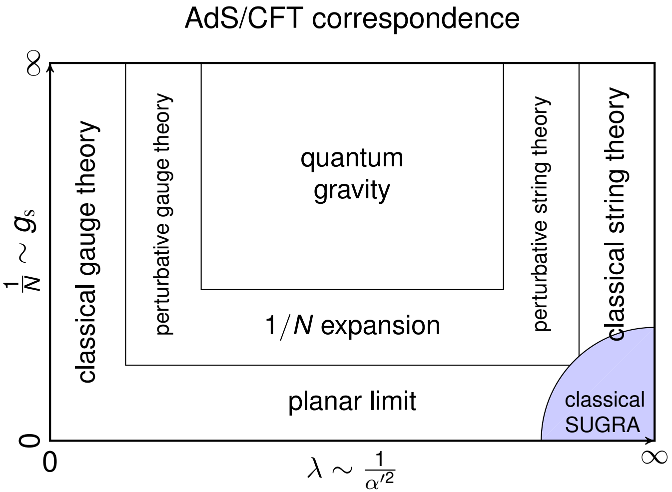

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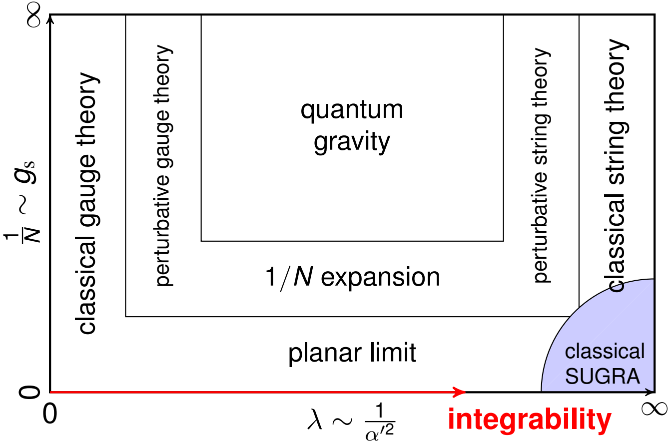
AdS/CFT correspondence



Motivation: Dualities

β-deformation

λ-deformation

$AdS_5 \times S^5$

η-deformation

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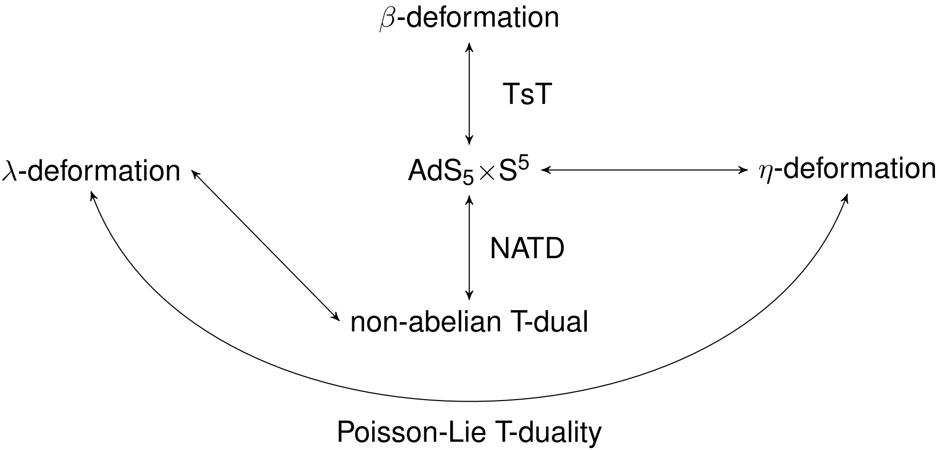

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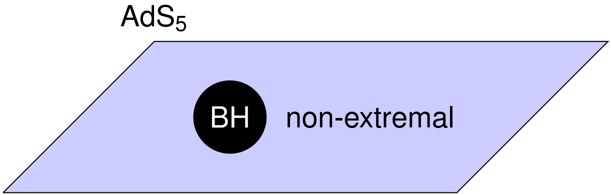

