Quantum corrections for generalised T-dualities

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based on work in progress with

Thomas Rochais

October 2nd, 2020

Motivation

generalised
T-duality*

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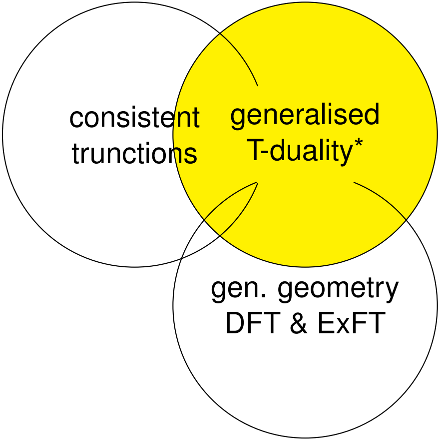

consistent
trunctions

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gen. geometry
DFT & ExFT

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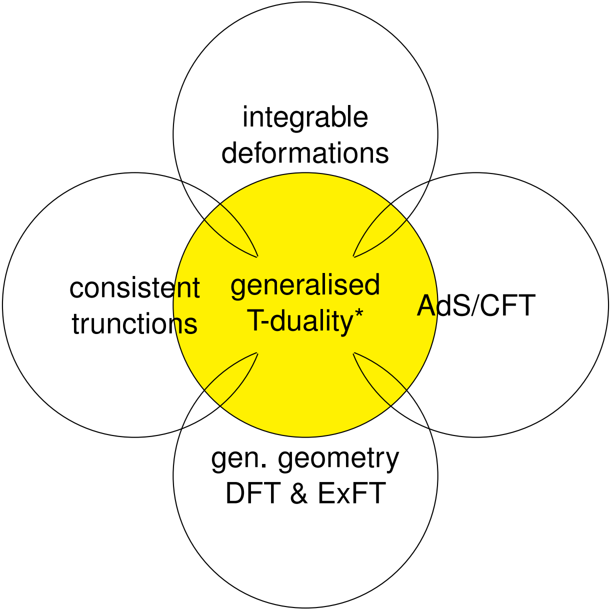

integrable
deformations

consistent
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generalised
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AdS/CFT

gen. geometry
DFT & ExFT

Fine print

*) While abelian T-duality is a symmetry of full string theory [Roček, Verlinde 91],

currently generalised T-duality is just proven to be a symmetry of the classical string.

¿quantum corrections?

Outline

1. Abelian vs. generalised T-duality

2. Relation to DFT & consistent truncations

classical

- -

quantum

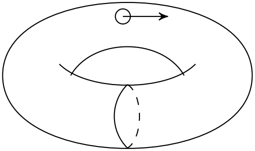
3. One- and two-loop RG flows

4. Application to integrable deformations

5. Open questions

Abelian T-duality

- target space with abelian isometries



winding ↔ momentum

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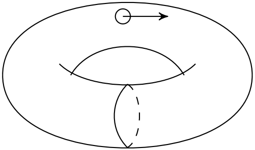


winding ↔ momentum

- on the worldsheet Buscher procedure [Buscher 87]

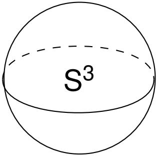
1. gauge global U(1) symmetry
2. use Lagrange multiplier λ to impose $F = \mathrm{d}A = 0$
3. integrate out λ or A

original model ← λ A → dual model

↯

4. Wilson loops $\oint A$ fix periodicity of λ

Non abelian T-duality [de la Ossa, Quevedo 93]



- idea: gauge non-abelian symmetry on worldsheet
- problem: λ now in the adjoint & <u>not</u> a singlet

  1. global properties of dual model do not arise as in the abelian case
  2. isometry group of the dual target space is smaller

$\downarrow$

¿NATD is not invertible?

