Generalized Parallelizable Spaces from Exceptional Group Manifolds

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in collaboration with

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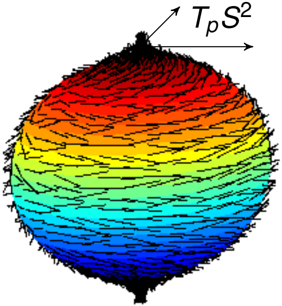
- Scherk-Schwarz compactifications on M do not break any SUSY

S^2 is not parallelizable



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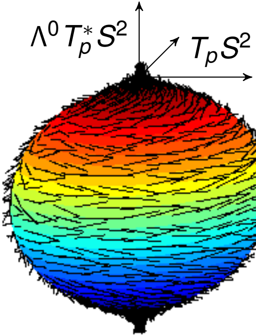

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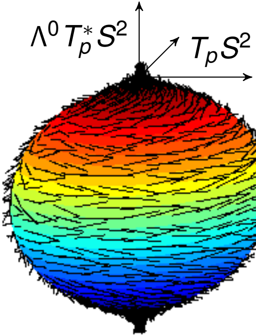

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- all spheres are generalized parallelizable on $TM \oplus \Lambda^{d-2} T^* M$
- generalized frame field $\hat{E}_A$ fulfilling $L_{\hat{E}_A} \hat{E}_B = F_{AB}{}^C \hat{E}_C$
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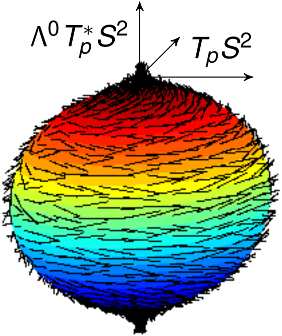

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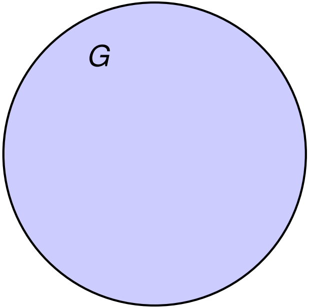
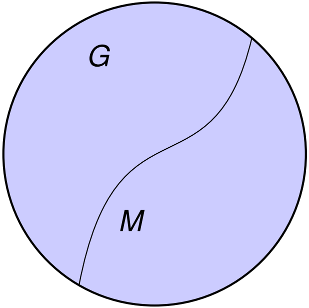
¿ Is there a systematic way to construct them ?



- embed M in group manifold G



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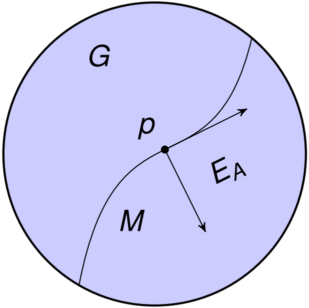

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- vielbein $E_A \in GL(D)$, D=dim G

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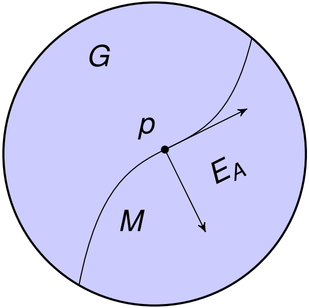

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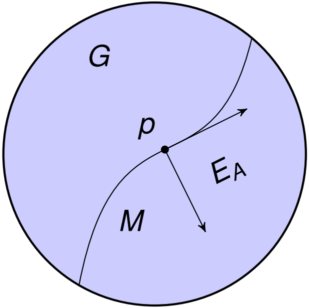
Structures on G:

1. flat der. $D_A V^B = E_A{}^I \partial_I V^B$



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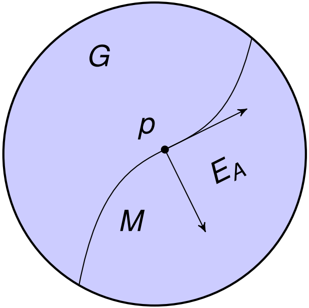

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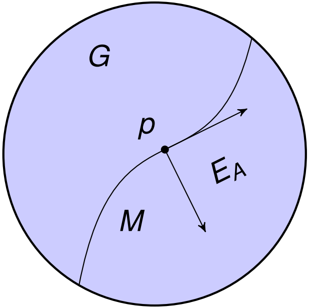
- Which groups G? Which generalized tangent bundle?
- adapt gen. Lie of $E_{d(d)}$ EFT to D-dim. diffeomorphisms of G

$$\mathcal{L}_\xi V^I = \xi^J \partial_J V^I - V^J \partial_J \xi^A + Y^{IJ}{}_{KL} \partial_J \xi^K V^L \,.$$

- closure requires *section condition* $Y^{IJ}{}_{KL} \partial_I \otimes \partial_J = 0$