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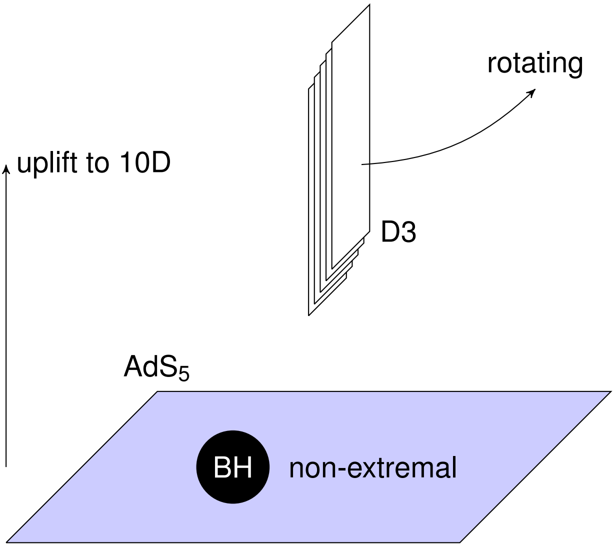

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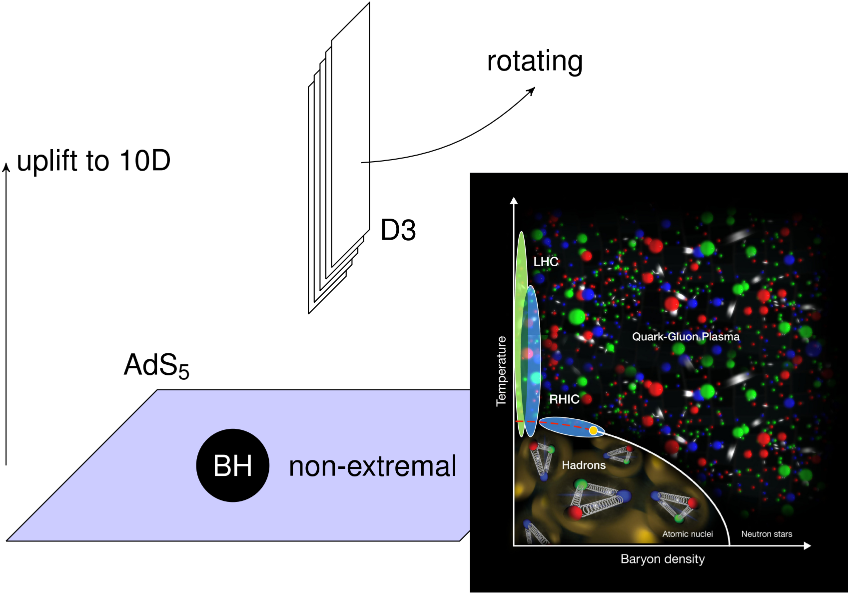

Outline

1. Motivation

2. Consistent truncations

3. Poisson-Lie T-duality

4. Outlook

Consistent truncations

- What are consistent truncations of SUGRA?
- Why are they useful?
- How are they related to Poisson-Lie T-duality?

Motivation: 1-loop quantum corrections

- σ-model $S = \frac{1}{2}\int \mathrm{d}^2\sigma\sqrt{-h}\left[h^{\mu\nu}\partial_\mu X^i(G_{ij}+B_{ij})\partial_\nu X^j + \phi R^{(2)}\right]$
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- β-functions match the field equations of the target space action

  $$S_{\mathrm{NS}} = \int \mathrm{d}^d x\,\sqrt{-G}e^{-2\phi}\left(R^{(d)} + 4\partial_i\phi\partial^i\phi - \tfrac{1}{12}H_{ijk}H^{ijk}\right)$$

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- symmetries:
  1. diffeomorphisms: $\delta G = L_x G \quad \delta B = L_x B$
  2. B-field gauge transformation: $B \to B - \mathrm{d}\phi$

- both captured by generalized Lie derivative

$$\delta\mathcal{H} = \mathcal{L}_{\left(x \quad \phi\right)}\mathcal{H}$$

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- various applications: AdS/CFT, phenomenology, cosmology, ...
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  3. apply dualities to known solutions
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- a prominent idea: reduce dimensions

= get ride of some degrees of freedom

→ simpler to find solution

New challenge: find consistent truncations

1. **consistent** ansatz for fields in 10/11D
2. reduce action with this ansatz
3. solve field equations of reduced action
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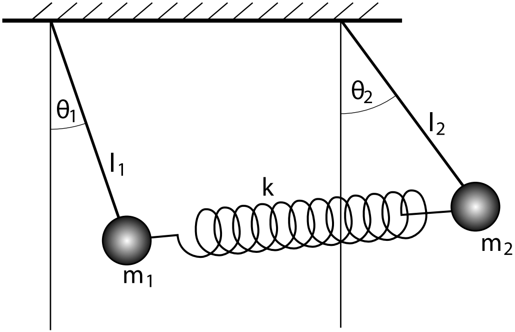

