String Geometry Beyond the Torus

Falk Haßler

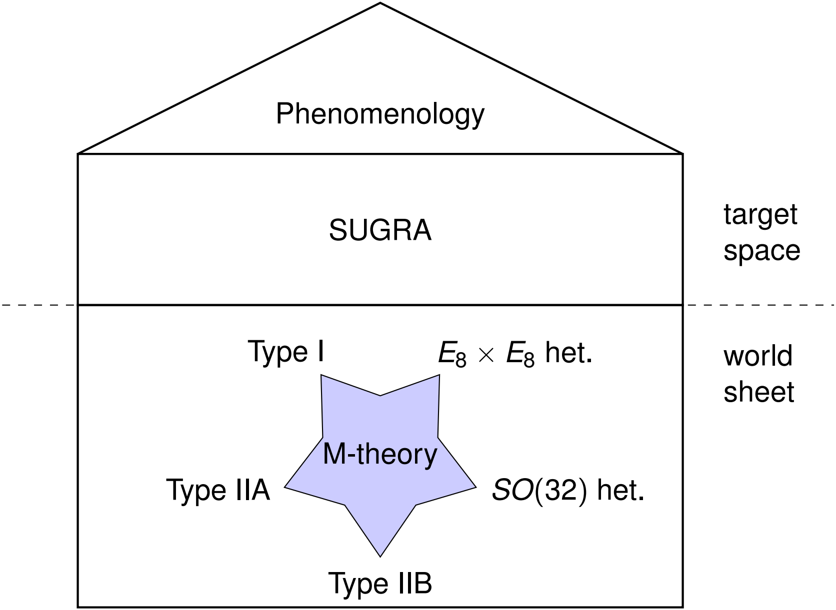
based on arXiv:1410.6374 with

Ralph Blumenhagen and Dieter Lüst

Arnold Sommerfeld Center
LMU Munich

October 25, 2014

The big picture



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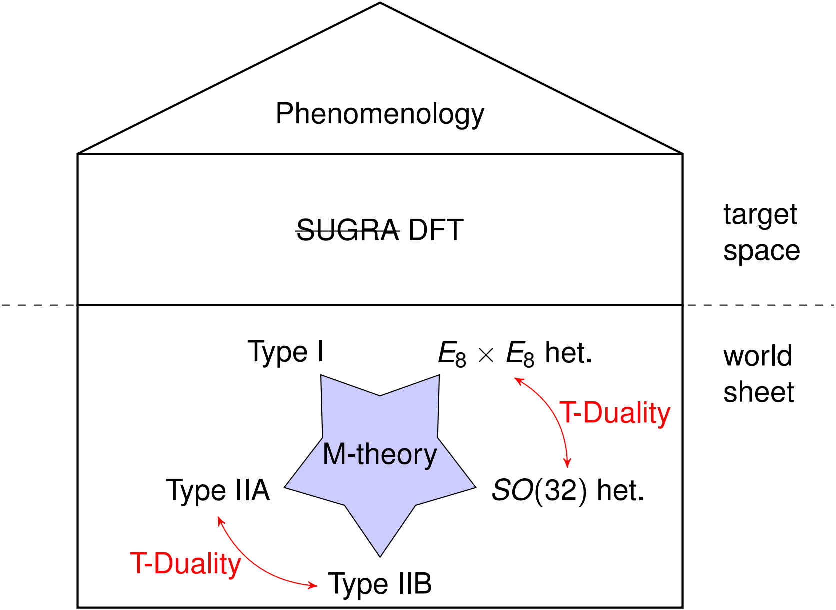

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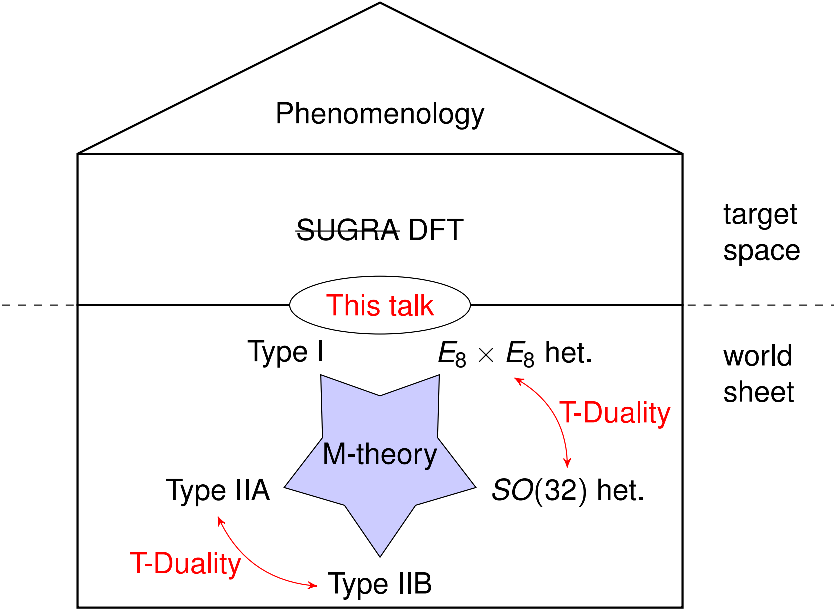