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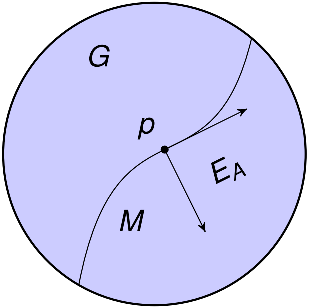

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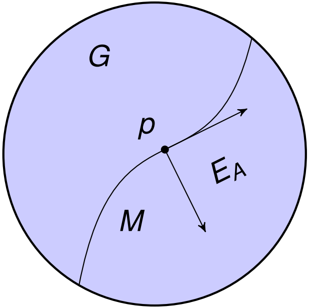
$G \in$ "Exceptional Group Manifold"



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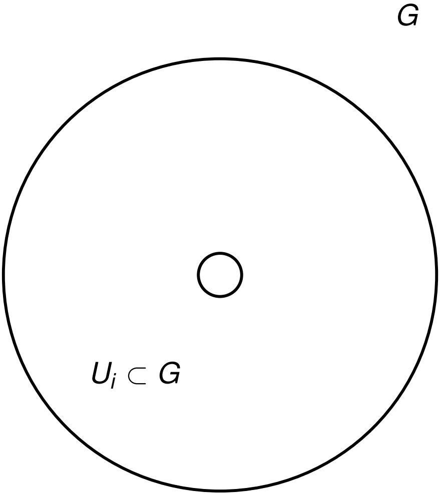
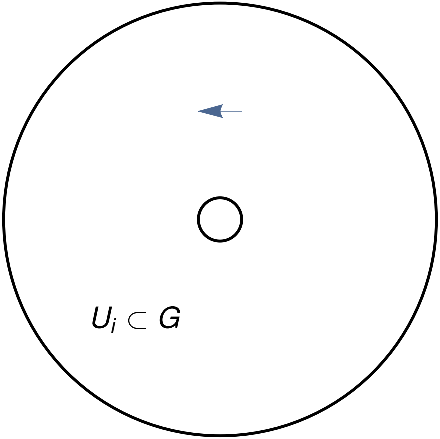
- closure requires *section condition* $Y^{AB}{}_{CD} D_A \otimes D_B = 0$
- embedding tensor classifies all possible groups G

Solving the section condition



Solving the section condition

$$\begin{array}{c} TG \\ \downarrow \\ G \end{array}$$



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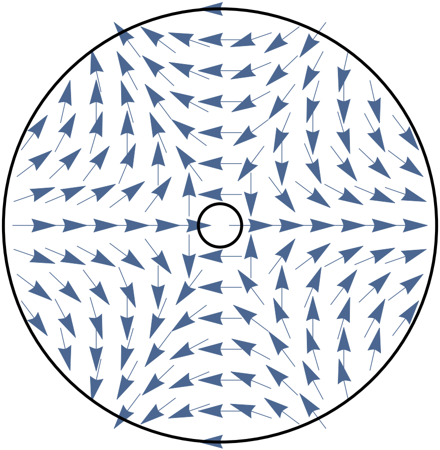
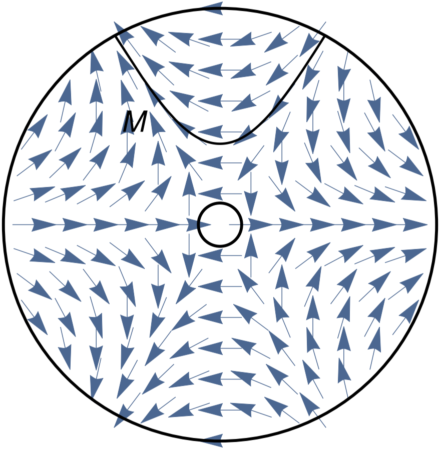


Solving the section condition

$$\begin{array}{ccc} TG & & \\ \downarrow & & \\ G & \underset{\sigma_i}{\overset{\pi}{\rightleftarrows}} & M \end{array}$$



Solving the section condition

$$
\begin{array}{ccc}
TG & \underset{\sigma_{i*}}{\overset{\pi_*}{\rightleftarrows}} & TM \\
\downarrow & & \downarrow \\
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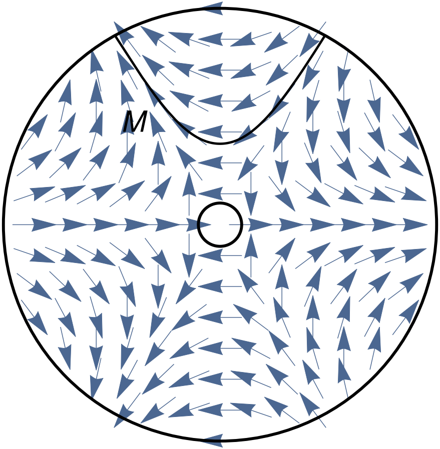