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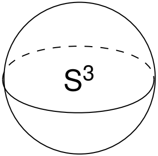

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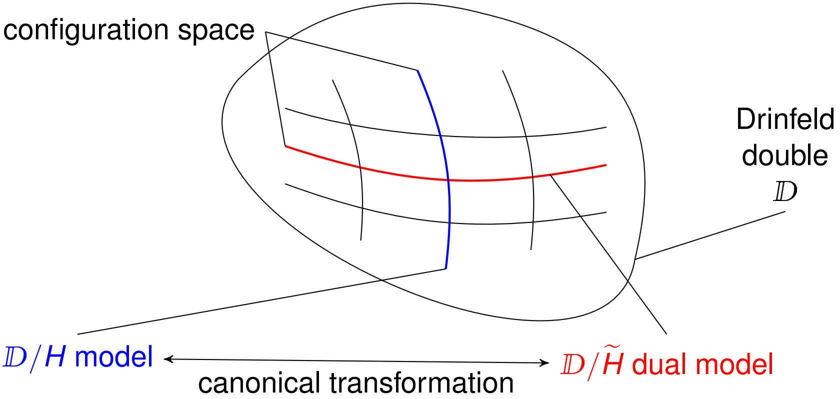
¿NATD is not invertible?

[Giveon, Roček 93]

> We argue that, except for “accidents”, there is no reason to expect nonabelian duality to be a symmetry of a CFT; at best, it can be a transformation between different CFT’s.

Poisson-Lie T-duality [Klimčík, Ševera 95]

NATD is invertible & we should look at the phase space



configuration space

Drinfeld double $\mathbb{D}$

$\mathbb{D}/H$ model

canonical transformation

$\mathbb{D}/\widetilde{H}$ dual model

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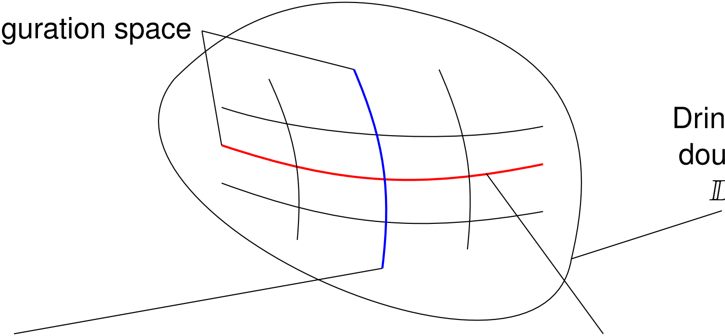

configuration space

Drinfeld double $\mathbb{D}$

$\mathbb{D}/H$ model $\longleftrightarrow$ $\mathbb{D}/\widetilde{H}$ dual model

canonical transformation

$\mathbb{D}$ = Lie group with two max. iso. subgroups; $\mathbb{D} = H \ltimes \widetilde{H}$

H and $\widetilde{H}$ are Poisson-Lie groups

Generalised T-duality

dressing coset construction — double coset $F \backslash \mathbb{D} / H$ [Klimčík, Ševera 96]
most general
includes $\mathrm{AdS}_5 \times \mathrm{S}^5$

$\cup$

Poisson-Lie + WZW — $\mathbb{D}$ with <u>one</u> max. iso. subgroup

$\cup$

Poisson-Lie — H and $\widetilde{H}$ are non-abelian

$\cup$

non-abelian — H or $\widetilde{H}$ is abelian

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classical! quantum corrections?

Poisson-Lie symmetry & DFT [Hassler 17, Çatal-Özer, Demulder, Sakatani, Thompson, …]

Definition: Poisson-Lie symmetric σ-model ($\mathcal{E}$-model)

- 2 D-dimensional Lie algebra $\mathfrak{d}$ with $T_A \in \mathfrak{d}$
  $$[T_A, T_B] = F_{AB}{}^C T_C$$
- ad-invariant O(D,D) pairing
  $$\langle T_A, T_B \rangle = \langle T_B, T_A \rangle = \eta_{AB}$$
- involution $\mathcal{E} : \mathfrak{d} \to \mathfrak{d}$, $\mathcal{E}^2 = 1$
  $$\langle T_A, \mathcal{E} T_B \rangle = \langle \mathcal{E} T_A, T_B \rangle = \mathcal{H}_{AB}$$

⋆ element from the center of $\mathfrak{d}$
  $$F^A t_A \,, \qquad F_{AB}{}^C F_C = 0$$

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¿How to get the metric g, the B-field B and the dilaton ϕ?