Outline

1. SUGRA & DFT in a nutshell

2. String geometry by violating the strong constraint

3. Deriving DFT_{WZW} from CSFT

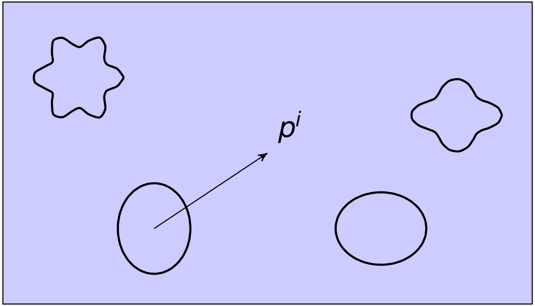
4. Applications

5. Summary and outlook

SUGRA

- closed strings in *D*-dim. flat space
- truncate all massive excitations
- match scattering amplitudes of strings with EFT

$$S_{\mathrm{NS}} = \int \mathrm{d}^D x \sqrt{g} e^{-2\phi} \left(\mathcal{R} + 4\partial_i \phi \partial^i \phi - \frac{1}{12} H_{ijk} H^{ijk} \right)$$



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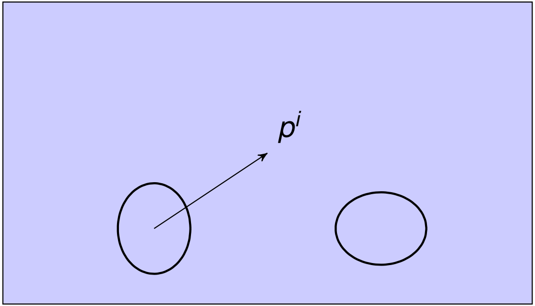

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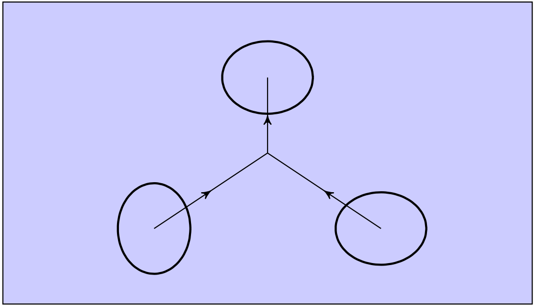
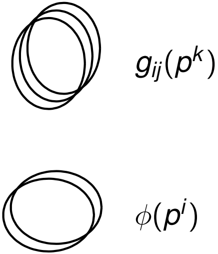




Manifest & hidden symmetries

- $S_{\rm NS}$ = action for NS/NS sector of Type IIA and Type IIB
- manifest invariant under

  diffeomorphisms $\quad g_{ij} = \mathcal{L}_\xi g_{ij}$

  gauge transformations $\quad B_{ij} = \mathcal{L}_\xi B_{ij} + \partial_i \alpha_j - \partial_j \alpha_i$

- compactification on circle $\to U(1)$ isometry
- Buscher rules implement T-duality [Buscher, 1987]

$$\tilde{g}_{\theta\theta} = \frac{1}{g_{\theta\theta}}\,, \quad \tilde{g}_{\theta i} = \frac{1}{g_{\theta\theta}} B_{\theta i}\,, \quad \tilde{g}_{ij} = g_{ij} + \frac{1}{g_{\theta\theta}}(g_{\theta i} g_{\theta j} - B_{\theta i} B_{\theta j})\,, \ \ldots$$

  from NS/NS sector of IIA to IIB

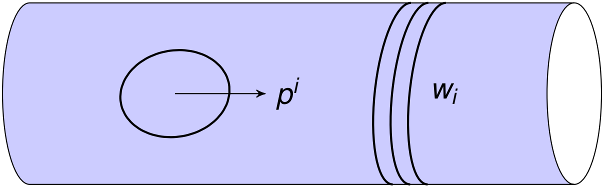
- T-duality is a hidden symmetry

DFT (Double Field Theory) [Siegel, 1993,Hull and Zwiebach, 2009,Hohm, Hull, and Zwiebach, 2010]

- closed strings on a flat torus
- combine conjugated variables x_i and $\tilde{x}^i$ into $X^M = \begin{pmatrix} \tilde{x}_i & x^i \end{pmatrix}$
- repeat steps from SUGRA derivation

$$S_{\mathrm{DFT}} = \int \mathrm{d}^{2D} X \, e^{-2d} \mathcal{R}(\mathcal{H}_{MN}, d)$$