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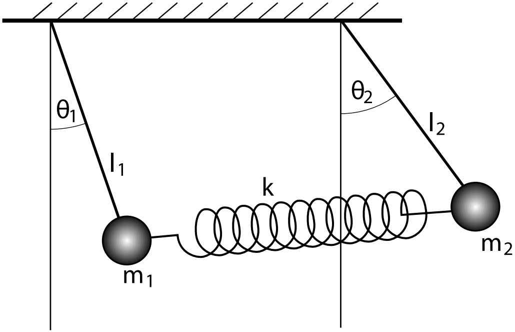


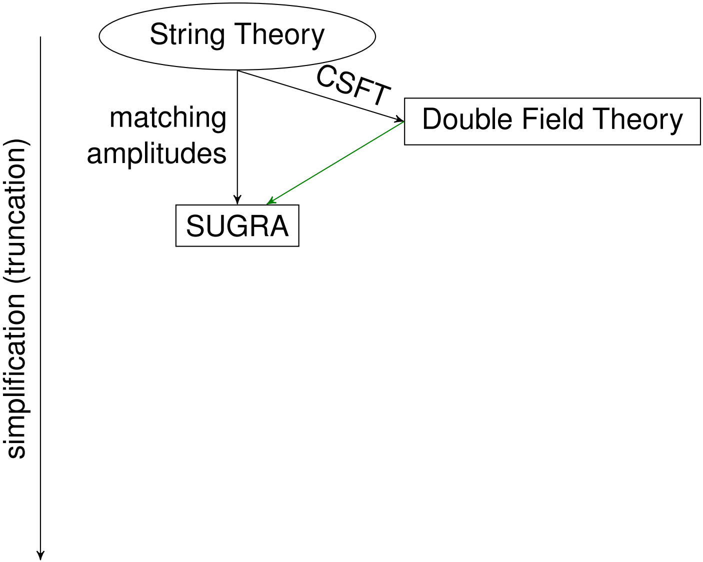
↯ $\theta_2 = 0$

✓ for $m_2 \to \infty$ set $\theta_2 = 0$

✓ for $m_1 = m_2$ set $\theta_1 = \theta_2$

Generalized Scherk-Schwarz compactification

[Aldazabal, Baron, Marques, and Nunez, 2011,Geissbuhler, 2011]



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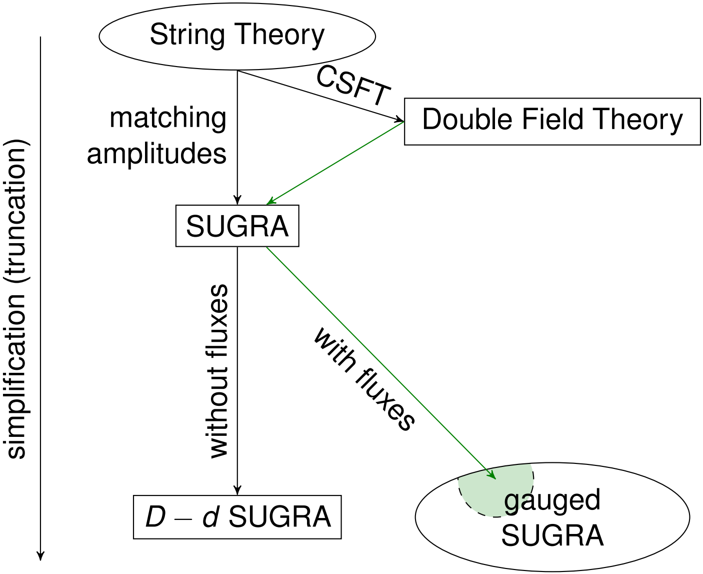

