String Geometry Beyond the Torus

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based on arXiv:1410.6374 with

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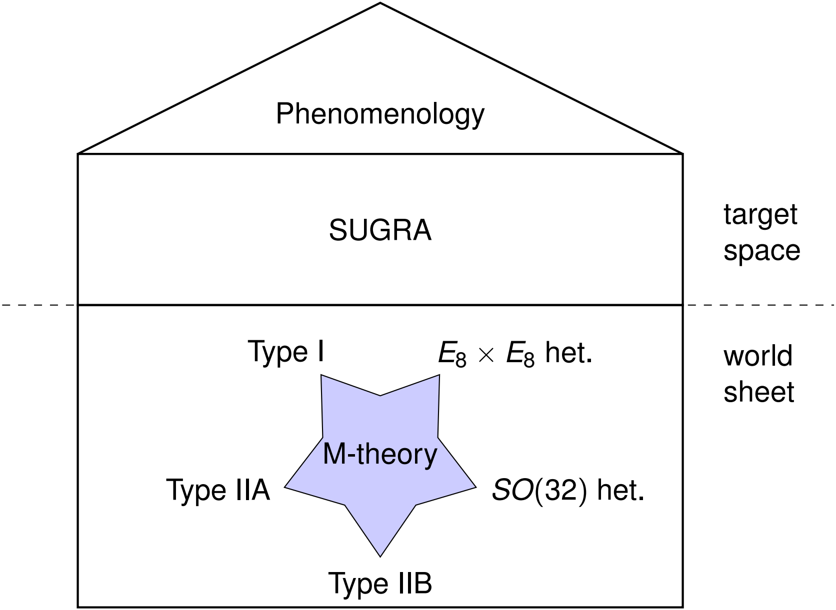
Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

The big picture



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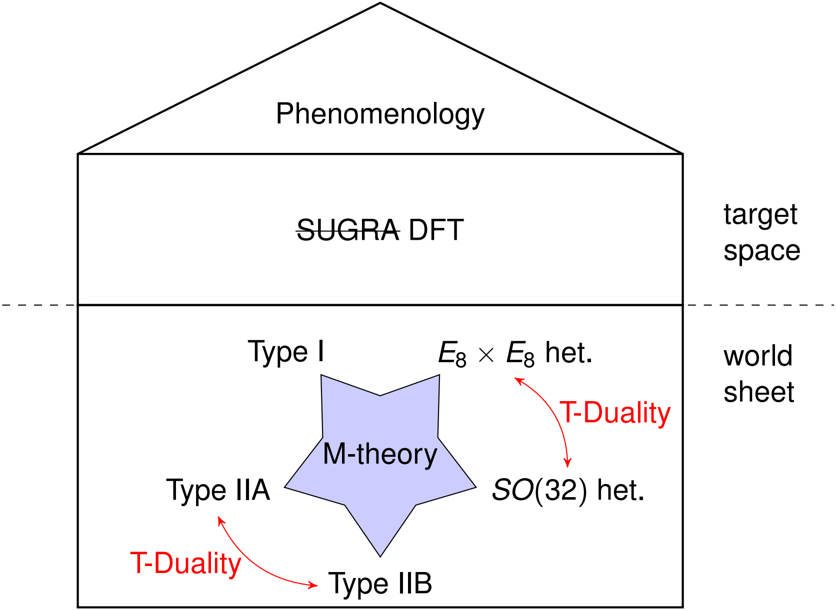

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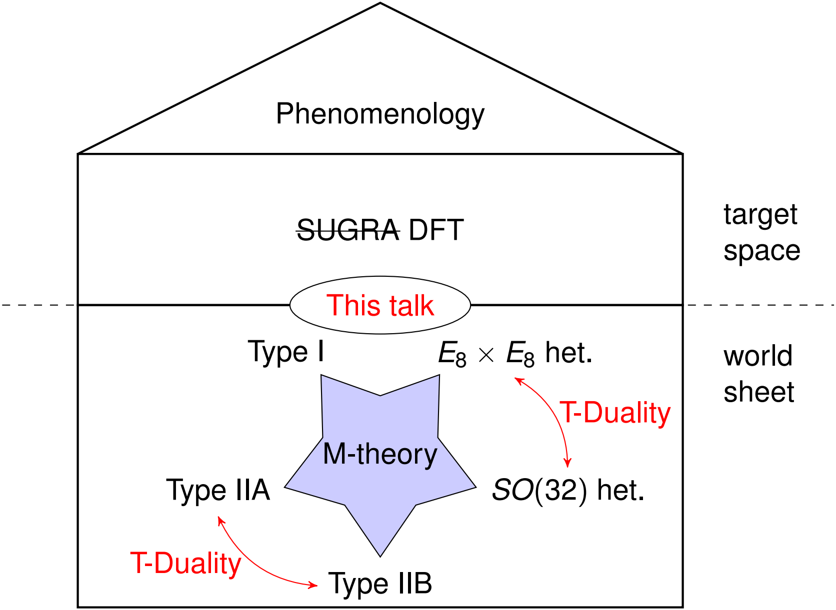


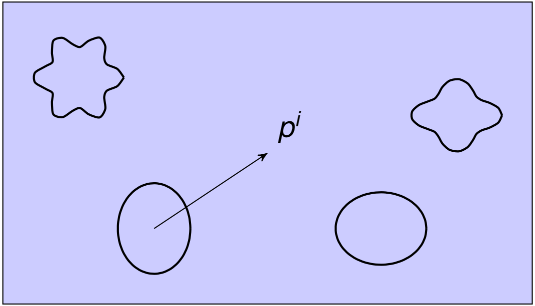
Outline

1. SUGRA & DFT in a nutshell
2. String geometry by violating the strong constraint
3. Deriving DFT_{WZW} from CSFT
4. Applications
5. Summary and outlook

SUGRA

- closed strings in D-dim. flat space
- truncate all massive excitations
- match scattering amplitudes of strings with EFT

$$S_{\text{NS}} = \int \mathrm{d}^D x \, \sqrt{g} e^{-2\phi} \left(\mathcal{R} + 4\partial_i \phi \partial^i \phi - \frac{1}{12} H_{ijk} H^{ijk} \right)$$



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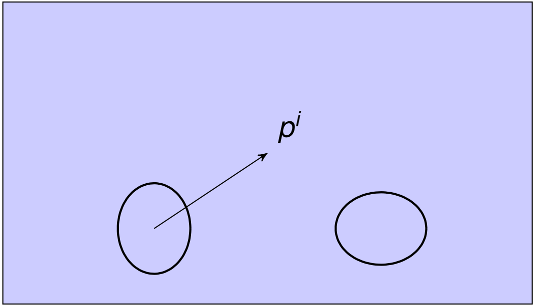

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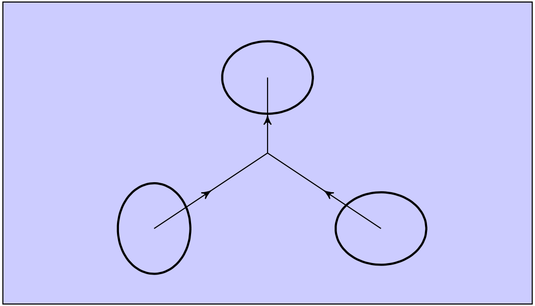
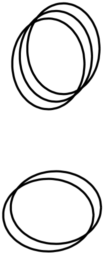