Solving the section condition

$\mathfrak{h}$

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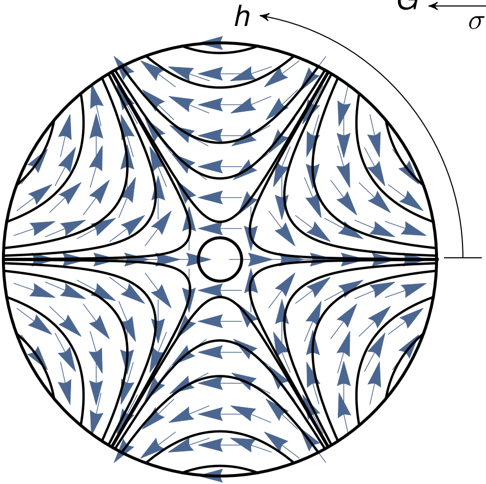
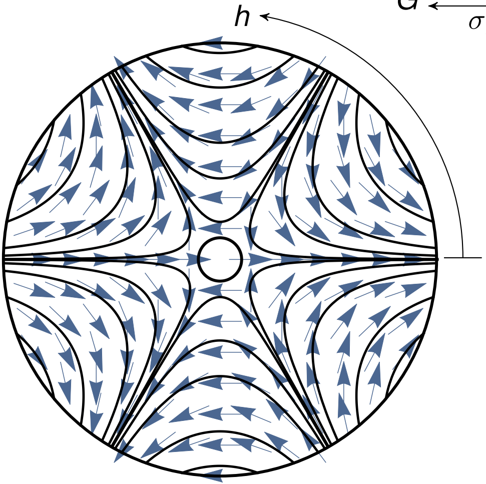


$$\mathfrak{g} = \mathfrak{m} \oplus \mathfrak{h}$$

Solving the section condition

$$\begin{array}{ccccc} \mathfrak{h} & \underset{\omega}{\overset{\sharp}{\rightleftarrows}} & TG & \underset{\sigma_{i*}}{\overset{\pi_*}{\rightleftarrows}} & TM \\ & & \downarrow & & \downarrow \\ & & G & \underset{\sigma_i}{\overset{\pi}{\rightleftarrows}} & M \end{array}$$



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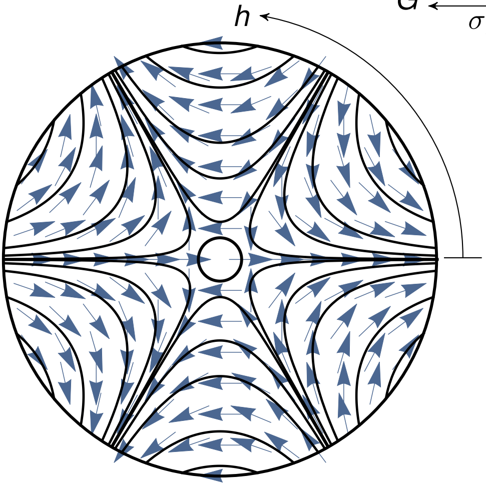

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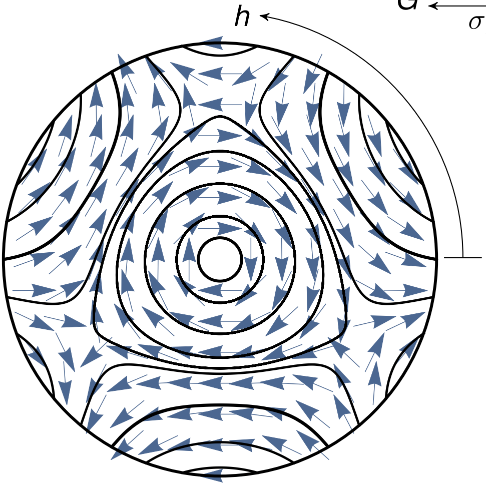

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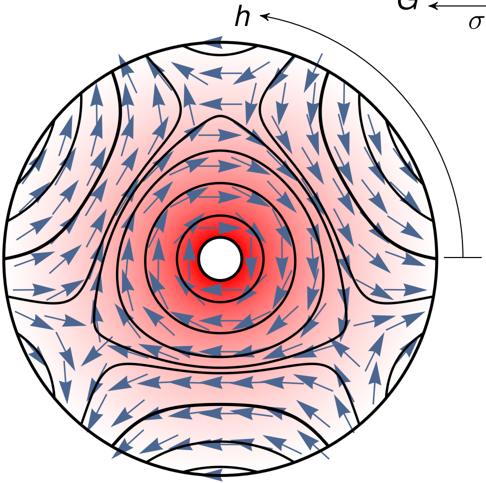