Generalised frame fields

- ${E^A}_I(x)$ with ${E^A}_I \eta_{AB} {E^B}_J = \begin{pmatrix} 0 & \delta_j^i \\ \delta_i^j & 0 \end{pmatrix}$

  such that $\boxed{\begin{array}{c} \mathcal{L}_{E_A} E_B = {F_{AB}}^C E_C \\ \mathcal{L}_{E_A} e^{-2d} = -F_A e^{-2d} \end{array}}$ holds (frame algebra)

  generalised Lie derivative

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  generalised Lie derivative

- g, B and ϕ are encoded in

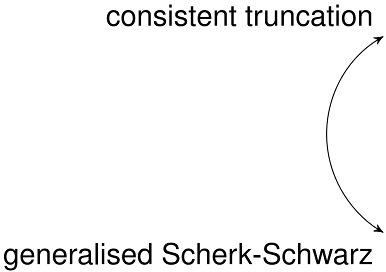
  gen. dilaton $d = \phi - \frac{1}{4} \log \det g$

  gen. metric $\mathcal{H}_{IJ} = \begin{pmatrix} g^{ij} & g^{ik} B_{kj} \\ -B_{ik} g^{kj} & g_{ij} - B_{ik} g^{kl} B_{lj} \end{pmatrix} = E^A{}_I \mathcal{H}_{AB} E^B{}_J$

Generalised Scherk-Schwarz reductions

- $\exists$ explicit construction for ${E_A}^I(x)$ on $\mathbb{D}/H$ iff H is max. iso.

  ($\forall t^a, t^b \in \mathfrak{h}$ the Lie algebra generating H, $\langle t^a, t^b \rangle = 0$)



consistent truncation

generalised Scherk-Schwarz

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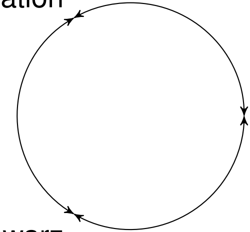


consistent truncation

generalised T-duality

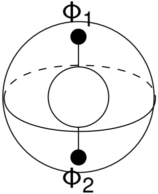
generalised Scherk-Schwarz

- Poisson-Lie T-duality (plurality) = different choices for *H*
- works for dressing cosets: gen. Scherk-Schwarz → gen. cosets

  [Demulder, Hassler, Piccinini, Thompson 19]

Quantum corrections

- loop corrections on fixed genus *g* wordsheets for correlator $\langle \Phi_1 \Phi_2 \rangle$
  $\underline{\alpha'\text{-corrections}}$



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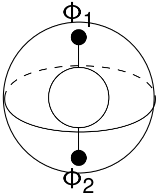
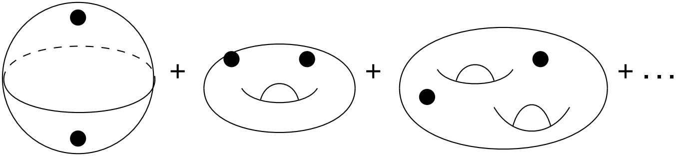


- loop corrections on fixed genus *g* wordsheets
  for correlator $\langle \Phi_1 \Phi_2 \rangle$
  α'-corrections

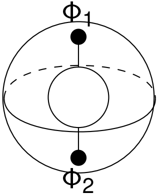
- string path integral genus expansion



g_s-corrections

should require some knowledge about global properties ????

Quantum corrections



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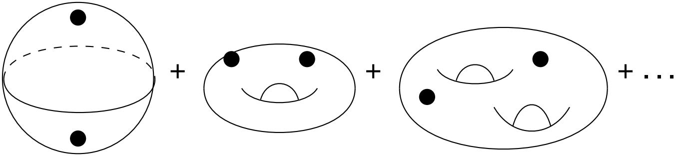

$\underline{g_s\text{-corrections}}$

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$\rightarrow$ α'-corrections are a good starting point

RG flows

- the σ-model is a 2-dim. QFT with ∞ number of couplings
- How do they flow from the UV → IR?

$$\frac{\mathrm{d}g_{ij}}{\mathrm{d}t} = R_{ij} \qquad t = \log \mu \qquad \text{(one-loop)}$$

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- with B-field and dilaton generalised Ricci flow

$$\frac{\mathrm{d}\mathcal{H}_{IJ}}{\mathrm{d}t} = \mathcal{R}_{IJ} \quad \text{———} \quad \text{generalised Ricci tensor}$$ [Hohm, Hull, Zwiebach 10]

