Double Field Theory

Double Fun?

Falk Haßler

based on

..., 1410.6374, 1502.02428, 1509.04176, ...

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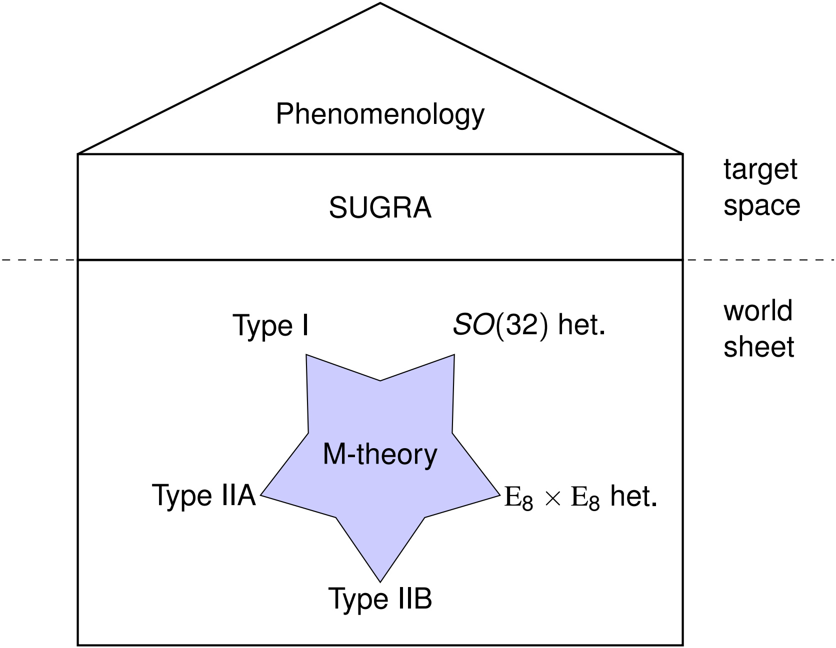
The big picture



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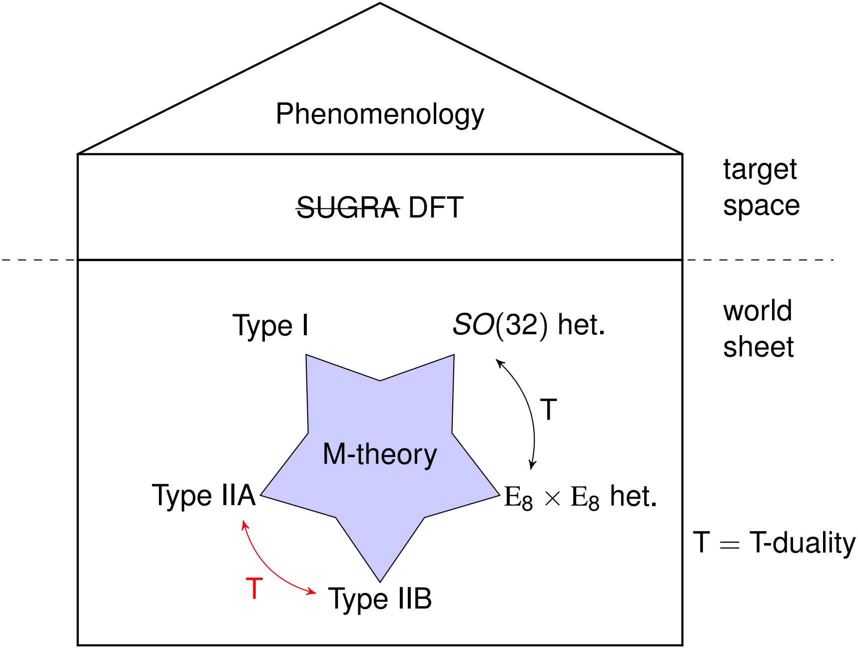

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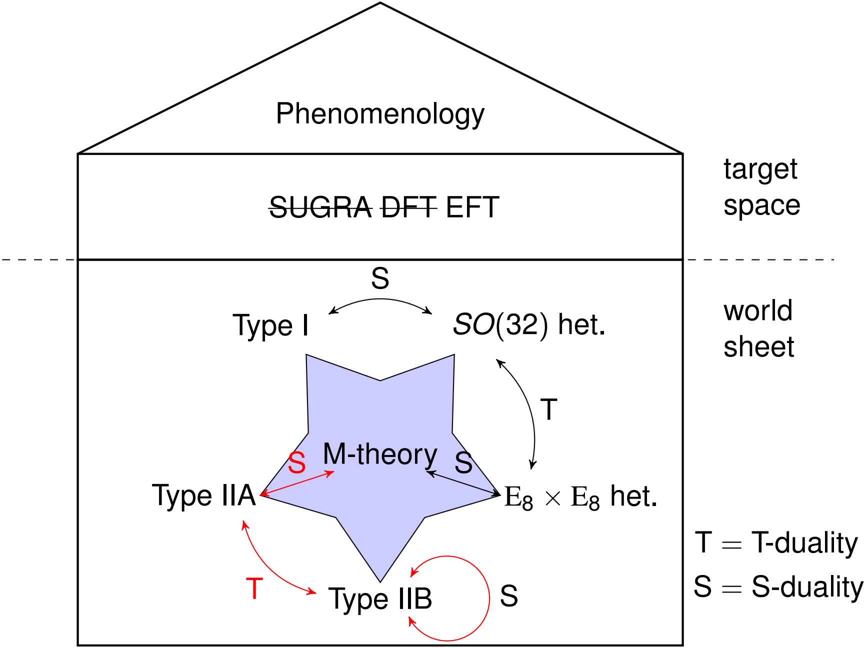

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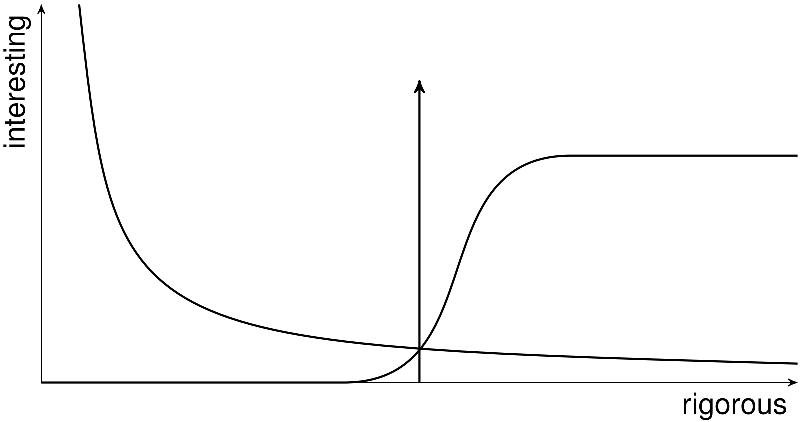

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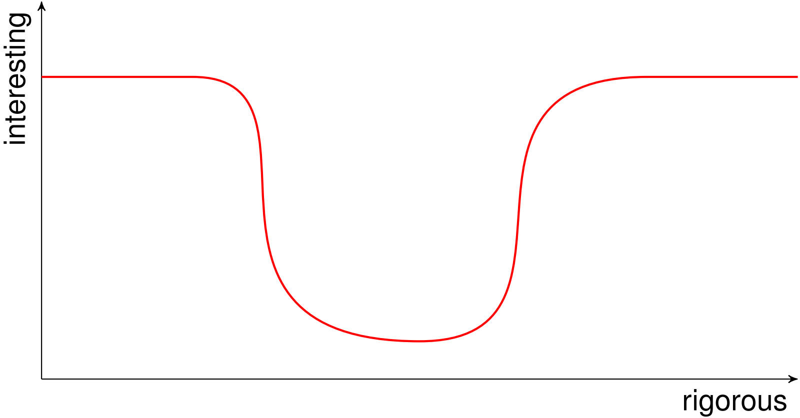

- closure constraint
- generalized Scherk-Schwarz
- non-geometric backgrounds
- strong constraint
- Closed String Field Theory
- generalized geometry