$g_{ij}(p^k)$

$\phi(p^i)$

Manifest & hidden symmetries

- S_{NS} = action for NS/NS sector of Type IIA and Type IIB
- manifest invariant under

diffeomorphisms	$g_{ij} = \mathcal{L}_\xi g_{ij}$
gauge transformations	$B_{ij} = \mathcal{L}_\xi B_{ij} + \partial_i \alpha_j - \partial_j \alpha_i$

- compactification on circle $\rightarrow U(1)$ isometry
- Buscher rules implement T-duality []

$$\tilde{g}_{\theta\theta} = \frac{1}{g_{\theta\theta}}\,, \quad \tilde{g}_{\theta i} = \frac{1}{g_{\theta\theta}} B_{\theta i}\,, \quad \tilde{g}_{ij} = g_{ij} + \frac{1}{g_{\theta\theta}}(g_{\theta i} g_{\theta j} - B_{\theta i} B_{\theta j})\,, \ldots$$

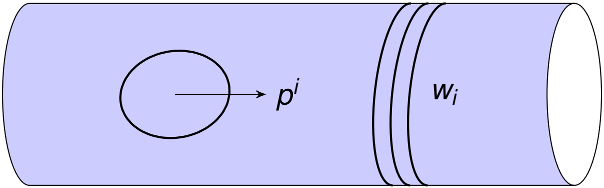
from NS/NS sector of IIA to IIB
- T-duality is a hidden symmetry

DFT (Double Field Theory) [,,]

- closed strings on a flat torus
- combine conjugated variables x_i and $\tilde{x}^i$ into $X^M = (\tilde{x}_i \quad x^i)$
- repeat steps from SUGRA derivation

$$S_{\mathrm{DFT}} = \int \mathrm{d}^{2D} X \, e^{-2d} \mathcal{R}(\mathcal{H}_{MN}, d)$$

- fields are constrained by strong constraint $\boxed{\partial_M \partial^M \cdot = 0}$



DFT (Double Field Theory) [,,]

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$$\mathcal{R} = 4\mathcal{H}^{MN}\partial_M\partial_N d - \partial_M\partial_N\mathcal{H}^{MN} - 4\mathcal{H}^{MN}\partial_M d\partial_N d + 4\partial_M\mathcal{H}^{MN}\partial_N d$$
$$+\frac{1}{8}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\partial_N\mathcal{H}_{KL} - \frac{1}{2}\mathcal{H}^{MN}\partial_N\mathcal{H}^{KL}\partial_L\mathcal{H}_{MK}$$

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$$\mathcal{H}^{MN} = \begin{pmatrix} g_{ij} - B_{ik}g^{kl}B_{lj} & -B_{ik}g^{kj} \\ g^{ik}B_{kj} & g^{ij} \end{pmatrix} \in O(D,D) \to \text{T-duality}$$

DFT (Double Field Theory) [,,]

- lower/raise indices with $\eta_{MN} = \begin{pmatrix} 0 & \delta^i_j \\ \delta_i^j & 0 \end{pmatrix}$ and $\eta^{MN} = \begin{pmatrix} 0 & \delta_i^j \\ \delta^i_j & 0 \end{pmatrix}$

$$X^M = \begin{pmatrix} \tilde{x}_i & x^i \end{pmatrix}$$

$$d = \phi - \tfrac{1}{2} \log \sqrt{g}$$

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Gauge transformations []

- generalized Lie derivative combines

  1. diffeomorphisms
  2. *B*-field gauge transformations
  3. β-field gauge transformations

  (1. and 2.: available in SUGRA)

$$\mathcal{L}_\lambda \mathcal{H}^{MN} = \lambda^P \partial_P \mathcal{H}^{MN} + (\partial^M \lambda_P - \partial_P \lambda^M)\mathcal{H}^{PN} + (\partial^N \lambda_P - \partial_P \lambda^N)\mathcal{H}^{MP}$$

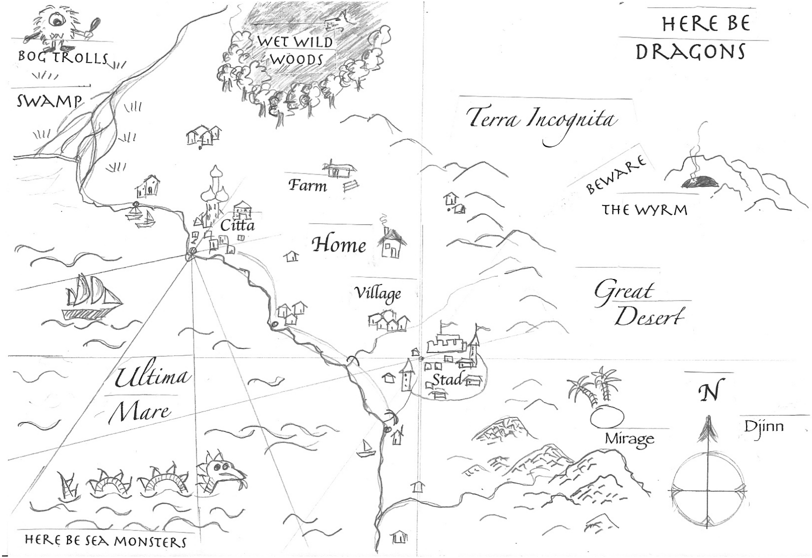
$$\mathcal{L}_\lambda d = \lambda^M \partial_M d + \frac{1}{2}\partial_M \lambda^M$$

- closure of algebra

$$\mathcal{L}_{\lambda_1}\mathcal{L}_{\lambda_2} - \mathcal{L}_{\lambda_2}\mathcal{L}_{\lambda_1} = \mathcal{L}_{\lambda_{12}} \quad \text{with} \quad \lambda_{12} = [\lambda_1, \lambda_2]_\mathrm{C}$$

- only if strong constraint holds

A landscape of string backgrounds [,]



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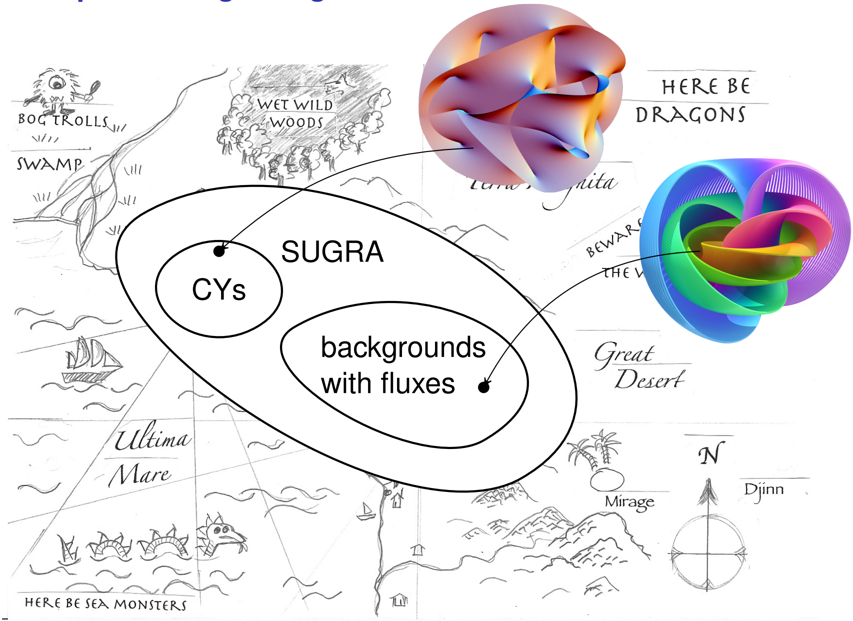