PL symmetry & one-loop RG flow

- go to adapted frame: $\mathcal{H}_{IJ} \xrightarrow{E_A{}^I} \mathcal{H}_{AB}$
- restrictions on $\dot{\mathcal{H}}_{AB} = \langle T_A, \dot{\mathcal{E}} T_B \rangle = \mathcal{R}_{AB}$

$$\mathcal{E}^2 = 1 \qquad \to \qquad 0 = P\dot{\mathcal{E}}P = \overline{P}\dot{\mathcal{E}}\overline{P} \qquad\qquad P = \frac{1}{2}(1 + \mathcal{E})$$

$$\overline{P} = \frac{1}{2}(1 - \mathcal{E})$$

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$$\mathcal{R}_{AB} = \begin{pmatrix} 0 & \mathcal{R}_{a\bar{b}} \\ \mathcal{R}_{\bar{a}b} & 0 \end{pmatrix} \qquad P^A{}_B = \begin{pmatrix} \delta^a_b & 0 \\ 0 & 0 \end{pmatrix}$$

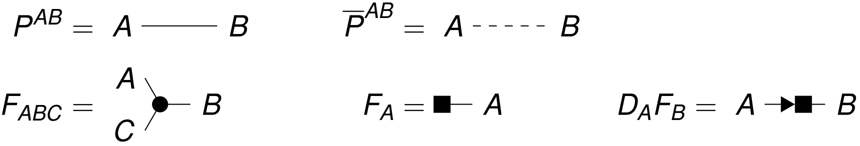
encodes
one-loop
RG flow

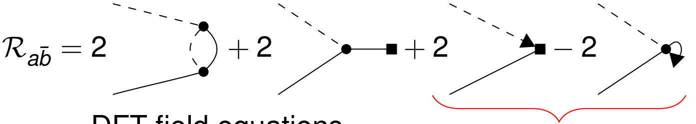
$$\overline{P}^A{}_B = \begin{pmatrix} 0 & 0 \\ 0 & \delta^{\bar{a}}_{\bar{b}} \end{pmatrix}$$

"Feynman"-diagrams

$$\mathcal{R}_{a\bar{b}} = 2 P_a{}^C \overline{P}_{\bar{b}}{}^D \left(F_{CEG} F_{DFH} P^{EF} \overline{P}^{GH} + F_{CDE} F_E P^{EF} + D_D F_C - D_E F_{CDF} \overline{P}^{EF} \right)$$

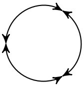
with $D_A = E_A{}^I \partial_I$ and $\partial_I = \begin{pmatrix} 0 & \partial_i \end{pmatrix}$ [Geissbuhler, Marqués, Nuñez, Penas 13]





= DFT field equations

killed by PL symmetry

consistent trunction  PL symmetry preserved under RG flow

PL σ-model renormalisable

Challenges beyond one-loop I

β-functions are scheme dependent!

scheme = coordinate choice on coupling space

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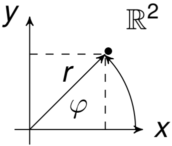

diffeomorphism

$x = r\cos\varphi$

$y = r\sin\varphi$

an infinitesimal coordinate change

$x^\mu \to x^\mu - \xi^\mu$

changes a vector v^μ

$v^\mu \to v^\mu + L_\xi v^\mu$

$$L_\xi v^\mu = \xi^\nu \partial_\nu v^\mu - v^\nu \partial_\nu \xi^\mu$$

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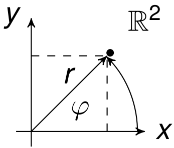

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$$x^\mu \sim \begin{pmatrix} g_{ij} & B_{ij} & \phi \end{pmatrix}$$

$$\xi^\mu \sim \begin{pmatrix} \Delta g_{ij} & \Delta B_{ij} & \Delta\phi \end{pmatrix} = \Psi$$

$$v^\mu \sim \begin{pmatrix} \beta^g_{ij} & \beta^B_{ij} & \beta^\phi \end{pmatrix} = \beta$$

$$\xi^\mu \partial_\mu = \delta_\Psi = \Delta g_{ij} \frac{\delta}{\delta g_{ij}} + \dots$$

$$\beta \to \beta + L_\Psi \beta$$

$$\boxed{L_\Psi = \delta_\Psi \beta - \delta_\beta \Psi - T(\Psi, \beta)}$$

δ can have torsion

The “right” scheme [Marqués, Nuñez 15]

- observables are scheme independent

  BUT action of symmetries depend on the scheme

- keep frame algebra unmodified $\boxed{\mathcal{L}_{E_A} E_B = F_{AB}{}^C E_C}$

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→ generalised Bergshoeff-de Roe scheme (Marqués-Nuñez scheme)