- fields are constrained by strong constraint $\boxed{\partial_M \partial^M \cdot = 0}$



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$$\mathcal{R} = 4\mathcal{H}^{MN}\partial_M\partial_N d - \partial_M\partial_N\mathcal{H}^{MN} - 4\mathcal{H}^{MN}\partial_M d\partial_N d + 4\partial_M\mathcal{H}^{MN}\partial_N d + \frac{1}{8}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\partial_N\mathcal{H}_{KL} - \frac{1}{2}\mathcal{H}^{MN}\partial_N\mathcal{H}^{KL}\partial_L\mathcal{H}_{MK}$$

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$$\mathcal{H}^{MN} = \begin{pmatrix} g_{ij} - B_{ik}g^{kl}B_{lj} & -B_{ik}g^{kj} \\ g^{ik}B_{kj} & g^{ij} \end{pmatrix} \in O(D,D) \to \text{T-duality}$$

DFT (Double Field Theory) [Siegel, 1993,Hull and Zwiebach, 2009,Hohm, Hull, and Zwiebach, 2010]

- lower/raise indices with $\eta_{MN} = \begin{pmatrix} 0 & \delta^i_j \\ \delta^j_i & 0 \end{pmatrix}$ and $\eta^{MN} = \begin{pmatrix} 0 & \delta^j_i \\ \delta^i_j & 0 \end{pmatrix}$

$$X^M = \begin{pmatrix} \tilde{x}_i & x^i \end{pmatrix} \qquad d = \phi - \tfrac{1}{2} \log \sqrt{g}$$

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Gauge transformations [Hull and Zwiebach, 2009]

- generalized Lie derivative combines

  1. diffeomorphisms
  2. B-field gauge transformations
  3. β-field gauge transformations

  (1. and 2.: available in SUGRA)

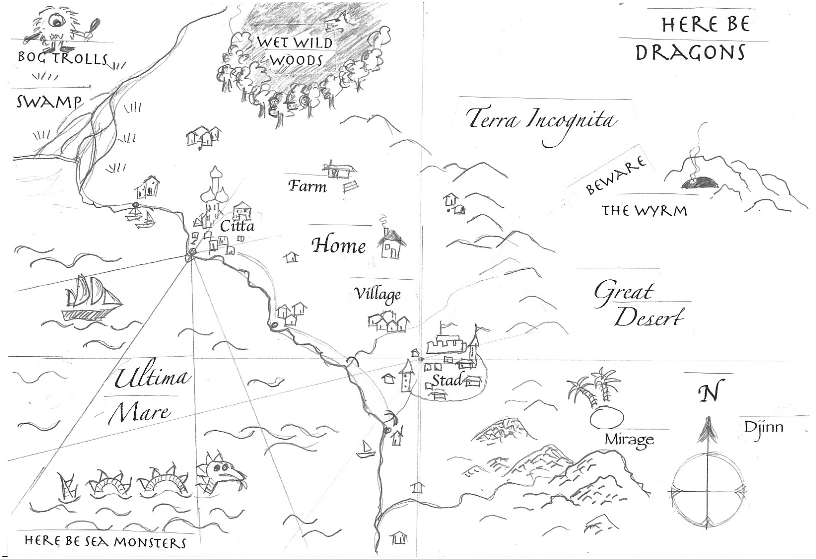
$$\mathcal{L}_\lambda \mathcal{H}^{MN} = \lambda^P \partial_P \mathcal{H}^{MN} + (\partial^M \lambda_P - \partial_P \lambda^M)\mathcal{H}^{PN} + (\partial^N \lambda_P - \partial_P \lambda^N)\mathcal{H}^{MP}$$
$$\mathcal{L}_\lambda d = \lambda^M \partial_M d + \frac{1}{2}\partial_M \lambda^M$$

- closure of algebra

$$\mathcal{L}_{\lambda_1}\mathcal{L}_{\lambda_2} - \mathcal{L}_{\lambda_2}\mathcal{L}_{\lambda_1} = \mathcal{L}_{\lambda_{12}} \quad \text{with} \quad \lambda_{12} = [\lambda_1, \lambda_2]_\mathrm{C}$$

- only if strong constraint holds

A landscape of string backgrounds [Douglas, 2003,Susskind, 2003]



