Poisson-Lie T-duality in Double Field Theory

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① Introduction (world sheet)

2D σ-model with target space

$$G \xleftrightarrow{\text{Poisson-Lie T-d.}} \tilde{G}$$

$$S = \int dz\, d\bar{z}\; \underbrace{E_{ij}}_{g_{ij} + B_{ij}}\, \partial x^i \bar{\partial} x^j \qquad \tilde{S} = \int dz\, d\bar{z}\; \tilde{E}_{ij}\, \partial \tilde{x}^i \bar{\partial} \tilde{x}^j$$

<u>If</u> $\quad L_{V_a} E_{ij} = - \tilde{F}^{bc}{}_a V_b{}^k V_c{}^l E_{ik} E_{lj}$

- $[V_a, V_b] = F_{ab}{}^c V_c$ (Lie algebra of G)
- $[t^a, t^b] = \tilde{F}^{ab}{}_c t^c \quad t^a \in \tilde{\mathfrak{g}}$
- F and $\tilde{F}$ satisfy cocycle condition $\quad t_a \in \mathfrak{g}$
  - $\rightarrow$ G and $\tilde{G}$ form Drinfeld double
  - $\hat{=}$ Lie group D with Lie algebra $\mathfrak{d} \ni t_A = (t^a \;\; t_a)$
  - $\begin{pmatrix} 0 & \delta^a_b \\ \delta_a^b & 0 \end{pmatrix} = \langle t_A, t_B \rangle = \eta_{AB}$ bilinear form

<u>then</u>

$$L_{\tilde{V}} \tilde{E}_{ij} = - F_{bc}{}^a \tilde{V}^{bk} \tilde{V}^{cl} \tilde{E}_{ik} \tilde{E}_{lj}$$

At the classical level both σ-model are equivalent

- related by a canonical transformation of phase space
- one-loop β-functions match
- winding $\Longleftrightarrow$ momentum mode exchange

e.g. hep-th/9710163, hep-th/9803019, hep-th/9605212

Covers: • abelian T-d G & $\tilde{G}$ abelian
• non-abelian T-d. G non-abelian & $\tilde{G}$ abelian

Applications: • solution generator for AdS/CFT
• Yang-Baxter σ-models
→ integrable η-deformations
→ Poisson-Lie T-d to λ-deformations
• central for $AdS_5 \times S^5 \Leftrightarrow \mathcal{N}=4$ SYM

② Double Field Theory (target space)

Ⓘ $S_{DFT} = \int d^{2D}X\, e^{-2d} \left(\frac{1}{8} \mathcal{H}^{IJ} \partial_I \mathcal{H}_{LK} \partial_J \mathcal{H}^{LK} + \dots \right)$

$\mathcal{H}^{IJ} = \begin{pmatrix} g - Bg^{-1}B & -Bg^{-1} \\ g^{-1}B & g^{-1} \end{pmatrix}^{IJ}$ — section condition $\partial_I \cdot \partial^I \cdot = 0$

Ⓘⓘ SUGRA: $S = \int d^D x \sqrt{g}\, e^{-2\phi} \left(R + 4(\partial\phi)^2 - \frac{1}{12} H^2 \right)$

abelian T-duality: • $\tilde{\mathcal{H}} = O^T \mathcal{H} O \qquad \eta = O^T \eta O$
+ $\mathcal{H}$ is independent of T-d. directions

→ $S_{DFT} \rightarrow \tilde{S} = \int d^D x \sqrt{\tilde{g}}\, e^{-2\tilde{\phi}} (\dots)$ SUGRA on dual backgr.

manifest T-duality

$$\boxed{\begin{array}{ccc} & S_{DFT} + \text{section cond} & \\ \swarrow & & \searrow \\ S & & \tilde{S} \end{array}}$$

? similar for Poisson-Lie T-d ?