↯ double Lorentz transformations are modified

→ keep track of $E_A{}^I$'s double Lorentz frame

finite generalised Green-Schwarz transformation
generate α'-correction for metric, B-field & dilaton

[Borsato, López, Wulff 20; Hassler, Rochais 20; Borsato, Wulff 20; Codina, Marqués 20]

Challenges beyond one-loop II

- in right scheme two-loop field equations exclusively depend on

  [Baron, Fernández-Melgarejo, Marqués, Nuñez 17]

$$F_{ABC}\,, \qquad F_A\,, \qquad P^{AB}\,, \qquad \overline{P}^{AB}\,, \qquad \text{and} \qquad D_A$$

↯

BUT field equations $\neq$ β-functions

instead $\delta_\Psi S = \int \mathrm{d}^D x e^{-2d} \Psi \cdot K(\beta)$

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- @ two-loops: $\delta_\Psi S^{(2)} = \int \mathrm{d}^D x e^{-2d} \left[\Psi \cdot K^{(2)}(\beta^{(1)}) + \Psi \cdot K^{(1)}(\beta^{(2)})\right]$
- $K^{(1)}_{AB;CD} = -\eta_{AC}\eta_{BD}$ Zamolodchikov metric

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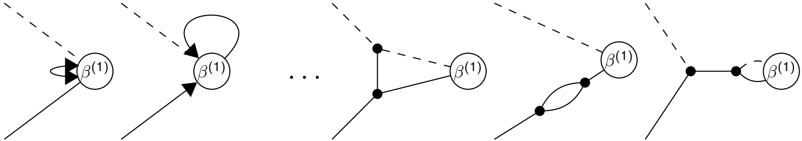
¿Is it possible to write $K^{(2)}$ just with $\bullet$?

Sketch of the computation

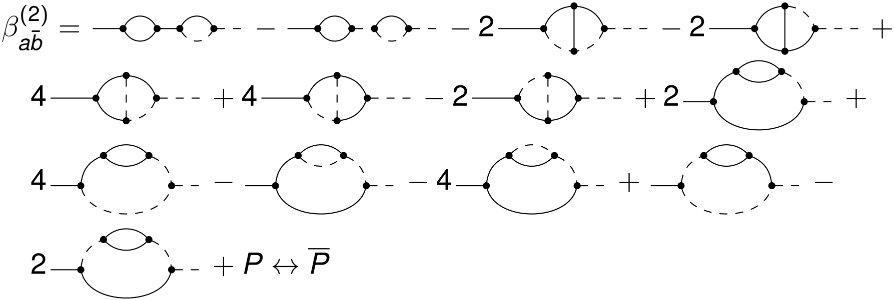
- obtain $K^{(2)}$ in the Metsaev-Tseytlin scheme
- transform it to the gen. BdR scheme
- write result in terms of H-flux H_{abc}, spin connection $\omega_{ab}{}^{c}$ and F_a
- match 77 terms with 19 doubled diagrams like



- works despite 4:1 overdetermined

Two-loop β-function

- 342 diagrams vs. 4 @ one-loop
- transform covariantly under gen. Green-Schwarz transformations
- imposing PL symmetry and $F_A = 0 \to$ 26 diagrams remain



PL symmetry is preserved under two-loop RG flows!

Beyond two-loops

- computations become much more complicated

  STILL no obvious conceptual road blocks

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- expand β-function for coupling λ^a around CFT fixed point

$$\beta^a = -(2 - \Delta_a)\lambda^a + \sum_{b,c} C^a{}_{bc}\lambda^b\lambda^c + \dots$$

anomalous dimension

OPE structure coefficients

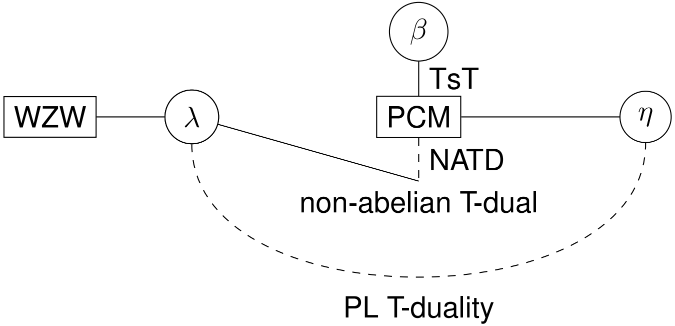
for operator $\mathcal{O}_a$ corresponding to λ^a (in OPE scheme)

Conjecture

CFTs related by PL T-duality share at least a common subsector.