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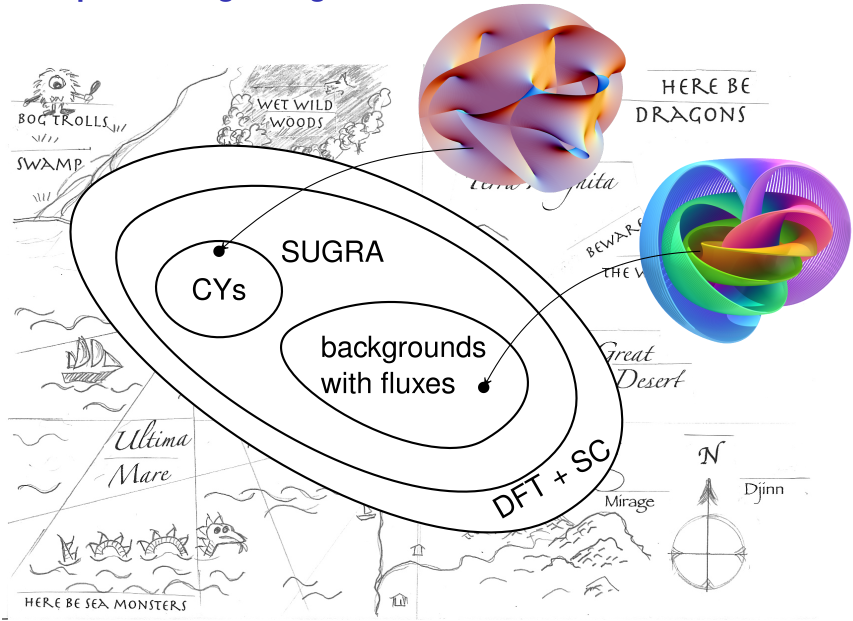

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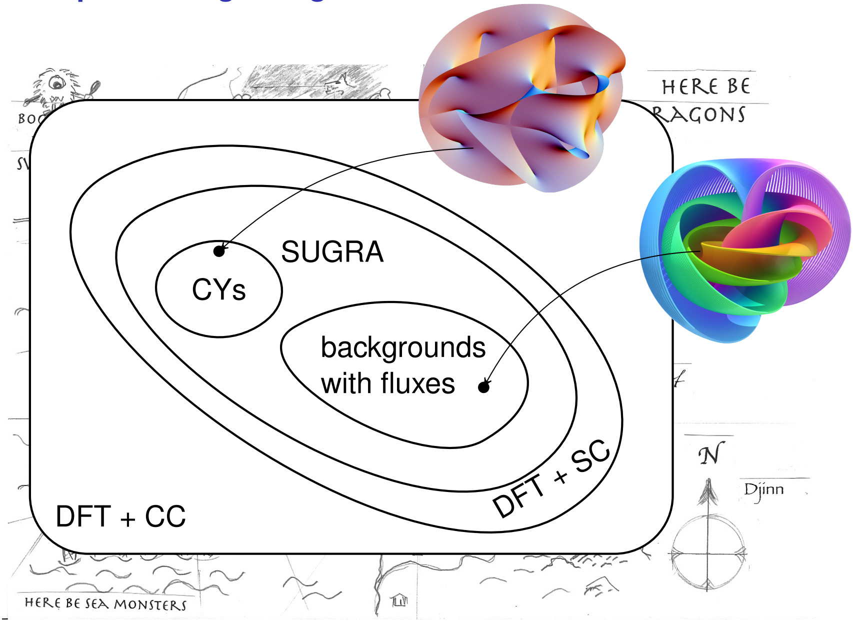

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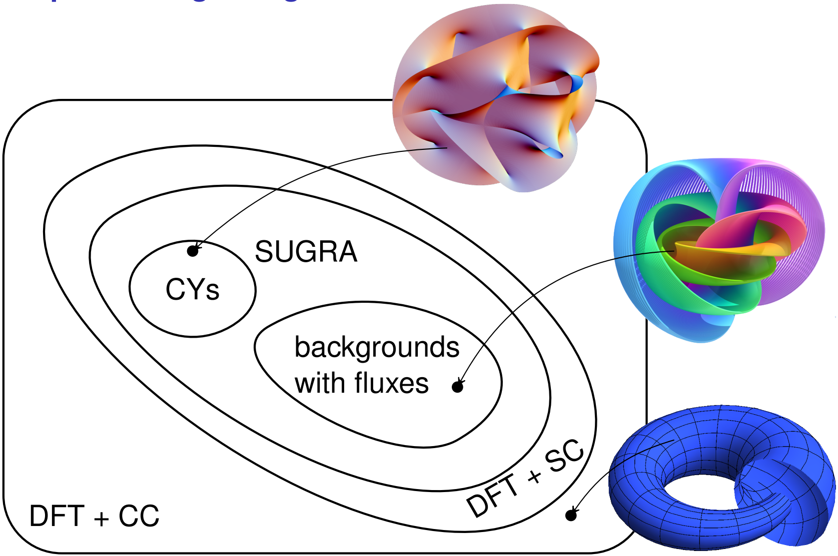

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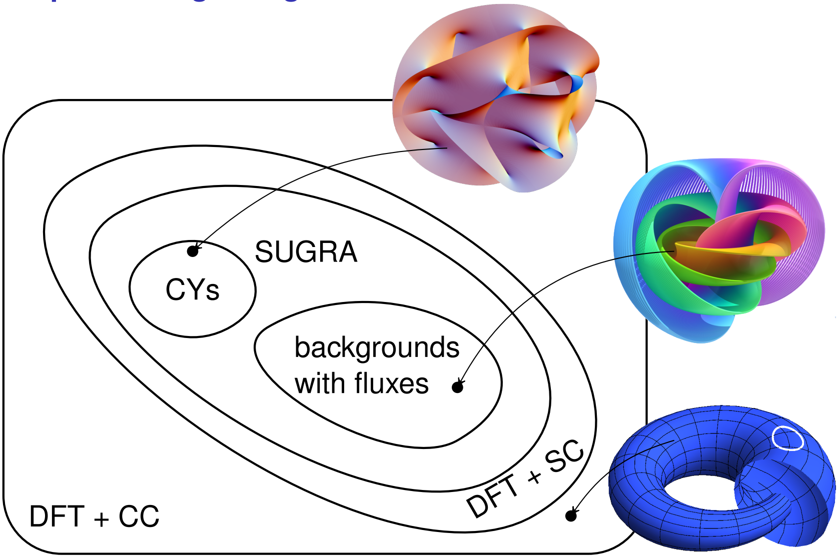