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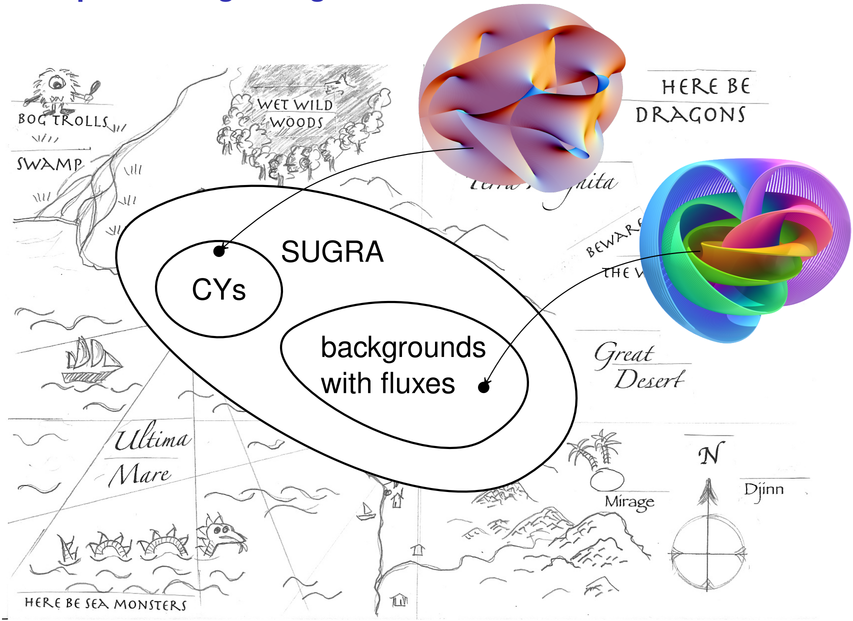

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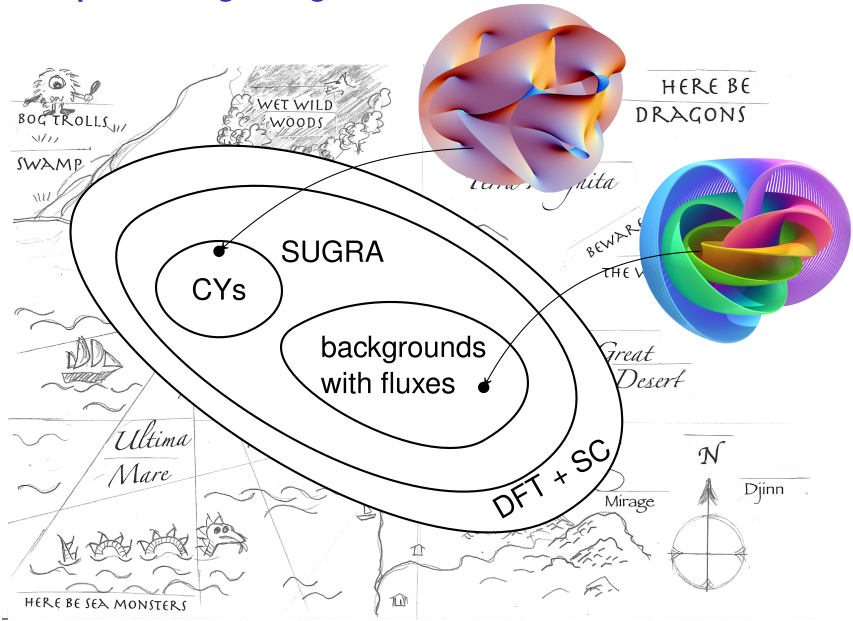

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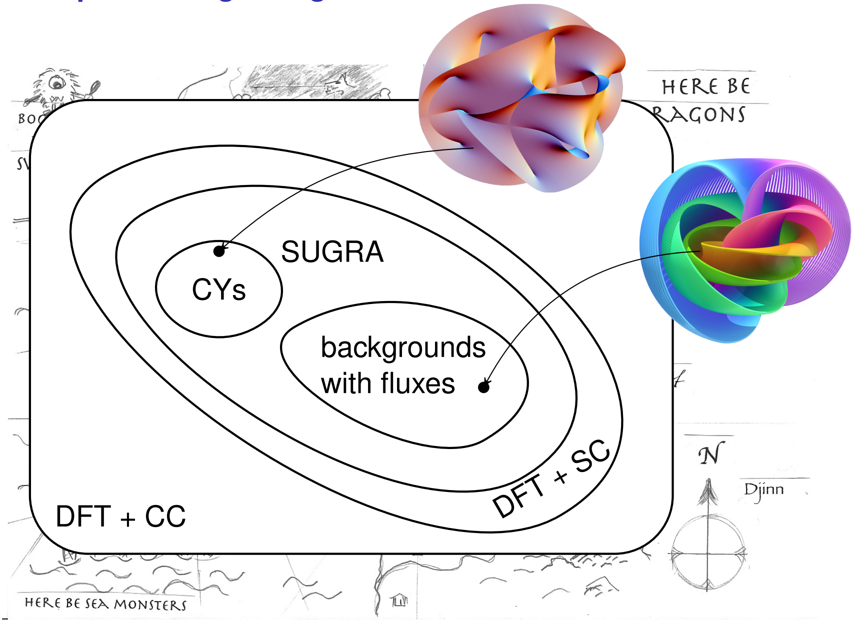