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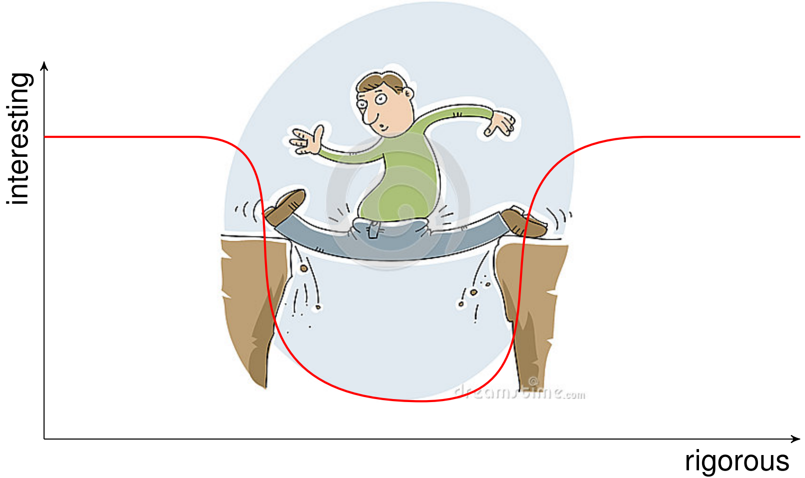

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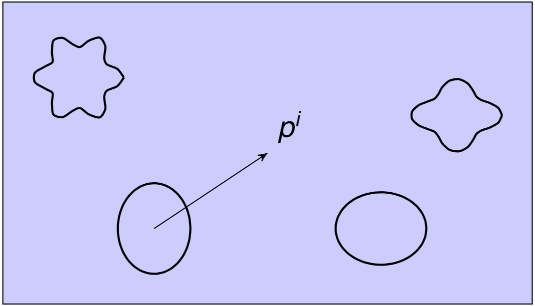
SUGRA

- closed strings in D-dim. flat space with momentum p^i
- truncate all massive excitations
- match scattering amplitudes of strings with EFT

$$S_{\rm NS} = \int \mathrm{d}^D x \sqrt{g} e^{-2\phi} \left(\mathcal{R} + 4 \partial_i \phi \partial^i \phi - \frac{1}{12} H_{ijk} H^{ijk} \right)$$



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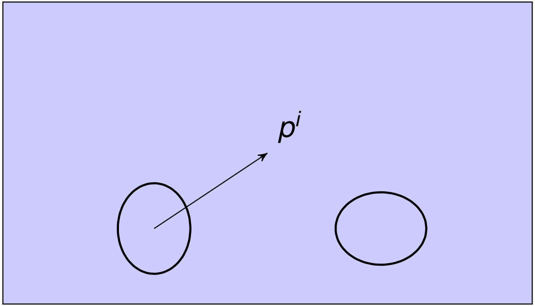

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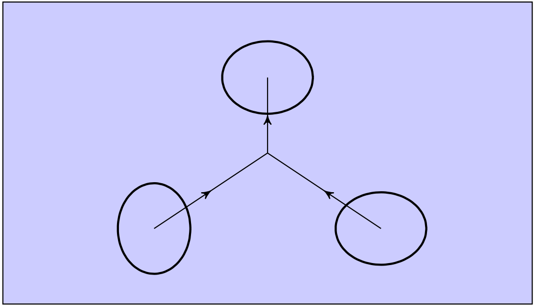
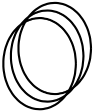
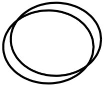




$g_{ij}(p^k)$



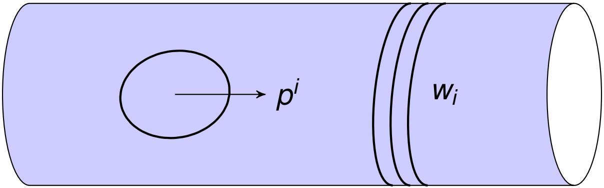
$\phi(p^i)$

DFT (Double Field Theory) [Siegel, 1993, Hull and Zwiebach, 2009, Hohm, Hull, and Zwiebach, 2010]

- closed strings on a flat torus with momentum p^i and winding w_i
- combine conjugated variables x_i and $\tilde{x}^i$ into $X^M = \begin{pmatrix} \tilde{x}_i & x^i \end{pmatrix}$
- repeat steps from SUGRA derivation

$$S_{\mathrm{DFT}} = \int \mathrm{d}^{2D} X \, e^{-2d} \mathcal{R}(\mathcal{H}_{MN}, d)$$

- fields are constrained by strong constraint $\boxed{\partial_M \partial^M \cdot = 0}$



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$$\mathcal{R} = 4\mathcal{H}^{MN}\partial_M\partial_N d - \partial_M\partial_N\mathcal{H}^{MN} - 4\mathcal{H}^{MN}\partial_M d\partial_N d + 4\partial_M\mathcal{H}^{MN}\partial_N d + \frac{1}{8}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\partial_N\mathcal{H}_{KL} - \frac{1}{2}\mathcal{H}^{MN}\partial_N\mathcal{H}^{KL}\partial_L\mathcal{H}_{MK}$$

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- lower/raise indices with $\eta_{MN} = \begin{pmatrix} 0 & \delta_j^i \\ \delta_i^j & 0 \end{pmatrix}$ and $\eta^{MN} = \begin{pmatrix} 0 & \delta_i^j \\ \delta_j^i & 0 \end{pmatrix}$

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Gauge transformations [Hull and Zwiebach, 2009]

- generalized Lie derivative combines

  1. diffeomorphisms
  2. *B*-field gauge transformations
  3. β-field gauge transformations

  (1. and 2.: available in SUGRA)

$$\mathcal{L}_\lambda \mathcal{H}^{MN} = \lambda^P \partial_P \mathcal{H}^{MN} + (\partial^M \lambda_P - \partial_P \lambda^M)\mathcal{H}^{PN} + (\partial^N \lambda_P - \partial_P \lambda^N)\mathcal{H}^{MP}$$

$$\mathcal{L}_\lambda d = \lambda^M \partial_M d + \frac{1}{2}\partial_M \lambda^M$$

- closure of algebra

$$\mathcal{L}_{\lambda_1}\mathcal{L}_{\lambda_2} - \mathcal{L}_{\lambda_2}\mathcal{L}_{\lambda_1} = \mathcal{L}_{\lambda_{12}} \quad \text{with} \quad \lambda_{12} = [\lambda_1, \lambda_2]_\mathrm{C}$$

- only if strong constraint holds

Rigorous, more rigorous, generalized geometry

- trivial solution of SC $\tilde{\partial}^i \cdot = 0 \to$ SUGRA
- x^i are coordinated on manifold M
- interprete components of doubled vector

$$\xi^M = \begin{pmatrix} \xi^m & \xi_m \end{pmatrix}$$

as $\xi^m \in \Gamma(TM)$ and $\xi_m \in \Gamma(T^*M)$

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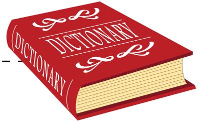

- generalized Lie derivative = Dorfman bracket

$$[X+\xi, Y+\eta] = [X, Y] + L_X\eta - i_Y d\xi$$

on generalized tangent space $TM \oplus T^*M$

- $O(D, D)$ metric = bilinear form

$$\langle X+\xi, Y+\eta\rangle = \eta(X) + \xi(Y)$$

- Courant algebroids, generalized complex structure, ...