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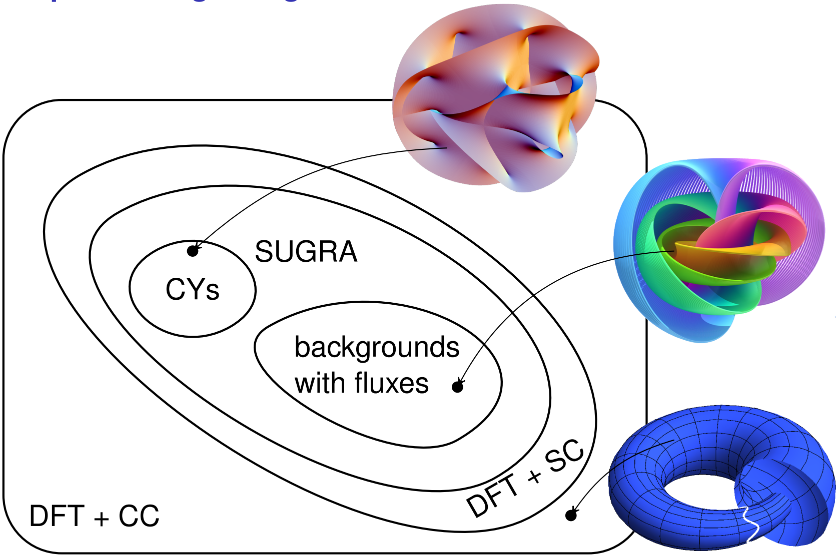

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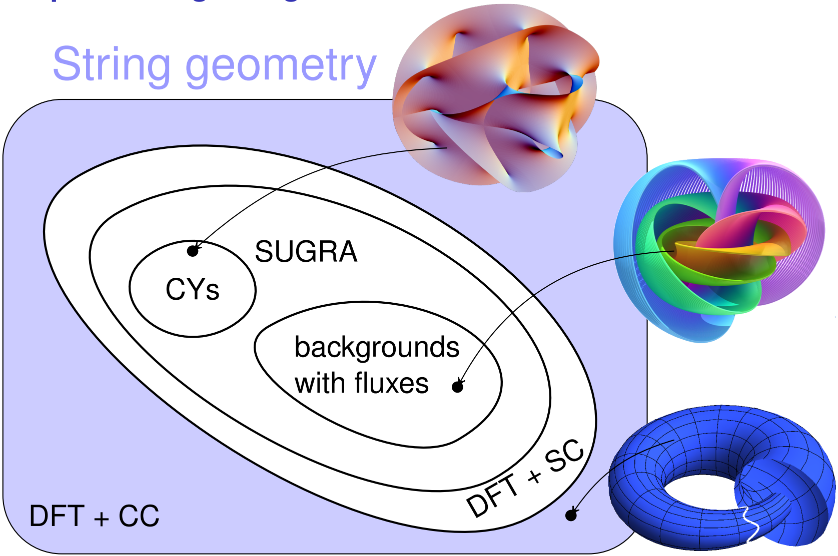
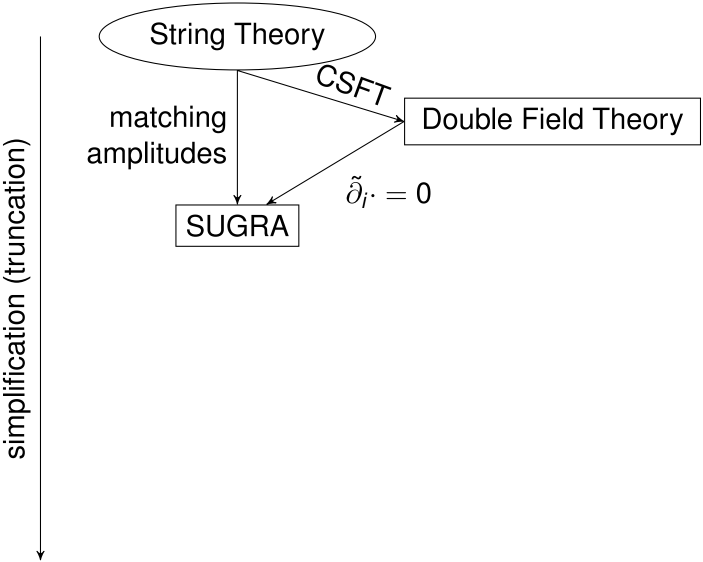


[,,]

Generalized Scherk-Schwarz compactification

[,]



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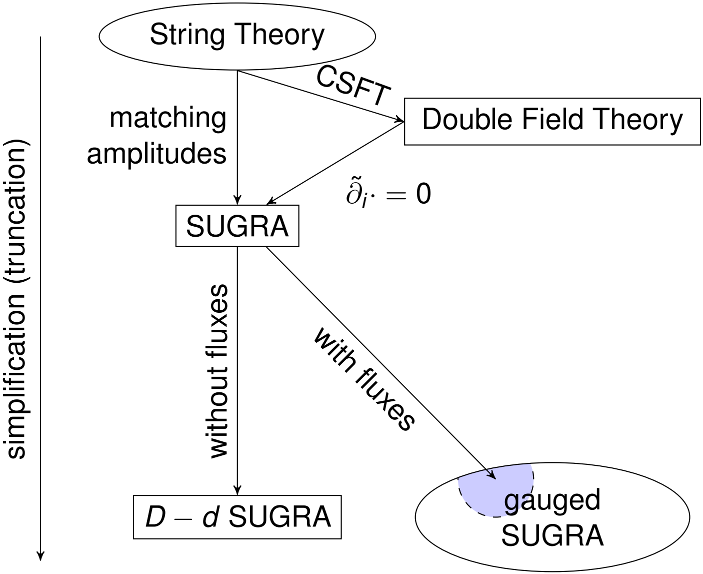

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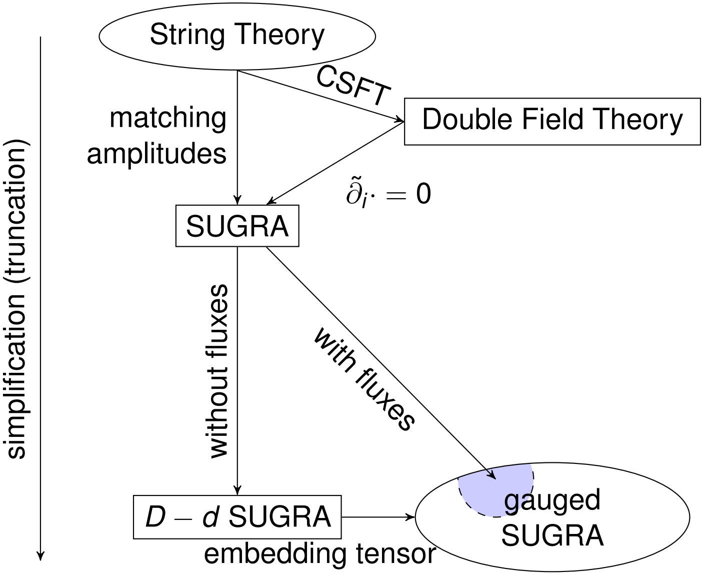