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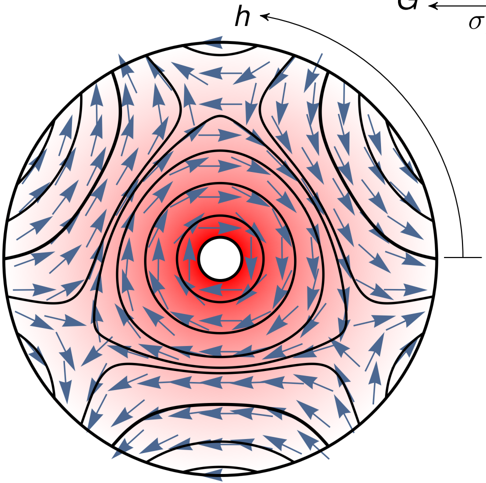

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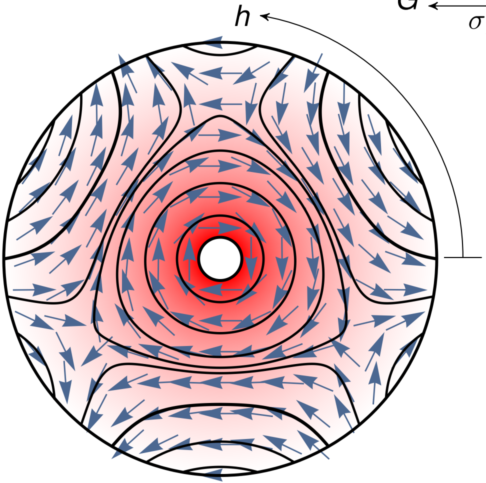

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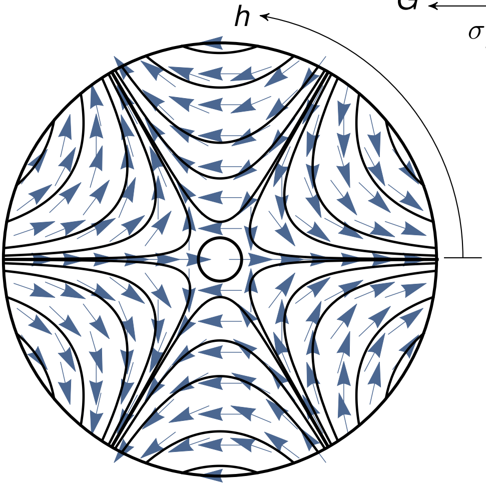

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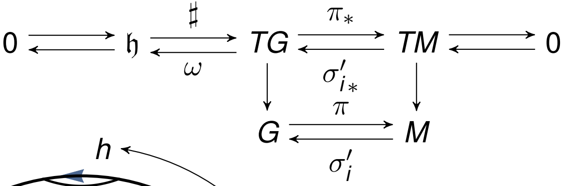

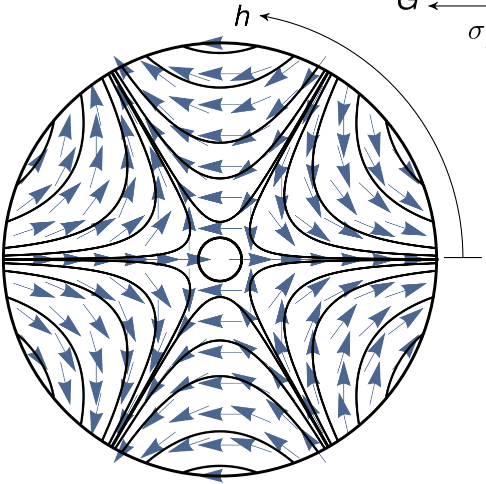

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e.g. for S^4 isomorphisms
$\eta : \mathfrak{h} \to \Lambda^2 T^* M$

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 & & & & \downarrow & & \downarrow & & \\
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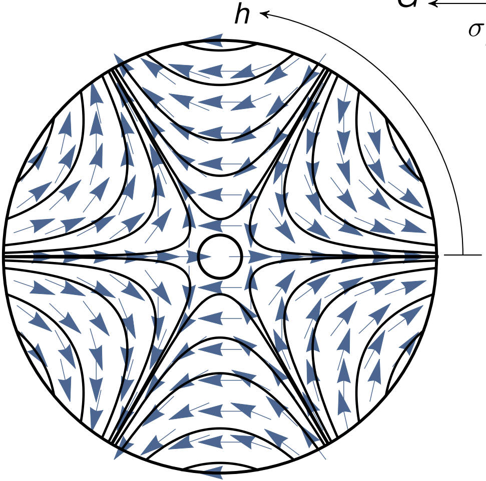

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