Doubled space = Drinfeld double 0902.4032

Ⅰ. adapt DFT! a) section condition $D_A \cdot D^A \cdot = 0$

$D_A = E_A{}^I \partial_I$ $\quad [D_A, D_B] = F_{AB}{}^C D_C$ Structurcoeff. of $\mathcal{D}$

b) $\nabla_A V^B = D_A V^B + \frac{1}{3} F_{AC}{}^B V^C$

$$\boxed{S^{\mathcal{D}}_{DFT} = S_{DFT} \Big|\, \partial_I \to \nabla_A,\ \mathcal{H}^{IJ} \to \mathcal{H}^{AB} + 1/6\, F_{ACD} F_B{}^{CD} \mathcal{H}^{AB}}$$

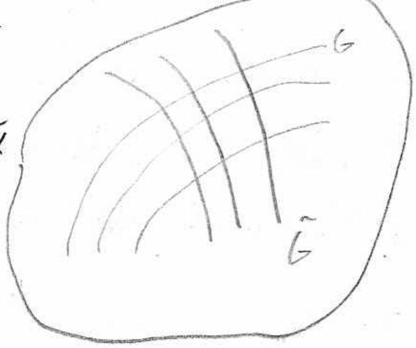
$\mathcal{L}^{\mathcal{D}}_{DFT} \to \mathcal{L}_{DFT} \big|\, \partial_I \to \nabla_A$

Ⅱ solve SC:

a) physical „direction" $\hat{=} M$ $\nearrow$ given by $G \hat{=} x$, $\searrow$ — " — $\tilde{G} \hat{=} \tilde{x}$

$\to \mathcal{H}^{AB}(x)$ or $\mathcal{H}^{AB}(\tilde{x})$

foliation of D



b) map to generalized tangent bundle

$$V^A = (V_a \ V^a) \xrightarrow{\hat{E}_A{}^{\hat{I}}(x)} (V_i \ V^i) = V^{\hat{I}} \in TM \oplus T^*M$$
$$(V^{\hat{I}} = \hat{E}_A{}^{\hat{I}} V^A)$$

with $\mathcal{L}_{DFT\, \hat{E}_A} \hat{E}_B = F_{AB}{}^C \hat{E}_C$ gen. para space

$$\mathcal{H}^{\hat{I}\hat{J}}_{(x)} = \begin{pmatrix} g - B g^{-1} B & \cdots \\ \cdots & \end{pmatrix} = \hat{E}_A{}^{\hat{I}} \mathcal{H}^{AB}_{(x)} \hat{E}_B{}^{\hat{J}}$$

$\hookrightarrow \mathcal{H}^{AB}(g, B)$, similar $d(g, \phi)$ $\to S^{\mathcal{D}}_{DFT}$

SUGRA: $S = \int d^D x \sqrt{g}\, e^{-2\phi} (R \ldots)$

same works for „dual background" after $x \leftrightarrow \tilde{x}$ $(G \leftrightarrow \tilde{G})$

Poisson - Lie T-d :

- $\mathcal{H}^{IJ}, d$ have freely acting isometries on D

isometry group $\subset D$ $\Rightarrow$ no spectators

$=$

$\rightarrow$ $\mathcal{H}^{AB}$ - const $\Rightarrow$ SC solved for both $\nearrow G$, $\searrow \tilde{G}$

$$\boxed{\begin{array}{ccc} & S^{\mathcal{D}}_{DFT} & \\ \swarrow & & \searrow \\ S & & \tilde{S} \end{array}} \qquad \boxed{\begin{array}{ccc} & \mathrm{eom}^{\mathcal{D}} & \\ \swarrow & & \searrow \\ \mathrm{eom} & & \widetilde{\mathrm{eom}} \end{array}}$$

+ gives correct transformation for g, B and ϕ ?
+ „hidden isometries" are now manifest
+ extends to R-R sector
+ if g, B, ϕ solve SUGRA eom also $\tilde{g}, \tilde{B}, \tilde{\phi}$
+ after compactification gauged SUGRA
+ G and $\tilde{G}$ have to be unimod $\hat{=}$ anomaly free on WS