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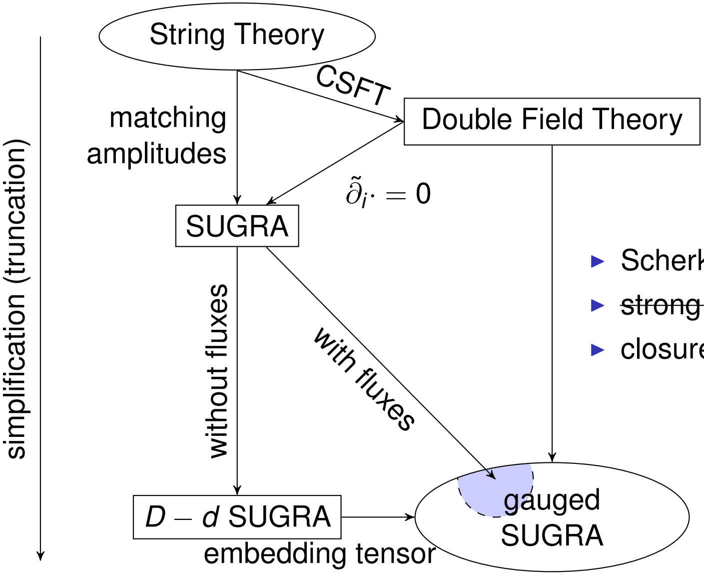

- Scherk-Schwarz
- ~~strong constraint~~
- closure constraint

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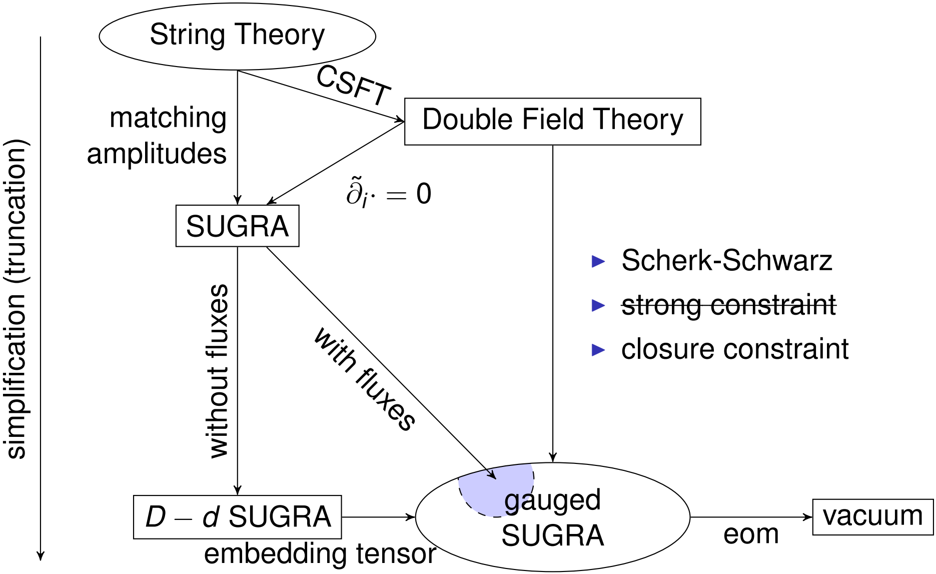

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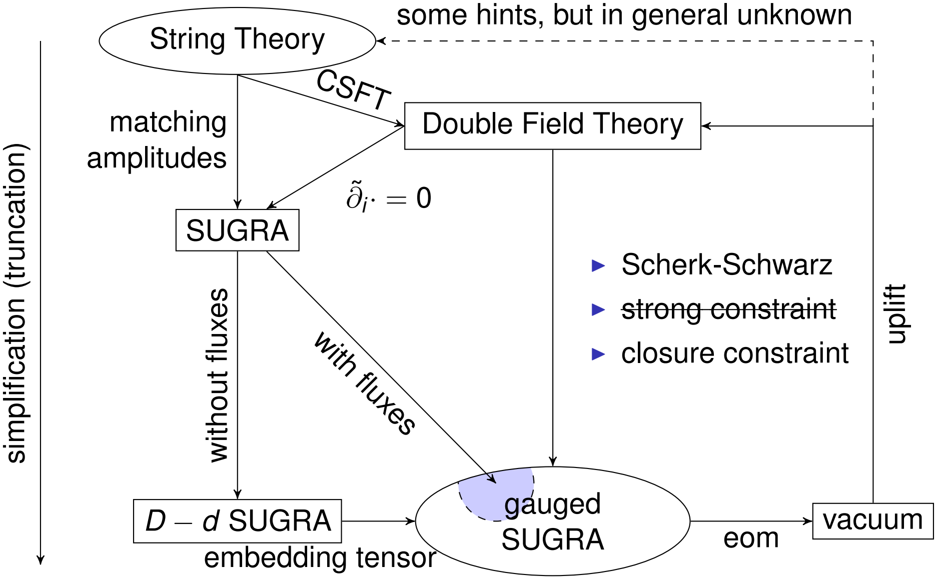

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DFT on group manifolds = DFT$_{\mathrm{WZW}}$

Use group manifold instead of a torus to derive DFT!

+ non-abelian gauge groups
+ cosmological constant
+ flux backgrounds with const. fluxes

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Questions about DFT_{WZW}

- What are the covariant objects?
- Does it make non-abelian duality manifest?
- How is it connected to DFT?

} not trivial

WZW model & Kač-Moody algebra [,]

- $g \in G$, a compact simply connected Lie group

$$S_{\mathrm{WZW}} = \frac{1}{2\pi\alpha'} \int_M d^2z \mathcal{K}(g^{-1}\partial g, g^{-1}\bar{\partial} g) + S_{\mathrm{WZ}}(g)$$

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- metric and 3-form flux in flat indices

$$\eta_{ab} := \mathcal{K}(t_a, t_b) \quad \text{and} \quad F_{abc} := \mathcal{K}([t_a, t_b], t_c)$$

- D chiral and D anti-chiral Noether currents (=2D indep. currents)

$$j_a(z) = \frac{2}{\alpha'}\mathcal{K}(\partial g g^{-1}, t_a) \quad \text{and} \quad j_{\bar{a}}(\bar{z}) = -\frac{2}{\alpha'}\mathcal{K}(g^{-1}\bar{\partial} g, t_{\bar{a}})$$

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- radial quantization

$$\boxed{j_a(z) j_b(w) = -\frac{\alpha'}{2}\frac{1}{(z-w)^2}\eta_{ab} + \frac{1}{z-w} F_{ab}{}^c j_c(z) + \dots}$$

Action

- tree level action in CSFT []

$$(2\kappa^2)S = \frac{2}{\alpha'}\left(\langle\Psi|c_0^- Q|\Psi\rangle + \frac{1}{3}\{\Psi,\Psi,\Psi\}_0 + \dots\right)$$