The quest for integrable σ-models

- deform known integrable σ-models to find new ones



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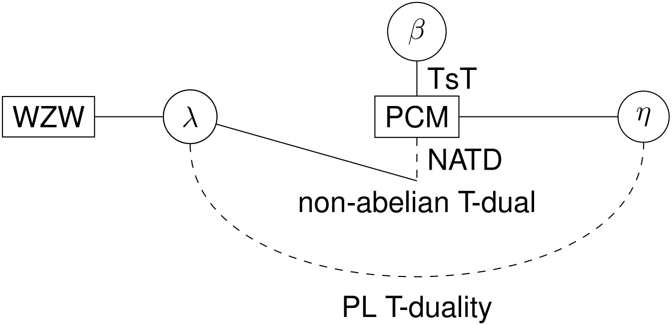
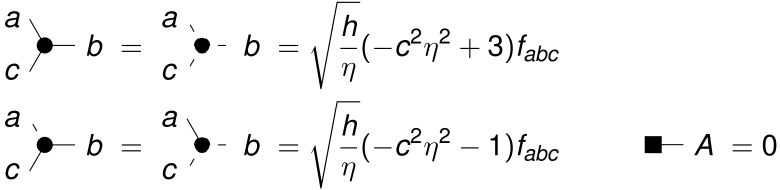


- they are:
  1. PL symmetric
  2. two-loop renormalisable [Hoare, Levine, Tseytlin 19; Georgiou, Sagkrioti, Sfetsos, Siampos 19]

(consistent truncation)^2

- on a semisimple Lie group G, generated by $t_a \in \mathfrak{g}$ with $[t_a, t_b] = f_{ab}{}^c t_c$ and $f_{ac}{}^d f_{bd}{}^c = c_G \kappa_{ab}$, they are captured by



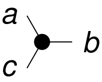
$$= \sqrt{\frac{h}{\eta}}(-c^2\eta^2 + 3)f_{abc}$$

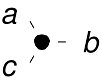
$$= \sqrt{\frac{h}{\eta}}(-c^2\eta^2 - 1)f_{abc}$$

$$= 0$$

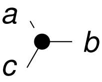
(consistent truncation)^2

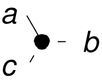
- on a semisimple Lie group G, generated by $t_a \in \mathfrak{g}$ with $[t_a, t_b] = f_{ab}{}^c t_c$ and $f_{ac}{}^d f_{bd}{}^c = c_G \kappa_{ab}$, they are captured by



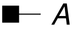
$=$


$= \sqrt{\frac{h}{\eta}}(-c^2\eta^2 + 3)f_{abc}$



$=$


$= \sqrt{\frac{h}{\eta}}(-c^2\eta^2 - 1)f_{abc}$



$= 0$

- furthermore $\beta^\eta \frac{\mathrm{d}F_{ABC}}{\mathrm{d}\eta} = 3\beta_{[A}{}^D F_{BC]D} \quad \leftrightarrow \quad \beta_{a\bar{b}} = -\frac{\beta^\eta}{2\eta}\kappa_{a\bar{b}}$
- only one parameter (η) out of $\dim G^2$ flows
- $\mathbb{Z}_2$ symmetry $\eta \leftrightarrow -\eta$ and $h \leftrightarrow -h$ manifest

Interpretation of c and h

- $c = 0$: β-deformation
- $c = i$: λ-deformation, $\quad h = \frac{\alpha'}{2k}, \quad \eta = \frac{1-\lambda}{1+\lambda}$
- $c = 1$: η-deformation, $\quad h = \frac{\eta t \alpha'}{2\pi}$

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¿Derive $\beta^{(l)}$, $l > 1$, from a q-deformation of $\beta^{(1)}$?

Open questions

- is there a “geometric” interpretation, like for $\mathcal{R}_{AB}$, for $\beta^{(2)}_{AB}$
- obtaining β-functions directly from the $\mathcal{E}$-model
- can dressing cosets be treated in the same way
- two-loop heterotic string should be manageable
- explore the integrability/RG flow correspondence further
- how do quantum groups fit into the picture
- is it possible to study irrelevant deformations, like $T\overline{T}$
- operator map under PL T-duality
- …