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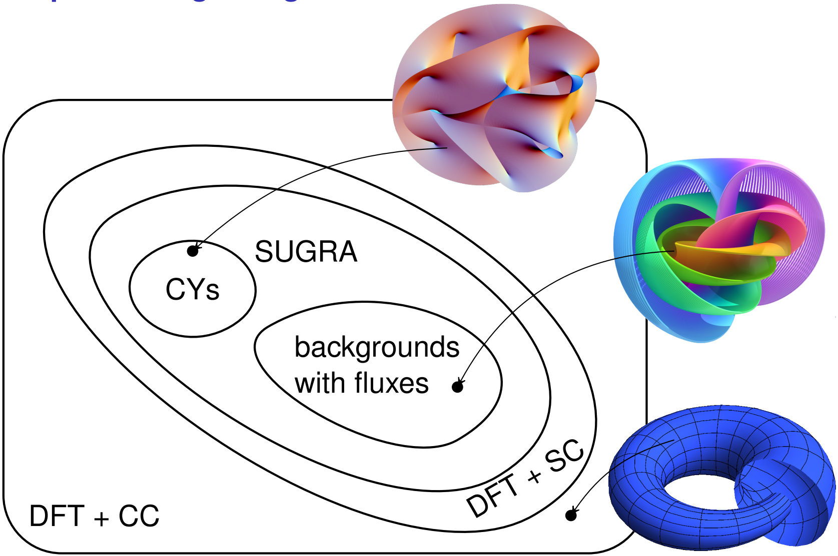

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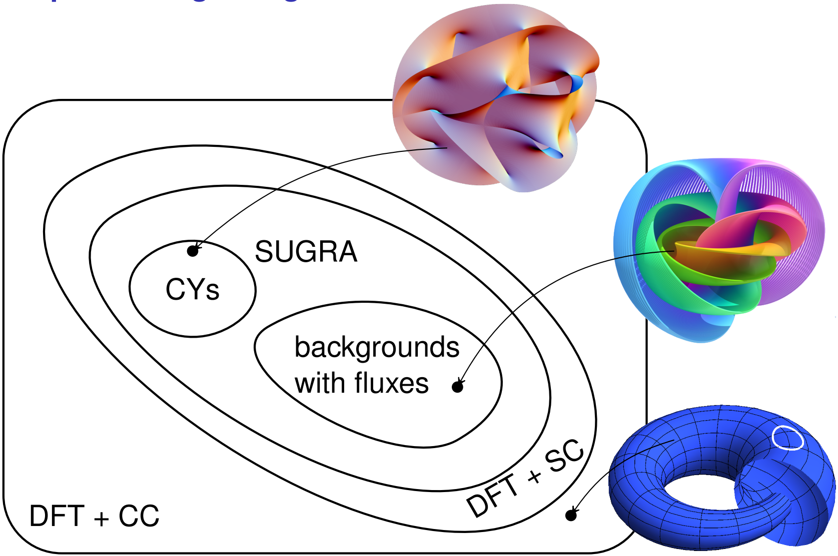

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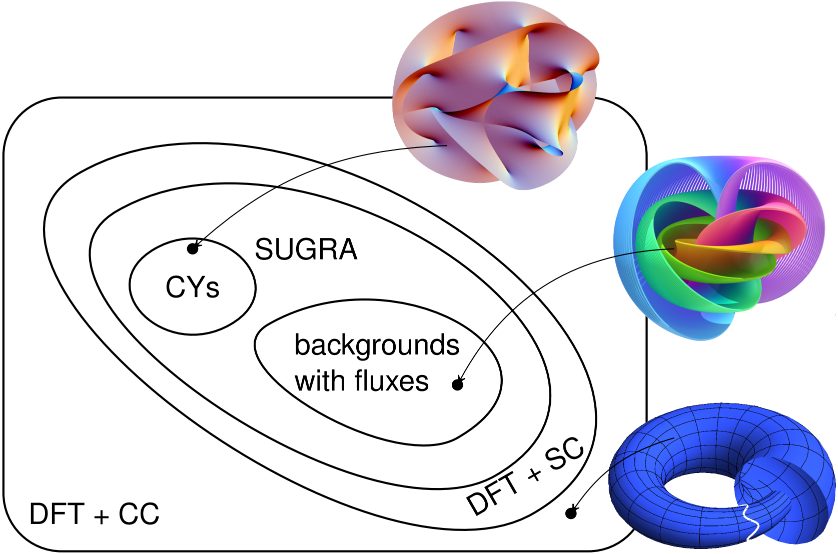