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- string field for massless excitations []

$$|\Psi\rangle = \sum_R \Big[\frac{\alpha'}{4}\epsilon^{a\bar{b}}(R) j_{a\,-1} j_{\bar{b}\,-1} c_1 \bar{c}_1 + e(R) c_1 c_{-1} + \bar{e}(R)\bar{c}_1 \bar{c}_{-1} + \\ \frac{\alpha'}{2}\left(f^a(R) c_0^+ c_1 j_{a\,-1} + f^{\bar{b}}(R) c_0^+ \bar{c}_1 j_{\bar{b}\,-1}\right)\Big]|\phi_R\rangle$$

- R is highest weight of $\mathfrak{g}\times\mathfrak{g}$ representation

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- R is highest weight of $\mathfrak{g}\times\mathfrak{g}$ representation
- BRST operator (L_m from Sugawara construction)

$$Q = \sum_m \left(: c_{-m} L_m : + \frac{1}{2} : c_{-m} L_m^{gh} :\right) + \text{anti-chiral}$$

Geometric representation of primary fields ($k \to \infty$)

- flat derivative $\boxed{D_a = e_a{}^i \partial_i \quad \text{with} \quad e_a{}^i = \mathcal{K}(g^{-1}\partial^i g, t_a)}$

operator algebra	geometry ($j_{a\,0} \to D_a$)
$L_0 \vert\phi_R\rangle = j_{a\,0} j_0^a \vert\phi_R\rangle = h_R \vert\phi_R\rangle$	$D_a D^a Y_R(x^i) = h_R Y_R(x^i)$
$[j_{a\,0}, j_{b\,0}] = F_{ab}{}^c j_{c\,0}$	$[D_a, D_b] = F_{ab}{}^c D_c$
$\sum_R e(R) \vert\phi_R\rangle$	$\sum_R e(R) Y_R(x^i) := e(x^i)$

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$\sum_R e(R) \lvert\phi_R\rangle$	$\sum_R e(R) Y_R(x^i) := e(x^i)$

$$E_A{}^I = \begin{pmatrix} e_a{}^i & 0 \\ 0 & e_{\bar a}{}^{\bar i} \end{pmatrix} \qquad S_{AB} = \begin{pmatrix} \eta_{ab} & 0 \\ 0 & \eta_{\bar a \bar b} \end{pmatrix} \qquad \eta_{AB} = \begin{pmatrix} \eta_{ab} & 0 \\ 0 & -\eta_{\bar a \bar b} \end{pmatrix}$$

Weak constraint (level matching)

- level matched string field $(L_0 - \bar{L}_0)|\Psi\rangle = 0$ requires

$$(D_a D^a - D_{\bar{a}} D^{\bar{a}}) \cdot = 0 \quad \text{with} \quad \cdot \in \{\epsilon^{a\bar{b}}, e, \bar{e}, f^a, f^{\bar{b}}\}$$

- rewritten in terms of η^{AB} and $D_A = \begin{pmatrix} D_a & D_{\bar{a}} \end{pmatrix}$

$$\eta^{AB} D_A D_B \cdot = D_A D^A \cdot = 0 \qquad \text{compare with} \qquad \partial_M \partial^M \cdot = 0$$

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- change to curved indices using $E_A{}^M$

$$(\partial_M \partial^M - 2\partial_M d \partial^M) \cdot = 0 \qquad \text{with} \qquad d = \phi - \frac{1}{2} \log \sqrt{g}$$

- additional term which is absent in DFT $\rightarrow$ adsorb in cov. derivative

$$\boxed{\nabla_M \partial^M \cdot = 0} \quad \text{with} \quad \nabla_M V^N = \partial_M V^N + \Gamma_{MK}{}^N V^K \,, \quad \Gamma_{MK}{}^M = -2\partial_K d$$

Results (leading order k^{-1})

- calculate quadratic and cubic string functions
- integrate out auxiliary fields f^a and $f^{\bar{b}}$
- perform field redefinition

$$
\begin{aligned}
(2\kappa^2)S = \int d^{2D}X\sqrt{H}\,\Big[&\frac{1}{4}\epsilon_{a\bar{b}}\Box\epsilon^{a\bar{b}} + \dots \\
&-\frac{1}{4}\epsilon_{a\bar{b}}\left(F^{ac}{}_{d}\bar{D}^{\bar{e}}\epsilon^{d\bar{b}}\epsilon_{c\bar{e}} + F^{\bar{b}\bar{c}}{}_{\bar{d}}D^{e}\epsilon^{a\bar{d}}\epsilon_{e\bar{c}}\right) \\
&-\frac{1}{12}F^{ace}F^{\bar{b}\bar{d}\bar{f}}\epsilon_{a\bar{b}}\epsilon_{c\bar{d}}\epsilon_{e\bar{f}} + \dots\Big]
\end{aligned}
$$

- additional terms e.g. potential
- vanish in abelian limit $F_{abc} \to 0$ and $F_{\bar{a}\bar{b}\bar{c}} \to 0$

Gauge transformations

- tree level gauge transformation in CSFT []

$$\delta_\Lambda|\Psi\rangle = Q|\Lambda\rangle + [\Lambda, \Psi]_0 + \cdots$$

- string field for gauge parameter []

$$|\Lambda\rangle = \sum_R \Big[\frac{1}{2}\lambda^a(R) j_{a\,-1} c_1 - \frac{1}{2}\lambda^{\bar{b}}(R) j_{\bar{b}\,-1} \bar{c}_1 + \mu(R) c_0^+\Big]|\phi_R\rangle$$

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- after field redefinition and μ gauge fixing

$$\delta_\lambda \epsilon_{a\bar{b}} = D_{\bar{b}}\lambda_a + \frac{1}{2}\big[D_a\lambda^c\epsilon_{c\bar{b}} - D^c\lambda_a\epsilon_{c\bar{b}} + \lambda_c D^c\epsilon_{a\bar{b}} + F_{ac}{}^d\lambda^c\epsilon_{d\bar{b}}\big]$$

$$D_a\lambda_{\bar{b}} + \frac{1}{2}\big[D_{\bar{b}}\lambda^{\bar{c}}\epsilon_{a\bar{c}} - D^{\bar{c}}\lambda_{\bar{b}}\epsilon_{a\bar{c}} + \lambda_{\bar{c}} D^{\bar{c}}\epsilon_{a\bar{b}} + F_{\bar{b}\bar{c}}{}^{\bar{d}}\lambda^{\bar{c}}\epsilon_{a\bar{d}}\big]$$

$$\delta_\lambda d = -\frac{1}{4}D_a\lambda^a + \frac{1}{2}\lambda_a D^a d - \frac{1}{4}D_{\bar{a}}\lambda^{\bar{a}} + \frac{1}{2}\lambda_{\bar{a}} D^{\bar{a}} d$$