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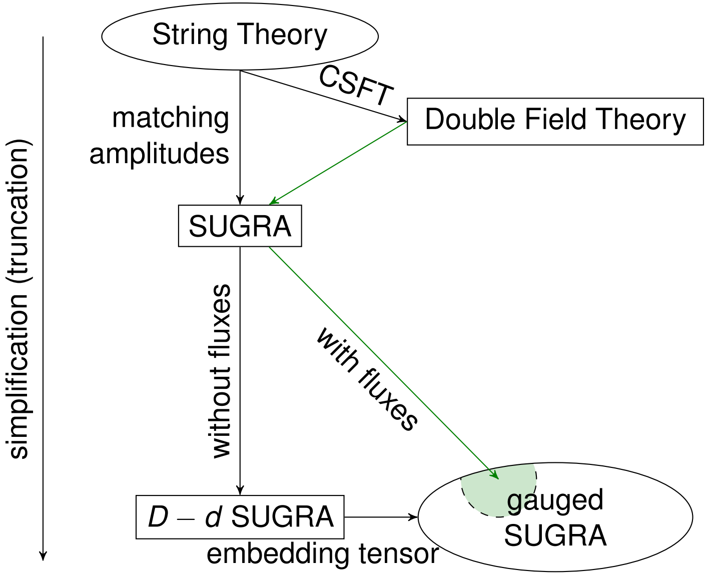

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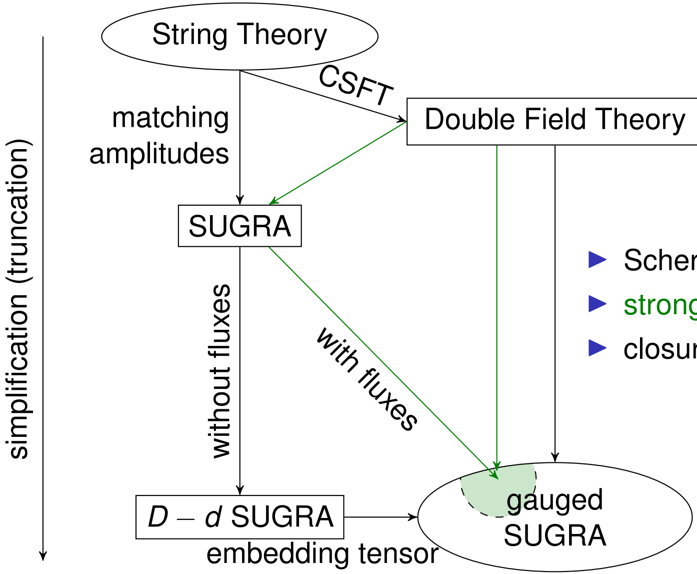

- Scherk-Schwarz
- strong constraint
- closure constraint

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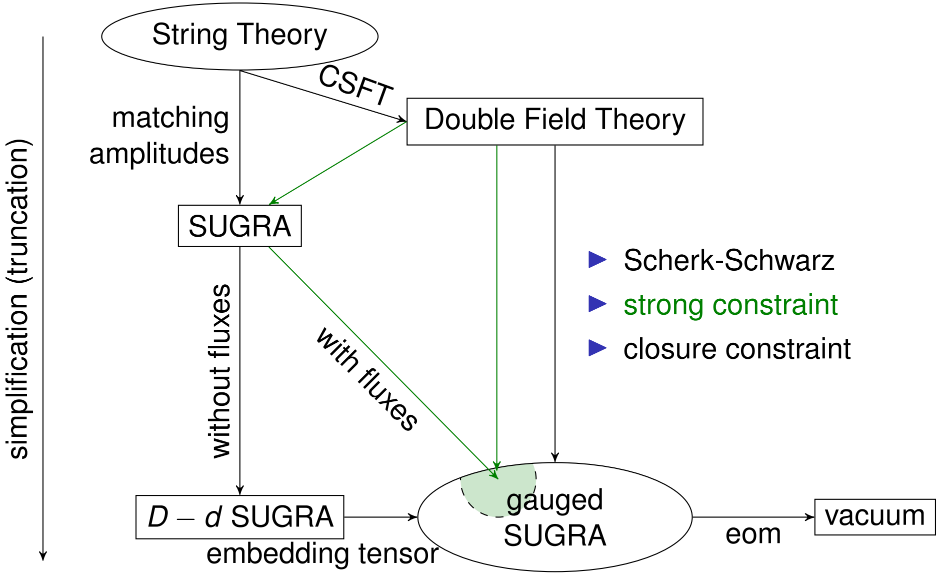