M-theory and Exceptional Field Theory

- winding of F1 string $\rightarrow$ wrapping of M2 brane
- example T^4: $X^I = \begin{pmatrix} x^i & \tilde{x}_{ij} \end{pmatrix}$ has $4 + 4 * 3/2 = 10$ components

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D	4	5	6	7
G	SL(5) = $E_{4(4)}$	SO(5, 5) = $E_{5(5)}$	$E_{6(6)}$	$E_{7(7)}$
X^I	**10**	**16**	**27**	**56**
SC	$\overline{\mathbf{5}}$	**10**	$\overline{\mathbf{27}}$	**133**

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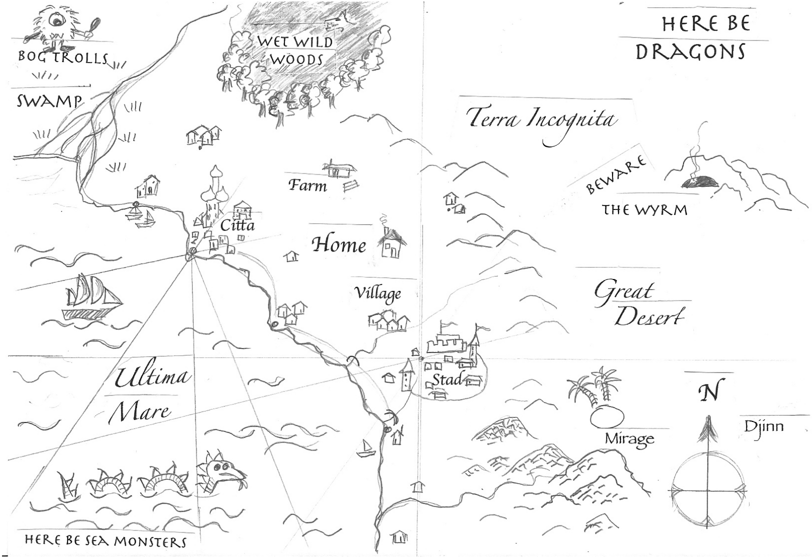
- SC = section condition

$$K_{IJ}^{MN} \partial_M \cdot \partial_N \cdot = 0$$

- generalized Lie derivative

$$\mathcal{L}_\xi V^I = \xi^J \partial_J V^I - V^J \partial_J \xi^I + K_{MN}^{IJ} \partial_J \xi^M V^N$$

A landscape of string backgrounds [Douglas, 2003,Susskind, 2003]



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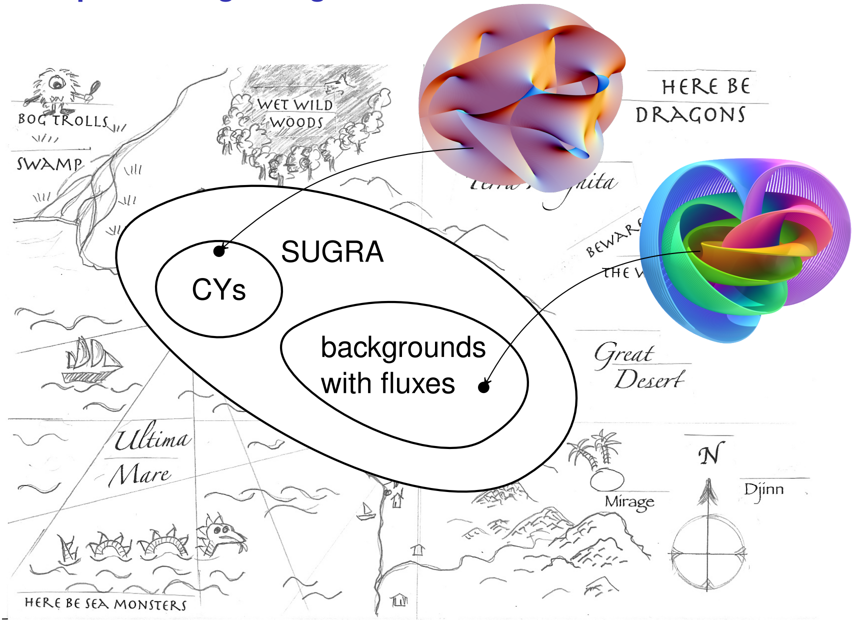

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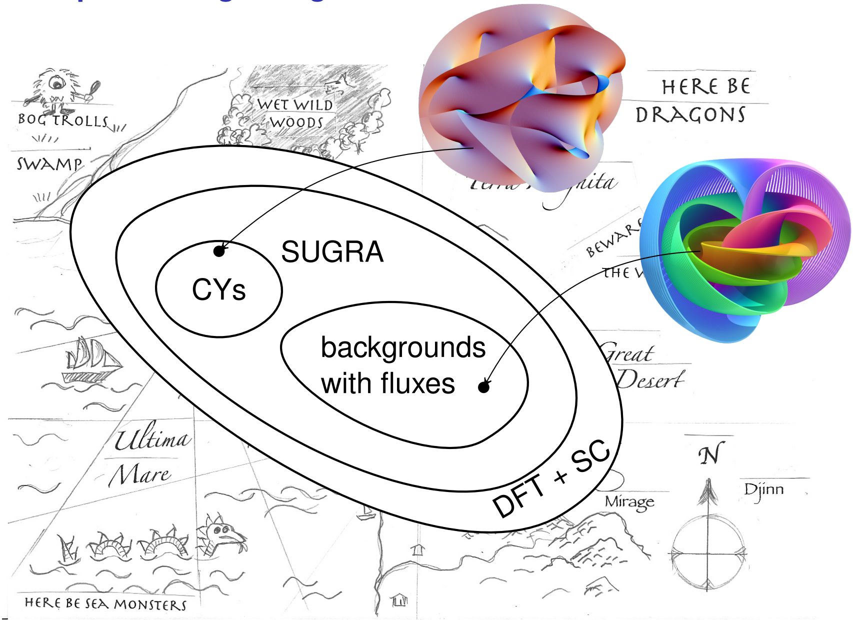

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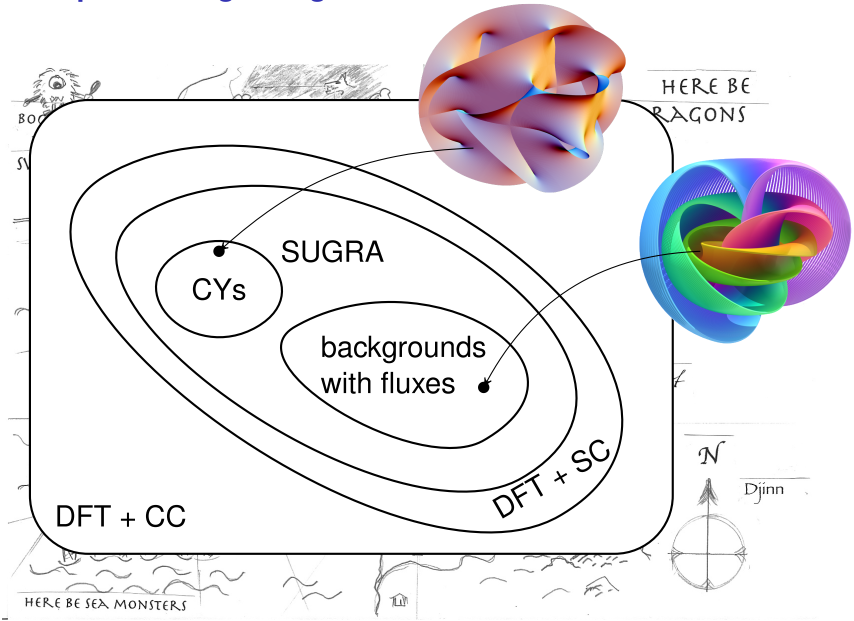

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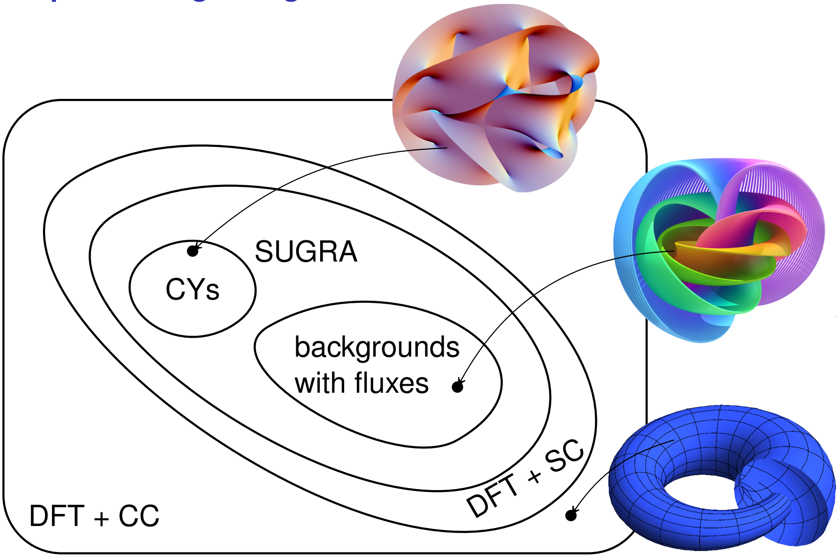