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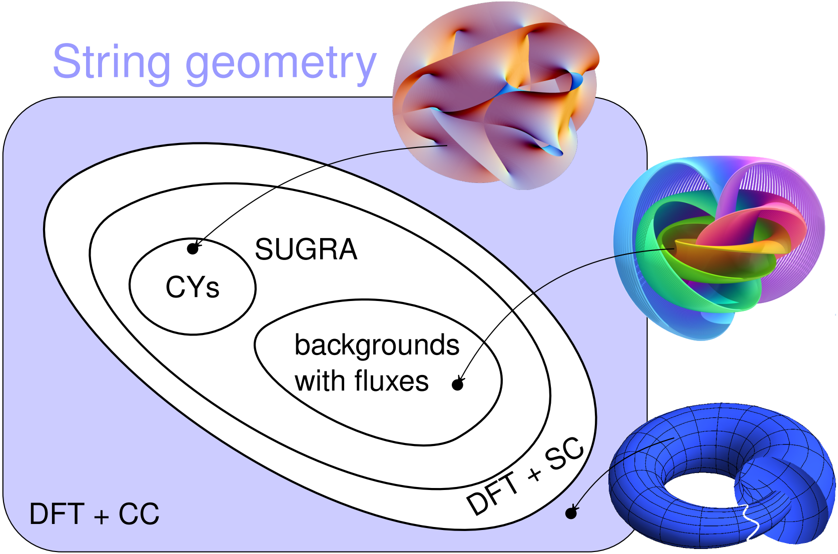

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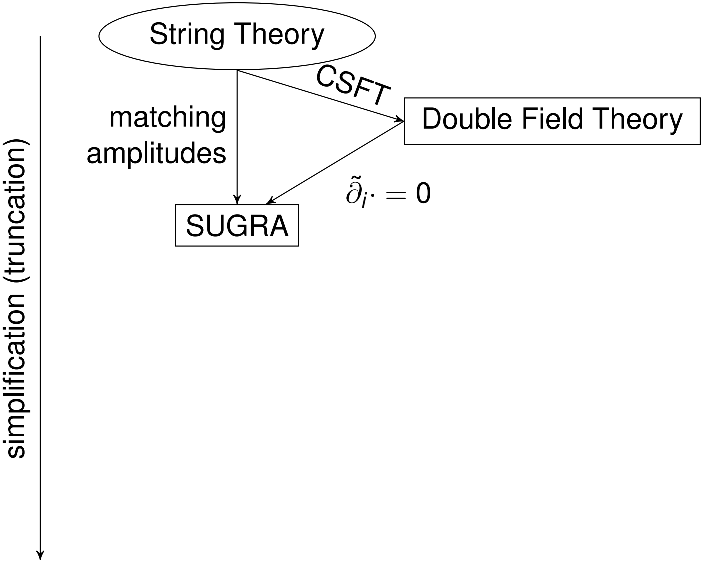
Generalized Scherk-Schwarz compactification

[Aldazabal, Baron, Marques, and Nunez, 2011,Geissbuhler, 2011]



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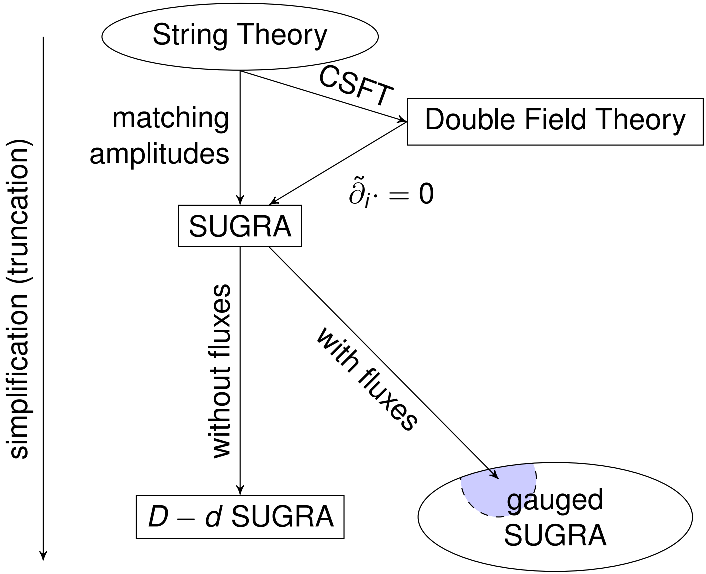