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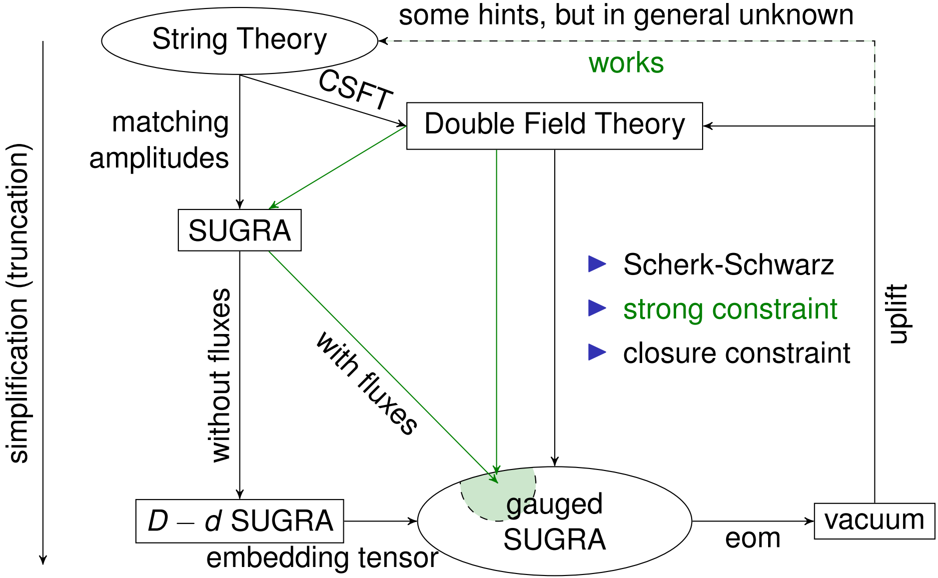

The compactification ansatz

- internal coordinates y, external coordinates x

$$\mathcal{H}_{IJ}(x,y) = E^A{}_I(y)\mathcal{H}_{AB}(x)E^B{}_J(y)$$

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1. $E_A{}^I \eta_{IJ} E_B{}^J = \eta_{AB} =$ constant
2. frame algebra

$$\mathcal{L}_{E_A} E_B{}^I = F_{AB}{}^C E_C{}^I$$

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- $F_{AB}{}^C$ is the embedding tensor; embeds gauge group $G \hookrightarrow$ O(d,d)
- ansatz is consistent
- remaining challenge:

find one E_A (unique?) for each $F_{AB}{}^C$

The solution

- the same structure as on the Poisson-Lie T-duality
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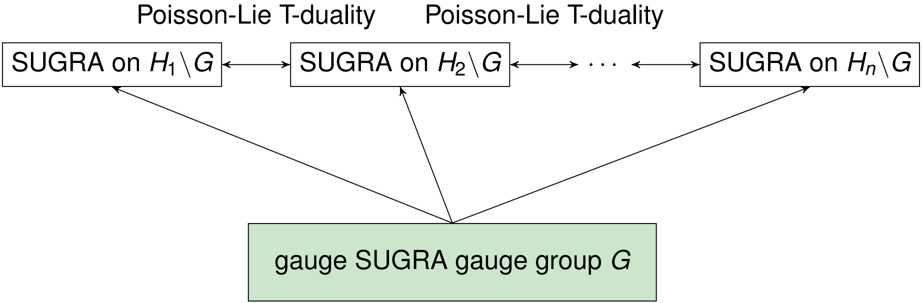

Dictionary

worldsheet		target space
bosonic closed string	⟷	NS/NS sector of SUGRA
$\mathcal{E}$-model	⟷	Double Field Theory
renormalizable	⟷	consistent trunction
Poisson-Lie T-duality	⟷	different uplifts
Green-Schwarz superstring	⇠ - - - - - ⇢	R/R sector
integrablitly		?
q-deformed symmetry		?
?		Exceptional Field Theory

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- TODO: explicit construction

Poisson-Lie T-duality

- What is Poisson-Lie T-duality?
- How does it connects to consistent truncations?