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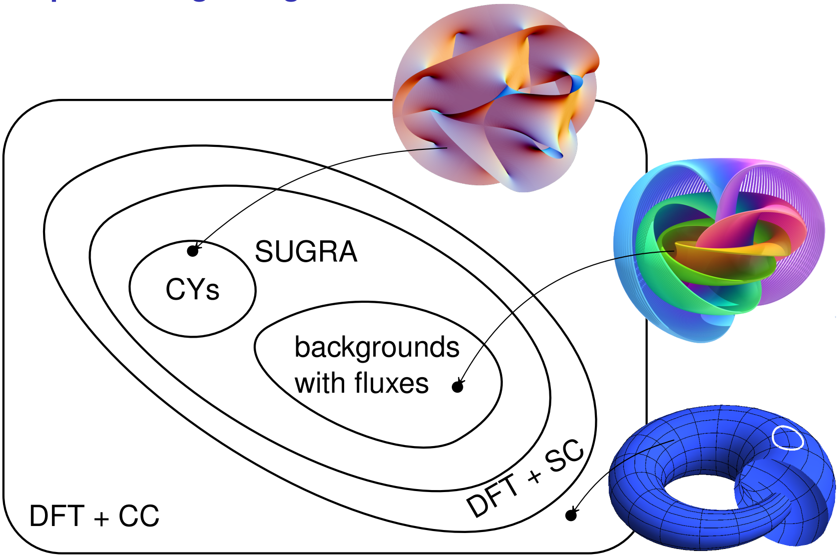

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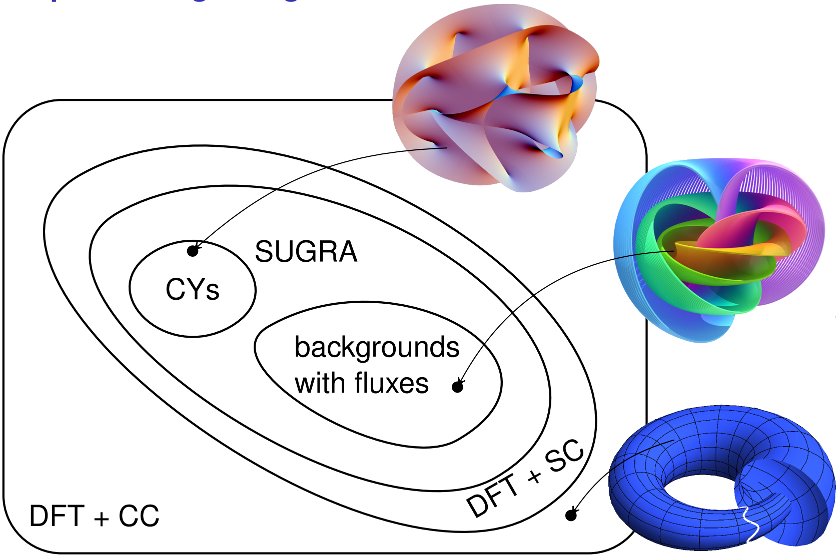

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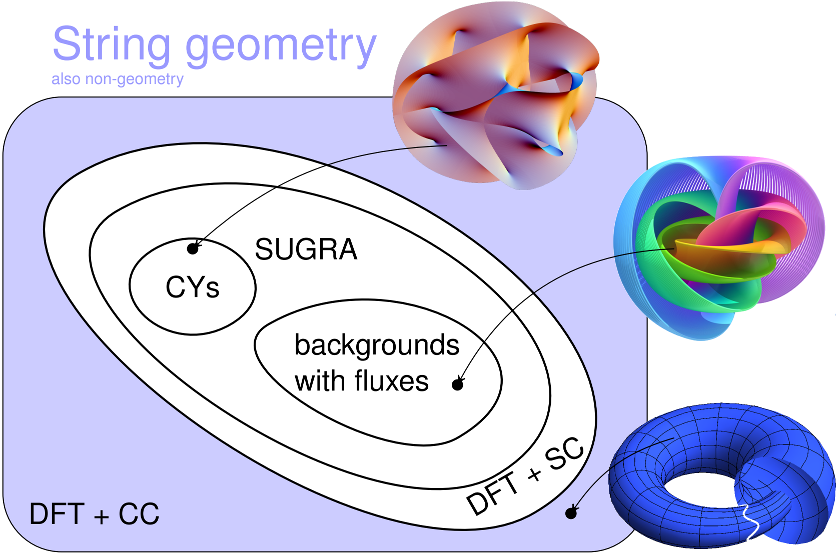

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Generalized Scherk-Schwarz compactification

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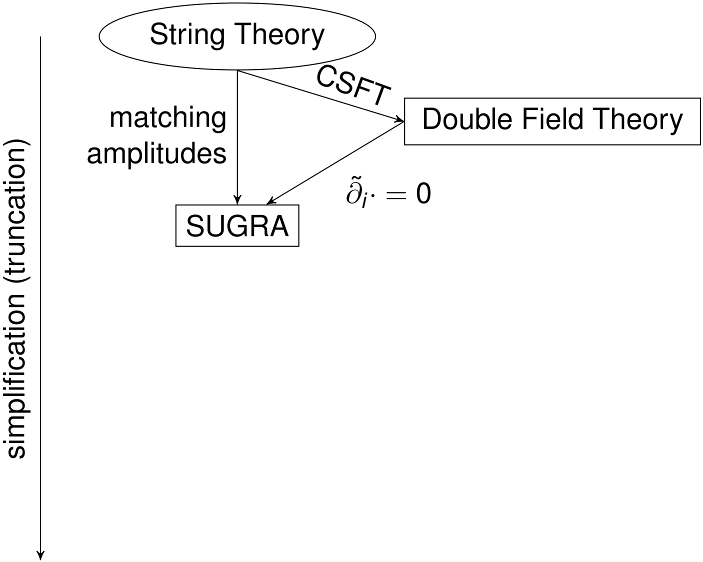

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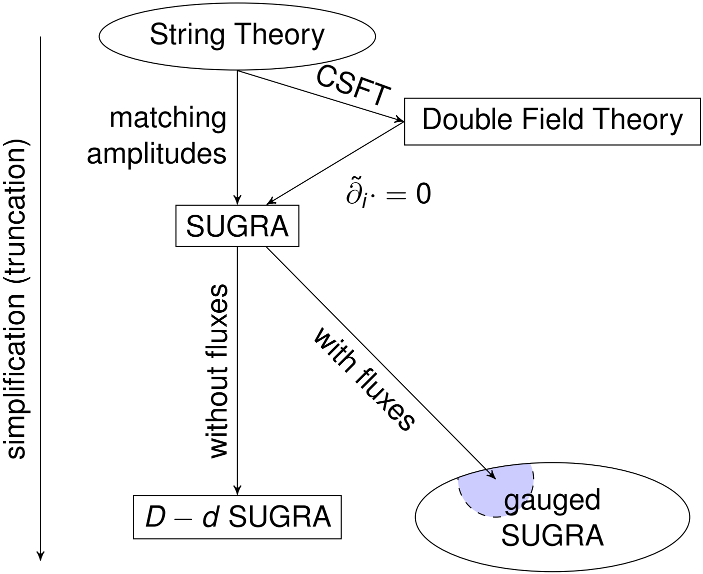