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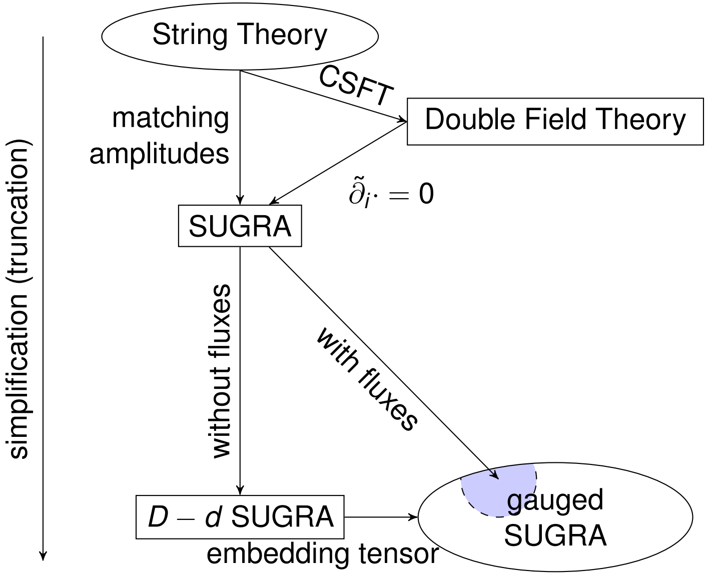

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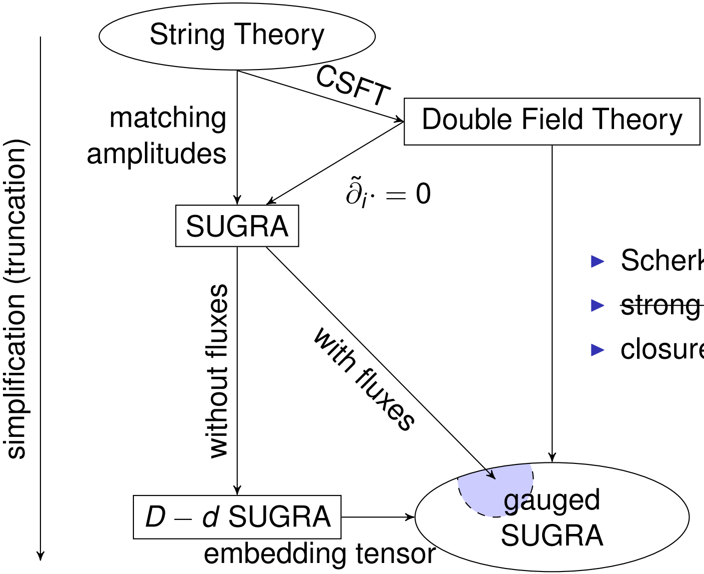

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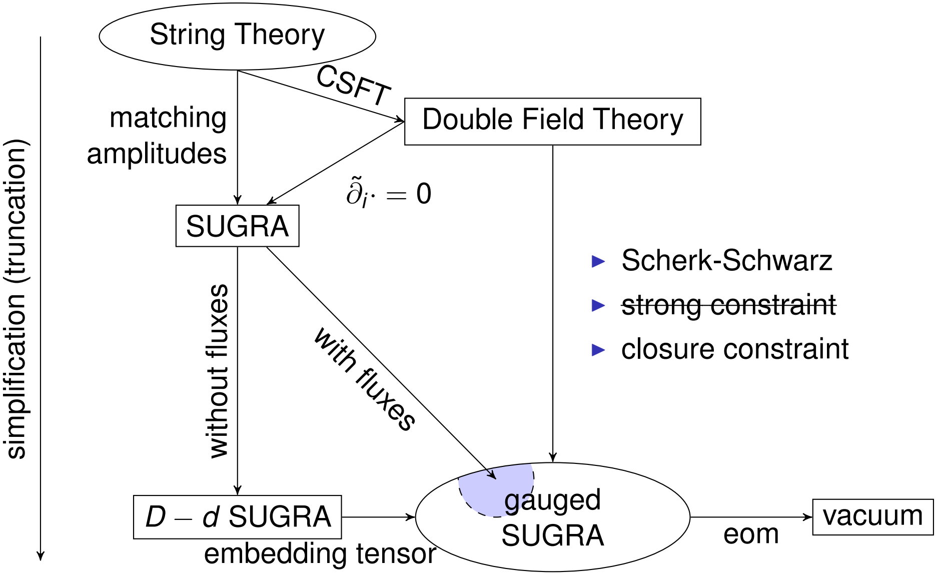