Generalized Lie derivative

- "doubled" version of fluctuations $\epsilon^{a\bar{b}}$

$$\epsilon^{AB} = \begin{pmatrix} 0 & -\epsilon^{a\bar{b}} \\ -\epsilon^{\bar{a}b} & 0 \end{pmatrix} \quad \text{with} \quad \epsilon^{a\bar{b}} = (\epsilon^T)^{\bar{b}a}$$

- generate generalized metric

$$\mathcal{H}^{AB} = S^{AB} + \epsilon^{AB} + \frac{1}{2}\epsilon^{AC} S_{CD} \epsilon^{DB} + \cdots = \exp(\epsilon^{AB})$$

with the defining property $\mathcal{H}^{AC}\eta_{CD}\mathcal{H}^{DB} = \eta^{AB}$

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- setting $\delta_\lambda S^{AB} := 0$ and using

$$\delta_\lambda \epsilon^{AB} = \frac{1}{2}\left(\mathcal{L}_\lambda S^{AB} + \mathcal{L}_\lambda \epsilon^{AB} + \mathcal{L}_\lambda S^{(A}{}_C S^{B)}{}_D \epsilon^{CD}\right) .$$

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Gauge algebra

- CSFT to cubic order fulfills

$$\delta_{\Lambda_1}\delta_{\Lambda_2} - \delta_{\Lambda_2}\delta_{\Lambda_1} = \delta_{\Lambda_{12}} \quad \text{with} \quad \Lambda_{12} = [\Lambda_2, \Lambda_1]_0$$

- after field redefinition and μ fixing $\lambda_{12}^A = \frac{1}{2}[\lambda_2, \lambda_1]_{\mathrm{C}}^A$ with

$$[\lambda_1, \lambda_2]_C^A = \lambda_1^B \nabla_B \lambda_2^A - \frac{1}{2}\lambda_1^B \nabla^A \lambda_{2\,B} - (1 \leftrightarrow 2)$$

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- algebra closes up to a trivial gauge transformation if
  1. fluctuations and parameter fulfill strong constraint $D_A D^A$.
  2. background fulfills closure constraint (CC)

$$F_{E[AB} F^E{}_{C]D} = 0$$

- no strong constraint required for background

Covariant derivative

- non-vanishing torsion and Riemann curvature

$$[\nabla_A, \nabla_B] V_C = R_{ABC}{}^D V_D - T^D{}_{AB} \nabla_D V_C \quad \text{with}$$

$$T^A{}_{BC} = -\frac{1}{3} F^A{}_{BC} \quad \text{and} \quad R_{ABC}{}^D = \frac{2}{9} F_{AB}{}^E F_{EC}{}^D$$

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$$\int d^{2D} X e^{-2d} U \nabla_M V^M = - \int d^{2D} X e^{-2d} \nabla_M U V^M$$

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Comparison DFT and DFT$_{\mathrm{WZW}}$

	DFT	DFT$_{\mathrm{WZW}}$
background	torus	group manifold

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$[\lambda_1, \lambda_2]^I_{\text{C}} =$	$\lambda^J_{[1} \partial_J \lambda^I_{2]} - \frac{1}{2} \lambda^J_{[1} \partial^I \lambda_{2]\,J}$	$\lambda^J_{[1} \nabla_J \lambda^I_{2]} - \frac{1}{2} \lambda^J_{[1} \nabla^I \lambda_{2]\,J}$

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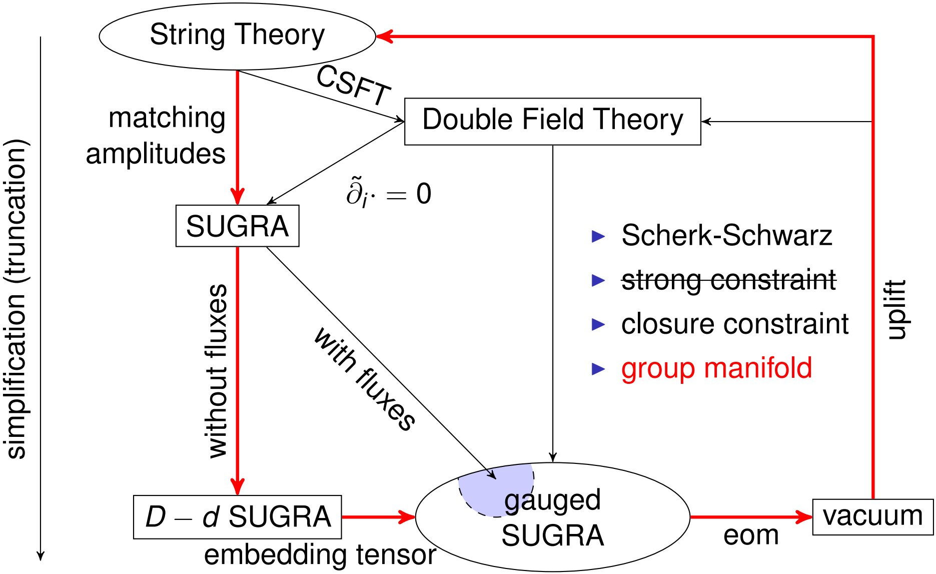
abelian limit

Reminder: Generalized Scherk-Schwarz compactification



- Scherk-Schwarz
- ~~strong constraint~~
- closure constraint
- group manifold

Embedding tensor

ID	$M_{mn}/\ \cos\alpha$	$\tilde{M}^{mn}/\ \sin\alpha$	range of α	gauging
1	diag$(1,1,1,1)$	diag$(1,1,1,1)$	$-\frac{\pi}{4} < \alpha \leq \frac{\pi}{4}$	$\begin{cases} \mathrm{SO}(4)\ , & \alpha \neq \frac{\pi}{4}\ , \\ \mathrm{SO}(3)\ , & \alpha = \frac{\pi}{4}\ . \end{cases}$
2	diag$(1,1,1,-1)$	diag$(1,1,1,-1)$	$-\frac{\pi}{4} < \alpha \leq \frac{\pi}{4}$	SO$(3,1)$
[illegible]	$(1,1$ [illegible]	diag$($ [illegible] $1)$	[illegible] $< \frac{\pi}{4}$	SO(2,2) , α [illegible] $\frac{\pi}{4}$, [illegible]

[]

- fluxes for embedding one

$$F_{abc} = \sqrt{2}\epsilon_{abc}(\cos\alpha + \sin\alpha) \quad \text{and} \quad F_{\bar{a}\bar{b}\bar{c}} = \sqrt{2}\epsilon_{abc}(\cos\alpha - \sin\alpha)$$

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2	diag(1,1,1,−1)	diag(1,1,1,−1)	$-\frac{\pi}{4} < \alpha \leq \frac{\pi}{4}$	SO(3,1)
	(1,1	diag(1 … 1)	$< \frac{\pi}{4}$	O(2,2) , α … $\frac{\pi}{4}$,