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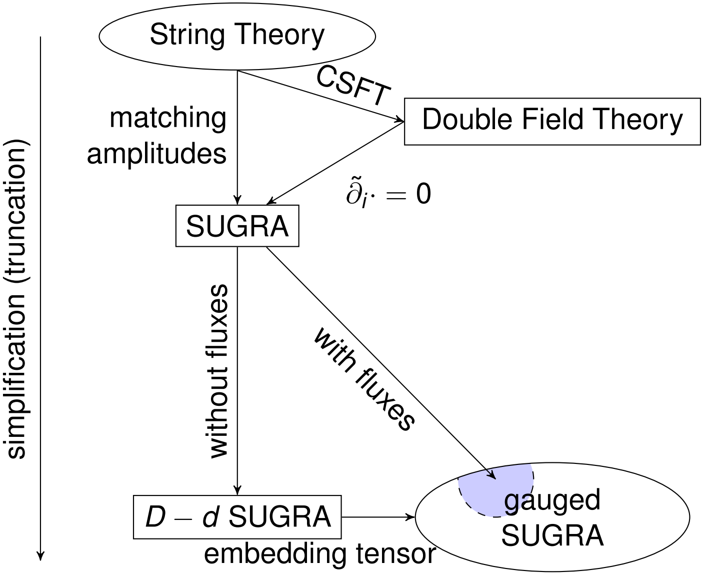

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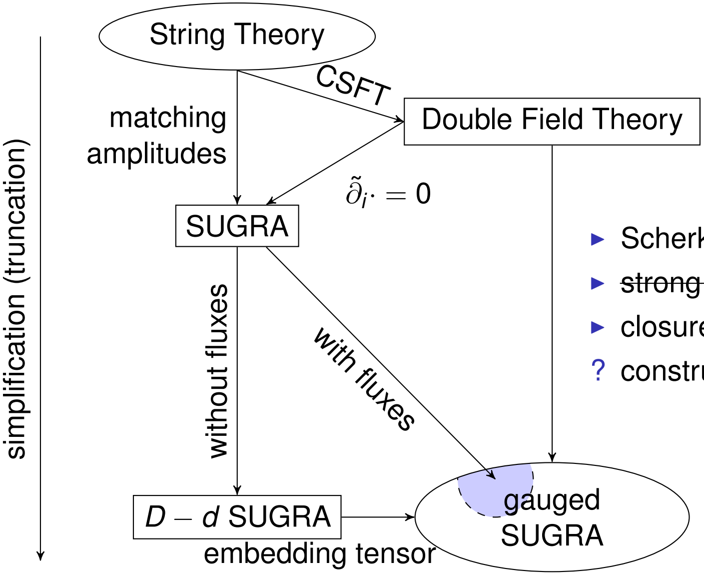

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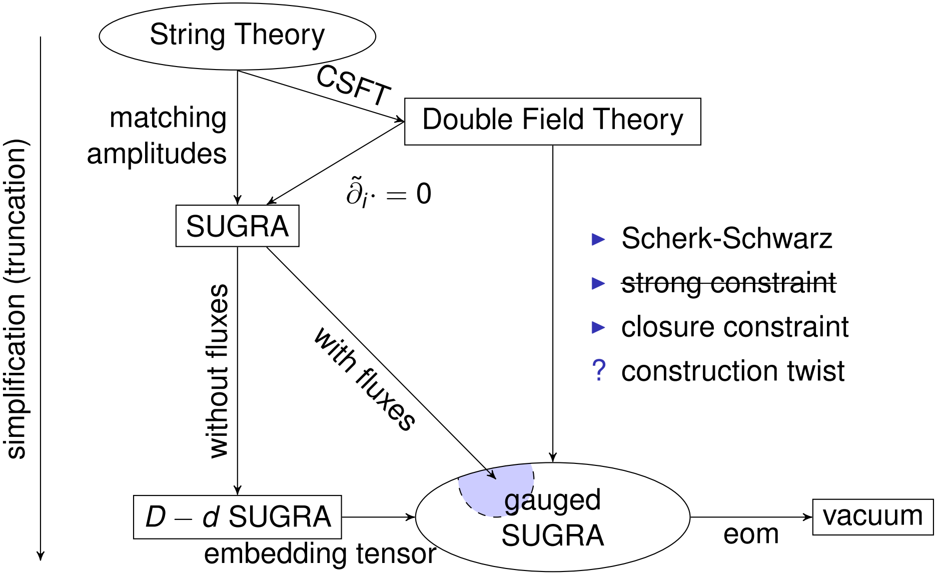

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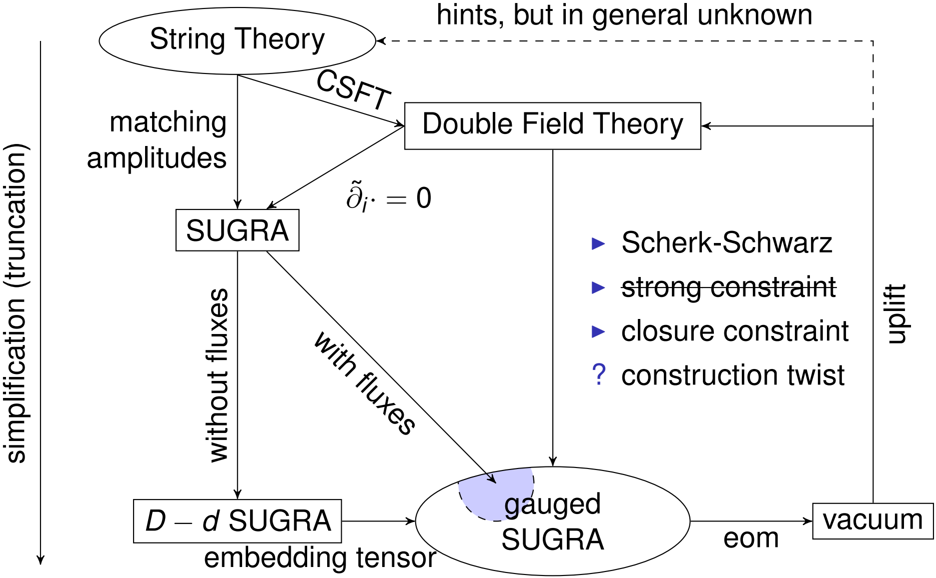

S^3, the elephant in the room

- switch on H-flux, solve eom

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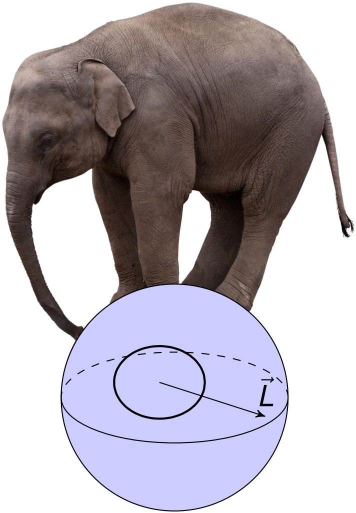

- switch on H-flux, solve eom
- no T^3 but S^3

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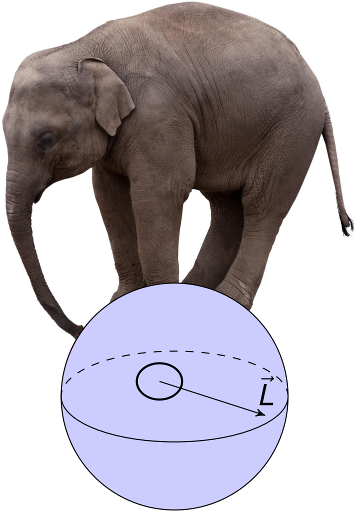

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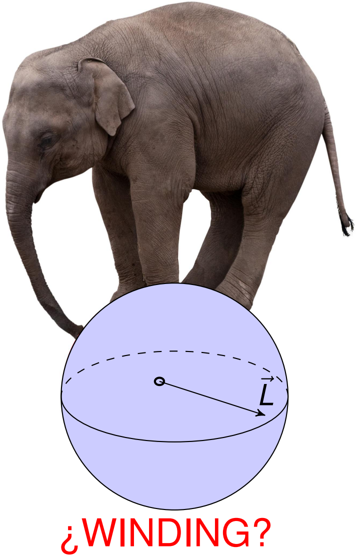