Two-dimensional σ-model: Lagrangian and Hamiltonian

- action $S = \frac{1}{2} \int \mathrm{d}^2\sigma \sqrt{-h} h^{\mu\nu} \partial_\mu X^i (G_{ij} + B_{ij}) \partial_\nu X^j$

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- alternatively use momentum

  $$\Pi_i = G_{ij}\partial_\tau X^j + B_{ij}\partial_\sigma X^j$$

  to write

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- with the Hamiltonian

  $$\mathrm{Ham}(X, \Pi) = \frac{1}{2}\int \mathrm{d}\sigma \, \begin{pmatrix} \partial_\sigma X & \Pi \end{pmatrix} \underbrace{\begin{pmatrix} G - BG^{-1}B & BG^{-1} \\ -G^{-1}B & G^{-1} \end{pmatrix}}_{\text{generalized metric } \mathcal{H}} \begin{pmatrix} \partial_\sigma X \\ \Pi \end{pmatrix}$$

[Tseytlin, 1990,Tseytlin, 1991]

Dynamics in the first order formulation

- time evolution for observable $\frac{\mathrm{d}}{\mathrm{d}\tau} f(X, \Pi) = \{f, \mathrm{Ham}\}$
- we need Poisson brackets

$$\{X^i(\sigma), X^j(\sigma')\} = 0$$

$$\{X^i(\sigma), \Pi_j(\sigma')\} = \delta^i_j \delta(\sigma - \sigma')$$

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When is it possible to

1. make the Hamiltonian quadratic
2. while keeping the "simple" Poisson brackets?

Current algebra. . .

- naive answer $\mathcal{H}$ is constant (independent of X)
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1. define the current $J_I = \begin{pmatrix} \partial_\sigma X^i & \Pi_i \end{pmatrix}$ resulting with bracket

$$\{J_I(\sigma), J_J(\sigma')\} = \eta_{IJ}\delta'(\sigma - \sigma')$$

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2. use ${E_A}^I(X)$ to transform

$$J'_A = {E_A}^I J_I\,, \quad \eta_{AB} = {E_A}^I \eta_{IJ} {E_B}^J\,, \quad \mathcal{H}_{AB} = {E_A}^I \mathcal{H}_{IJ} {E_B}^J$$

...and its transformation [Alekseev and Strobl, 2005]

- then we get the brackets

$$\{J_A(\sigma), J_B(\sigma')\} = F_{AB}{}^C J_C \delta(\sigma - \sigma') + \eta_{AB} \delta'(\sigma - \sigma')$$

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- the generalized Lie derivative

$$\mathcal{L}_{(x\quad \phi)}(y\quad \xi) = \left([x, y]_{\mathrm{Lie}}\quad L_x\xi - L_y\phi + \iota_y \mathrm{d}\phi\right)$$

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- use Lie group element $g \in D$ generated by $\mathfrak{d}$ to write

$$J_A = \langle T_A, g^{-1} \partial_\sigma g \rangle$$

$$\mathrm{Ham} = \frac{1}{2} \int \mathrm{d}\sigma \, \langle g^{-1} \partial_\sigma g, \mathcal{E} g^{-1} \partial_\sigma g \rangle \qquad \mathcal{H}_{AB} = \langle T_A, \mathcal{E} T_B \rangle$$

- coined as $\mathcal{E}$-model [Klimcik and Severa, 1996,Klimcik and Severa, 1996,Klimcik, 2015]

Poisson-Lie T-duality [Klimcik and Severa, 1995,Klimcik and Severa, 1996]

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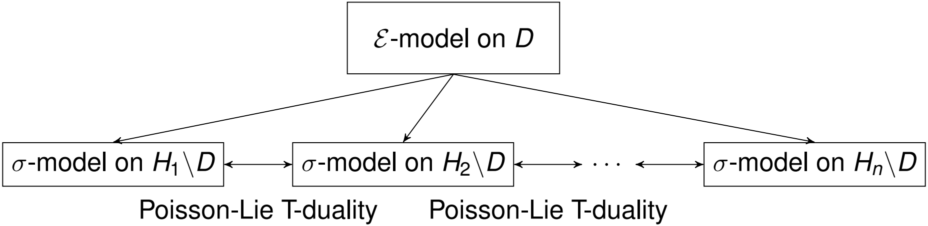
- physical target space $M = H \backslash D$
- in general different ways to choose H



Open questions

- complete the dictionary
- extension the Exceptional Field Theory
- include higher derivative corrections
- discuss branes
- what is the fade of supersymmetry
- applications to AdS/CFT

There is an intriguing web of relations between

Poisson-Lie symmetry, *integrable deformations* and *(g)SUGRA*.

It is quite likely the it will give rise to more interesting results in the future. Existing insights in one of them can lead to a better understanding of the others.