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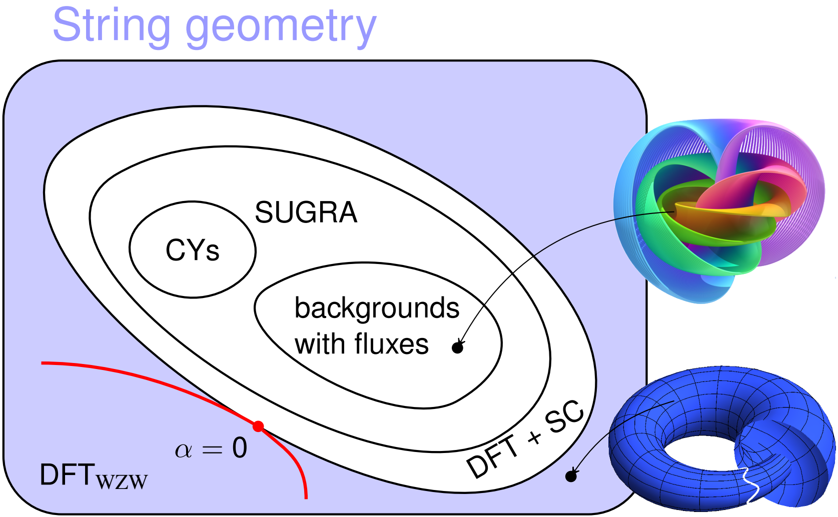
- DFT strong constraint holds only for

$$F_{ABC}F^{ABC} = 24\sin(2\alpha) = 0 \quad \to \alpha = \frac{\pi}{2}n \quad n \in \mathbb{Z}$$

- closure constraint holds always

The landscape again



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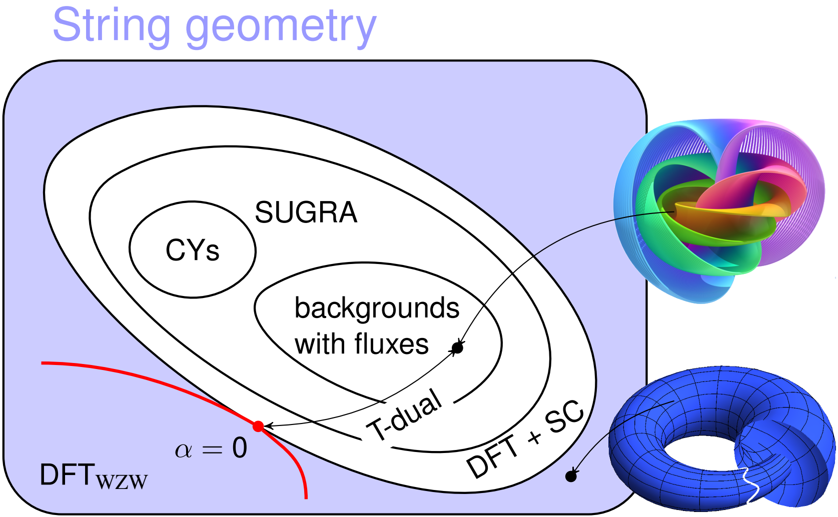

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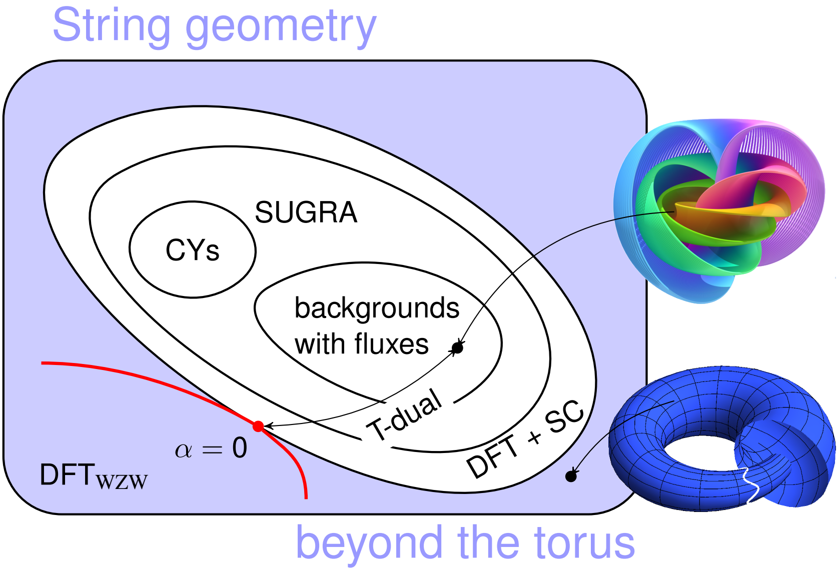

Summary

extended flat space	$\subset$	torus	$\subset$	group manifold
$\downarrow$	decomp. limit	$\downarrow$	abelian limit	$\downarrow$
SUGRA	$\subset$	DFT	?	$\mathrm{DFT}_{\mathrm{WZW}}$

Summary

extended flat space $\subset$ torus $\subset$ group manifold

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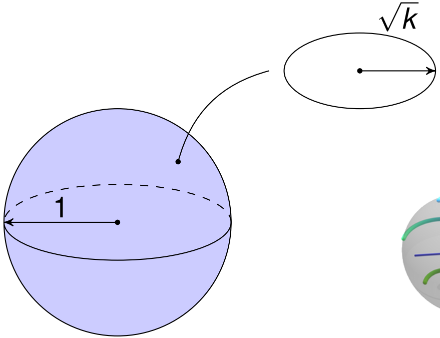
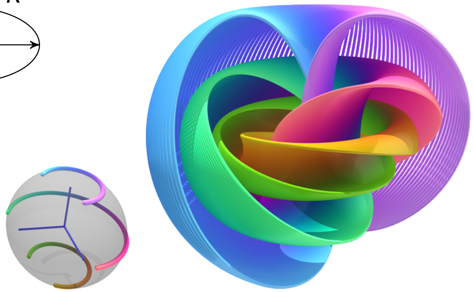
- phenomenology of non-geometric backgrounds []

Thank you for your attention. Are there any questions?

Example $SU(2)_k$ WZW model

- large volume limit $k \to \infty$
- S^3 with radius $\sqrt{k}$
- k units of H-flux $\sim$ volume 3-form of S^3
- Hopf fibration $S^1 \hookrightarrow S^3 \to S^2$

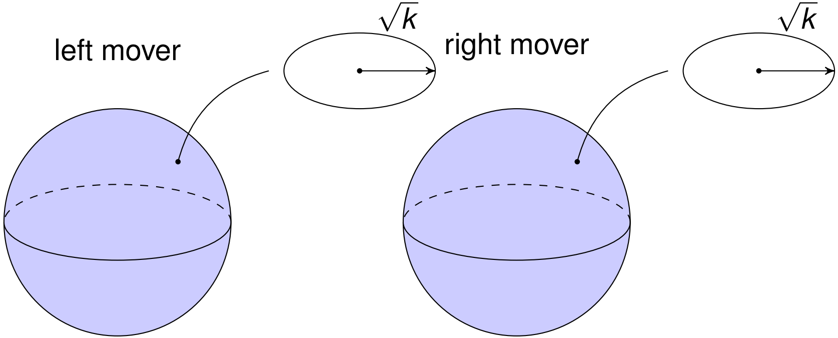




[Niles Johnson]

(T-)Duality [,]

- two copies of S^3 (left and right movers)
- Z_k orbifold of left movers
- still same partition function []



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