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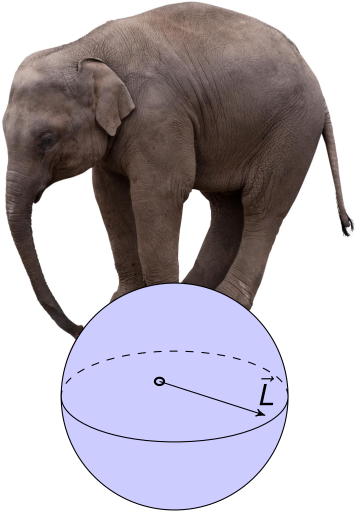

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$$\boxed{\mathrm{DFT}_{\mathrm{WZW}}} \supset \mathrm{DFT}$$

strong constraint

action

symmetries

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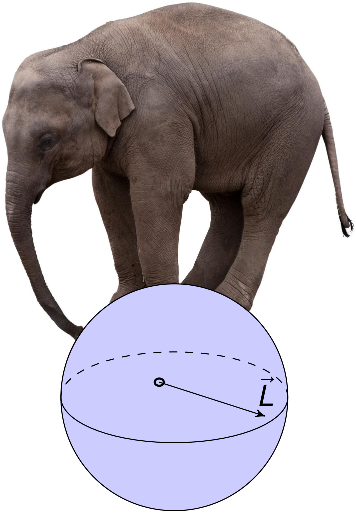

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$$\boxed{\mathrm{DFT}_{\mathrm{WZW}}} \supset \mathrm{DFT}$$

- NEW! strong constraint
- NEW! action
- NEW! symmetries
- FREE twist gen. Scherk-Schwarz
- FREE genuinely non-geometric backgr.

DFT$_{\mathrm{WZW}}$ = DFT on group manifolds

Use group manifold instead of a torus to derive DFT!

+ includes $\begin{cases} \mathrm{T}^D = \mathrm{U}(1)^D \\ S^3 = \mathrm{SU}(2) \end{cases}$
+ CFT exactly solvable
+ flux backgrounds with const. fluxes

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- treat left and right mover independently
- $2D$ independent coordinates

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$$x^i = \frac{1}{\sqrt{2}}(x_{\mathrm{L}}^i + x_{\mathrm{R}}^i)$$
$$\tilde{x}_i = \frac{1}{\sqrt{2}}(x_{\mathrm{L}i} - x_{\mathrm{R}i})$$

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DFT_{WZW} = DFT on group manifolds

Use group manifold instead of a torus to derive DFT!

+ includes $\begin{cases} T^D = U(1)^D \\ S^3 = SU(2) \end{cases}$
+ CFT exactly solvable
+ flux backgrounds with const. fluxes

$$x^i = \frac{1}{\sqrt{2}}(x_{\mathrm{L}}^i + x_{\mathrm{R}}^i)$$
$$\tilde{x}_i = \frac{1}{\sqrt{2}}(x_{\mathrm{L}i} - x_{\mathrm{R}i})$$

Double Field Theory =
- treat left and right mover independently
- $2D$ independent coordinates

Questions about DFT_{WZW}

- What are the covariant objects?
- How is it connected to DFT?
- Does it make non-abelian duality manifest?

} not trivial

WZW model & Kač-Moody algebra [Witten, 1983,Walton, 1999]

- $g \in G$, a compact simply connected Lie group

$$S_{\mathrm{WZW}} = \frac{1}{2\pi\alpha'} \int_M d^2 z \mathcal{K}(g^{-1}\partial g, g^{-1}\bar{\partial} g) + S_{\mathrm{WZ}}(g)$$

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- metric and 3-form flux in flat indices

$$\eta_{ab} := \mathcal{K}(t_a, t_b) \quad \text{and} \quad F_{abc} := \mathcal{K}([t_a, t_b], t_c)$$

- D chiral and D anti-chiral Noether currents (=2D indep. currents)

$$j_a(z) = \frac{2}{\alpha'}\mathcal{K}(\partial g g^{-1}, t_a) \quad \text{and} \quad j_{\bar{a}}(\bar{z}) = -\frac{2}{\alpha'}\mathcal{K}(g^{-1}\bar{\partial} g, t_{\bar{a}})$$

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- radial quantization

$$\boxed{j_a(z) j_b(w) = -\frac{\alpha'}{2}\frac{1}{(z-w)^2}\eta_{ab} + \frac{1}{z-w}F_{ab}{}^c j_c(z) + \dots}$$

Action

- tree level action in CSFT [Zwiebach, 1993]

$$(2\kappa^2)S = \frac{2}{\alpha'}\left(\langle\Psi|c_0^- Q|\Psi\rangle + \frac{1}{3}\{\Psi,\Psi,\Psi\}_0 + \dots\right)$$

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- string field for massless excitations [Hull and Zwiebach, 2009]

$$|\Psi\rangle = \sum_R \Big[\frac{\alpha'}{4}\epsilon^{a\bar{b}}(R) j_{a\,-1} j_{\bar{b}\,-1} c_1 \bar{c}_1 + e(R) c_1 c_{-1} + \bar{e}(R)\bar{c}_1 \bar{c}_{-1} + \frac{\alpha'}{2}\left(f^a(R) c_0^+ c_1 j_{a\,-1} + f^{\bar{b}}(R) c_0^+ \bar{c}_1 j_{\bar{b}\,-1}\right)\Big]|\phi_R\rangle$$

- R is highest weight of $\mathfrak{g}\times\mathfrak{g}$ representation

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- R is highest weight of $\mathfrak{g}\times\mathfrak{g}$ representation
- BRST operator (L_m from Sugawara construction)

$$Q = \sum_m \left(: c_{-m} L_m : + \frac{1}{2} : c_{-m} L_m^{gh} :\right) + \text{anti-chiral}$$

Geometric representation of primary fields ($k \to \infty$)

- flat derivative $\boxed{D_a = e_a{}^i \partial_i \quad \text{with} \quad e_a{}^i = \mathcal{K}(g^{-1}\partial^i g, t_a)}$

operator algebra	geometry ($j_{a\,0} \to D_a$)
$L_0\lvert\phi_R\rangle = j_{a\,0} j_0^a \lvert\phi_R\rangle = h_R\lvert\phi_R\rangle$	$D_a D^a Y_R(x^i) = h_R Y_R(x^i)$
$[j_{a\,0}, j_{b\,0}] = F_{ab}{}^c j_{c\,0}$	$[D_a, D_b] = F_{ab}{}^c D_c$
$\sum_R e(R)\lvert\phi_R\rangle$	$\sum_R e(R) Y_R(x^i) := e(x^i)$

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$$E_A{}^I = \begin{pmatrix} e_a{}^i & 0 \\ 0 & e_{\bar a}{}^{\bar i} \end{pmatrix} \qquad S_{AB} = 2\begin{pmatrix} \eta_{ab} & 0 \\ 0 & \eta_{\bar a \bar b} \end{pmatrix} \qquad \eta_{AB} = 2\begin{pmatrix} \eta_{ab} & 0 \\ 0 & -\eta_{\bar a \bar b} \end{pmatrix}$$

Weak constraint (level matching), later strong constraint

- level matched string field $(L_0 - \bar{L}_0)|\Psi\rangle = 0$ requires

$$(D_a D^a - D_{\bar{a}} D^{\bar{a}}) \cdot = 0 \quad \text{with} \quad \cdot \in \{\epsilon^{a\bar{b}}, e, \bar{e}, f^a, f^{\bar{b}}\}$$

- rewritten in terms of η^{AB} and $D_A = \begin{pmatrix} D_a & D_{\bar{a}} \end{pmatrix}$

$$\eta^{AB} D_A D_B \cdot = D_A D^A \cdot = 0$$

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- change to curved indices using $E_A{}^M$

$$(\partial_M \partial^M - 2\partial_M d \partial^M)\cdot = 0 \qquad \text{with} \qquad d = \phi - \frac{1}{2}\log\sqrt{g}$$

- NEW! term which is absent in DFT $\rightarrow$ adsorb in cov. derivative

$$\boxed{\nabla_M \partial^M \cdot = 0} \quad \text{with} \quad \nabla_M V^N = \partial_M V^N + \Gamma_{MK}{}^N V^K \,, \quad \Gamma_{MK}{}^M = -2\partial_K d$$

Results (leading order k^{-1})

- calculate quadratic and cubic string functions
- integrate out auxiliary fields f^a and $f^{\bar b}$
- perform field redefinition

$$
\begin{aligned}
(2\kappa^2)S = \int d^{2D}X\sqrt{H}\,\Big[&\frac{1}{4}\epsilon_{a\bar b}\Box\epsilon^{a\bar b} + \dots \\
&-\frac{1}{4}\epsilon_{a\bar b}\big(F^{ac}{}_d\bar D^{\bar e}\epsilon^{d\bar b}\epsilon_{c\bar e} + F^{\bar b\bar c}{}_{\bar d}D^e\epsilon^{a\bar d}\epsilon_{e\bar c}\big) \\
&-\frac{1}{12}F^{ace}F^{\bar b\bar d\bar f}\epsilon_{a\bar b}\epsilon_{c\bar d}\epsilon_{e\bar f} + \dots\Big]
\end{aligned}
$$

- NEW! terms e.g. potential
- vanish in abelian limit $F_{abc} \to 0$ and $F_{\bar a\bar b\bar c} \to 0$

Gauge transformations

- tree level gauge transformation in CSFT [Zwiebach, 1993]

$$\delta_\Lambda|\Psi\rangle = Q|\Lambda\rangle + [\Lambda, \Psi]_0 + \cdots$$

- string field for gauge parameter [Hull and Zwiebach, 2009]

$$|\Lambda\rangle = \sum_R \Big[\frac{1}{2}\lambda^a(R) j_{a\,-1} c_1 - \frac{1}{2}\lambda^{\bar{b}}(R) j_{\bar{b}\,-1} \bar{c}_1 + \mu(R) c_0^+\Big]|\phi_R\rangle$$

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- after field redefinition and μ gauge fixing

$$\begin{aligned}\delta_\lambda \epsilon_{a\bar{b}} =& D_{\bar{b}}\lambda_a + \frac{1}{2}\left[D_a\lambda^c\epsilon_{c\bar{b}} - D^c\lambda_a\epsilon_{c\bar{b}} + \lambda_c D^c\epsilon_{a\bar{b}} + F_{ac}{}^d\lambda^c\epsilon_{d\bar{b}}\right] \\ & D_a\lambda_{\bar{b}} + \frac{1}{2}\left[D_{\bar{b}}\lambda^{\bar{c}}\epsilon_{a\bar{c}} - D^{\bar{c}}\lambda_{\bar{b}}\epsilon_{a\bar{c}} + \lambda_{\bar{c}} D^{\bar{c}}\epsilon_{a\bar{b}} + F_{\bar{b}\bar{c}}{}^{\bar{d}}\lambda^{\bar{c}}\epsilon_{a\bar{d}}\right] \\ \delta_\lambda d =& -\frac{1}{4}D_a\lambda^a + \frac{1}{2}\lambda_a D^a d - \frac{1}{4}D_{\bar{a}}\lambda^{\bar{a}} + \frac{1}{2}\lambda_{\bar{a}} D^{\bar{a}} d\end{aligned}$$

Doubled objects

promising results, but bulky

Rewrite action/gauge trafo in terms of doubled object

- + simplifies expressions considerably
- + extrapolation from cubic to all order in fields

Doubled objects

object	doubled version
$\eta_{ab}, \eta_{\bar{a}\bar{b}}$	$\eta_{AB} = 2\begin{pmatrix}\eta_{ab} & 0 \\ 0 & -\eta_{\bar{a}\bar{b}}\end{pmatrix} \qquad S_{AB} = 2\begin{pmatrix}\eta_{ab} & 0 \\ 0 & \eta_{\bar{a}\bar{b}}\end{pmatrix}$
${e_a}^i, {e_{\bar{a}}}^{\bar{i}}$	${E_A}^I = \begin{pmatrix}{e_a}^i & 0 \\ 0 & {e_{\bar{a}}}^{\bar{i}}\end{pmatrix}$
$D_a, D_{\bar{a}}$	$D_A = \begin{pmatrix}D_a & D_{\bar{a}}\end{pmatrix} = {E_A}^I \partial_I$ with $\partial_I = \begin{pmatrix}\partial_i & \partial_{\bar{i}}\end{pmatrix}$

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$e_a{}^i$, $e_{\bar{a}}{}^{\bar{i}}$	$E_A{}^I = \begin{pmatrix} e_a{}^i & 0 \\ 0 & e_{\bar{a}}{}^{\bar{i}}\end{pmatrix}$
D_a, $D_{\bar{a}}$	$D_A = \begin{pmatrix} D_a & D_{\bar{a}}\end{pmatrix} = E_A{}^I \partial_I$ with $\partial_I = \begin{pmatrix}\partial_i & \partial_{\bar{i}}\end{pmatrix}$
ξ^i, $\xi^{\bar{i}}$	$\xi^I = \begin{pmatrix}\xi^i & \xi^{\bar{i}}\end{pmatrix}$
$F_{ab}{}^c$, $F_{\bar{a}\bar{b}}{}^{\bar{c}}$	$F_{AB}{}^C = \begin{cases} F_{ab}{}^c \\ F_{\bar{a}\bar{b}}{}^{\bar{c}} \\ 0 & \text{otherw.}\end{cases} \qquad [D_A, D_B] = F_{AB}{}^C D_C$

Gauge transformations

- "doubled" version of fluctuations $\epsilon^{a\bar{b}}$

$$\epsilon^{AB} = \begin{pmatrix} 0 & -\epsilon^{a\bar{b}} \\ -\epsilon^{\bar{a}b} & 0 \end{pmatrix} \quad \text{with} \quad \epsilon^{a\bar{b}} = (\epsilon^T)^{\bar{b}a}$$

- generate generalized metric [Hohm, Hull, and Zwiebach, 2010]

$$\mathcal{H}^{AB} = S^{AB} + \epsilon^{AB} + \frac{1}{2}\epsilon^{AC} S_{CD} \epsilon^{DB} + \dots = \exp(\epsilon^{AB})$$

with the defining property $\mathcal{H}^{AC}\eta_{CD}\mathcal{H}^{DB} = \eta^{AB}$

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with the defining property $\mathcal{H}^{AC}\eta_{CD}\mathcal{H}^{DB} = \eta^{AB}$

- generalized Lie derivative [Hull and Zwiebach, 2009,Grana and Marques, 2012]

$$\mathcal{L}_\lambda \mathcal{H}^{AB} = \lambda^C D_C \mathcal{H}^{AB} + (D^A\lambda_C - D_C\lambda^A)\mathcal{H}^{CB} + (D^B\lambda_C - D_C\lambda^B)\mathcal{H}^{AC} + F^A{}_{CD}\lambda^C\mathcal{H}^{DB} + F^B{}_{CD}\lambda^C\mathcal{H}^{AD}$$

$$\boxed{\delta_\lambda \mathcal{H}^{AB} = \mathcal{L}_\lambda \mathcal{H}^{AB} + \mathcal{O}(\epsilon^2)}$$

- similar for the generalized dilaton *d*

$$\boxed{\delta_\lambda \mathcal{H}^{AB} = \mathcal{L}_\lambda \mathcal{H}^{AB} + \mathcal{O}(\epsilon^2)}$$

- similar for the generalized dilaton d
- introduce covariant derivative

$$\nabla_A V^B = D_A V^B + \frac{1}{3} F^B{}_{AC} V^C$$

- NEW! generalized Lie derivative, e.g. for vector

$$\mathcal{L}_\lambda V^A = \lambda^B \nabla_B V^A + (\nabla^A \lambda_B - \nabla_B \lambda^A) V^B \quad \text{instead of}$$

$$\mathcal{L}_\lambda V^I = \lambda^J \partial_J V^I + (\partial^I \lambda_J - \partial_J \lambda^I) V^J \quad \text{in traditional DFT}$$

Gauge algebra

$$[\lambda_1, \lambda_2]^A_C = \lambda_1^B \nabla_B \lambda_2^A - \frac{1}{2} \lambda_1^B \nabla^A \lambda_{2\,B} - (1 \leftrightarrow 2)$$

- algebra closes up to a trivial gauge transformation if
    1. fluctuations and parameter fulfill NEW! strong constraint $D_A D^A \cdot$
    2. background fulfills Jacobi identity

$$F_{E[AB} F^E{}_{C]D} = 0$$

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strong constraint $D_A D^A \cdot$
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- no strong constraint required for background

Action

$$X^I = \begin{pmatrix} x^i & x^{\bar{i}} \end{pmatrix}$$

$$S = \int \mathrm{d}^{2D} X \, e^{-2d} \mathcal{R}(\mathcal{H}_{MN}, d)$$

Action

$$X^I = \begin{pmatrix} x^i & x^{\bar{i}} \end{pmatrix}$$

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$$\mathcal{R} = 4\mathcal{H}^{MN} \nabla_M \nabla_N d - \nabla_M \nabla_N \mathcal{H}^{MN} - 4\mathcal{H}^{MN} \nabla_M d \nabla_N d + 4 \nabla_M \mathcal{H}^{MN} \nabla_N d$$
$$+ \frac{1}{8}\mathcal{H}^{MN} \nabla_M \mathcal{H}^{KL} \nabla_N \mathcal{H}_{KL} - \frac{1}{2}\mathcal{H}^{MN} \nabla_N \mathcal{H}^{KL} \nabla_L \mathcal{H}_{MK} + \frac{1}{6} F_{MKL} F_N{}^{KL} H^{MN}$$

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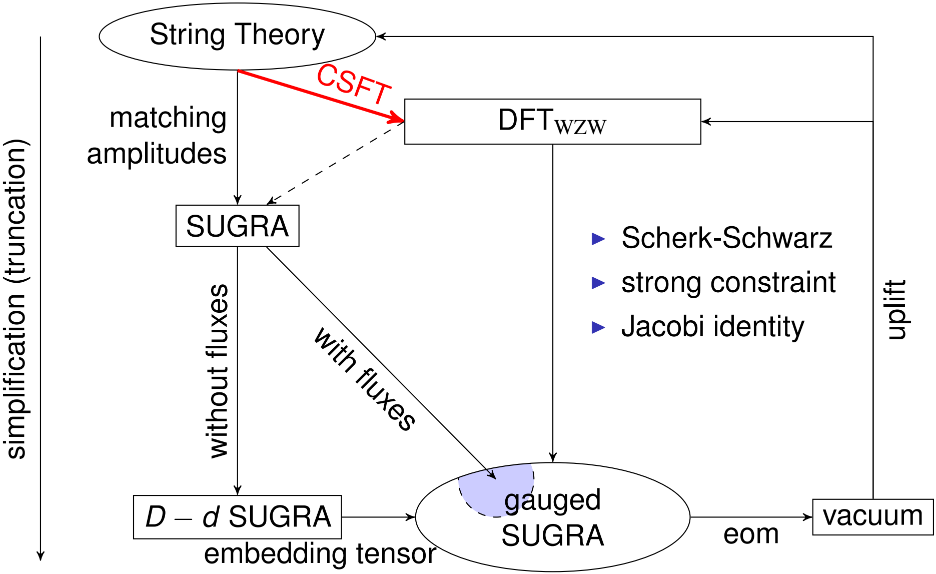
- lower indices with $\eta_{MN} = E^A{}_M E^B{}_N \eta_{AB} \neq \text{const.}$
- $H_{IJ} = E^A{}_M E^B{}_N S_{AB}$ background generalized metric

$$\nabla_M d = \partial_M \tilde{d}$$

$$\nabla_M \mathcal{H}^{KL} = \partial_M \mathcal{H}^{KL} + \Gamma_{MJ}{}^K \mathcal{H}^{JL} + \Gamma_{MJ}{}^L \mathcal{H}^{KJ}$$

Reminder: Generalized Scherk-Schwarz compactification



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- strong constraint
- Jacobi identity

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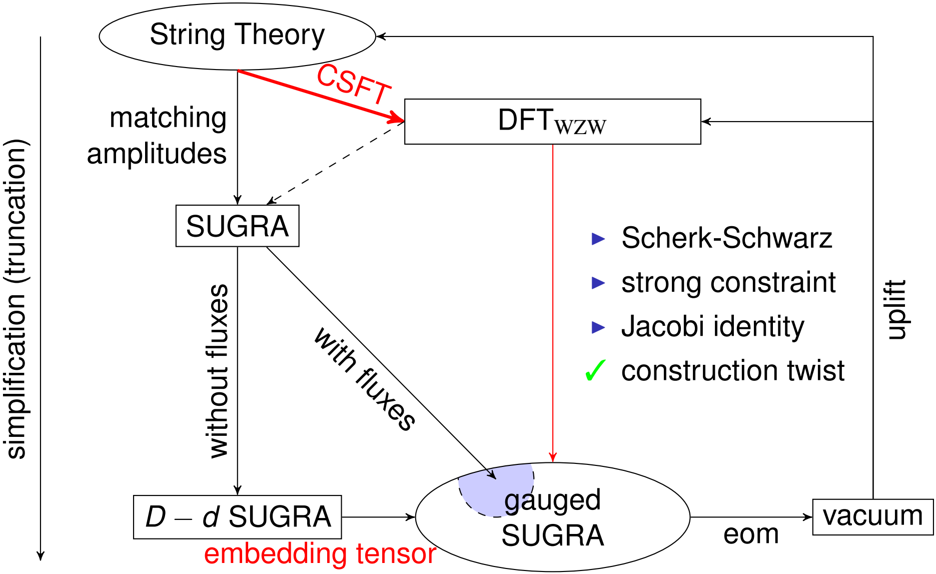

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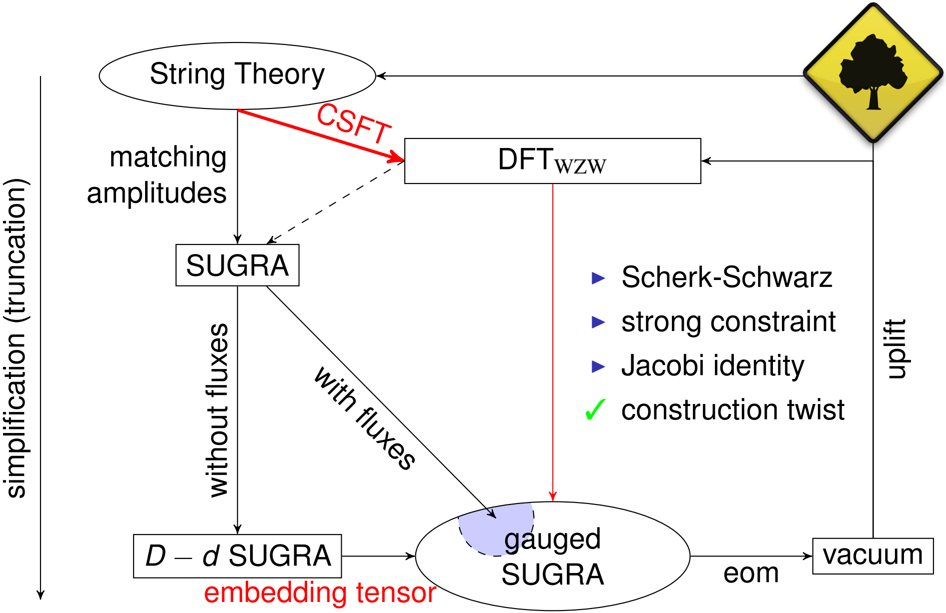

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Summary

DFT is able to satisfy various tastes

fancy mathematics

pure string theory

interesting phenomenology