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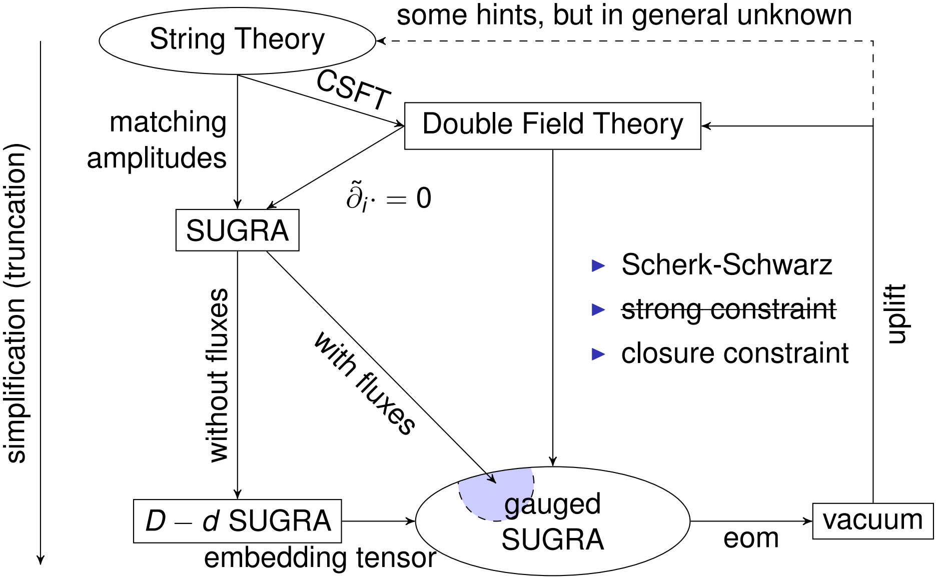

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Use group manifold instead of a torus to derive DFT!

+ non-abelian gauge groups
+ cosmological constant
+ flux backgrounds with const. fluxes

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Questions about DFT_{WZW}

- What are the covariant objects?
- Does it make non-abelian duality manifest?
- How is it connected to DFT?

} not trivial

WZW model & Kač-Moody algebra [Witten, 1983,Walton, 1999]

- $g \in G$, a compact simply connected Lie group

$$S_{\mathrm{WZW}} = \frac{1}{2\pi\alpha'}\int_M d^2z\,\mathcal{K}(g^{-1}\partial g, g^{-1}\bar{\partial} g) + S_{\mathrm{WZ}}(g)$$

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- metric and 3-form flux in flat indices

$$\eta_{ab} := \mathcal{K}(t_a, t_b) \quad \text{and} \quad F_{abc} := \mathcal{K}([t_a, t_b], t_c)$$

- D chiral and D anti-chiral Noether currents (=2D indep. currents)

$$j_a(z) = \frac{2}{\alpha'}\mathcal{K}(\partial g g^{-1}, t_a) \quad \text{and} \quad j_{\bar{a}}(\bar{z}) = -\frac{2}{\alpha'}\mathcal{K}(g^{-1}\bar{\partial}g, t_{\bar{a}})$$

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- radial quantization

$$\boxed{j_a(z) j_b(w) = -\frac{\alpha'}{2}\frac{1}{(z-w)^2}\eta_{ab} + \frac{1}{z-w}F_{ab}{}^c j_c(z) + \ldots}$$

Action

- tree level action in CSFT [Zwiebach, 1993]

$$(2\kappa^2)S = \frac{2}{\alpha'}\left(\langle\Psi|c_0^- Q|\Psi\rangle + \frac{1}{3}\{\Psi,\Psi,\Psi\}_0 + \dots\right)$$

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- string field for massless excitations [Hull and Zwiebach, 2009]

$$|\Psi\rangle = \sum_R \Big[\frac{\alpha'}{4}\epsilon^{a\bar{b}}(R) j_{a\,-1} j_{\bar{b}\,-1} c_1 \bar{c}_1 + e(R) c_1 c_{-1} + \bar{e}(R)\bar{c}_1 \bar{c}_{-1} + \frac{\alpha'}{2}\left(f^a(R) c_0^+ c_1 j_{a\,-1} + f^{\bar{b}}(R) c_0^+ \bar{c}_1 j_{\bar{b}\,-1}\right)\Big]|\phi_w\rangle$$

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$$\frac{\alpha'}{2}\big(f^a(R) c_0^+ c_1 j_{a\,-1} + f^{\bar{b}}(R) c_0^+ \bar{c}_1 j_{\bar{b}\,-1}\big)\Big]|\phi_w\rangle$$

- R is highest weight of $\mathfrak{g}\times\mathfrak{g}$ representation
- BRST operator (L_m from Sugawara construction)

$$Q = \sum_m (: c_{-m} L_m : + \frac{1}{2} : c_{-m} L_m^{gh} :) + \text{anti-chiral}$$

Geometric representation of primary fields ($k \to \infty$)

- flat derivative $$D_a = e_a{}^i \partial_i \quad \text{with} \quad e_a{}^i = \mathcal{K}(g^{-1}\partial^i g, t_a)$$

operator algebra	geometry
$L_0 \vert\phi_R\rangle = h_R \vert\phi_R\rangle$	$D_a D^a Y_R(x^i) = h_R Y_R(x^i)$
$j_{a\,0} \vert\phi_R\rangle$	$D_a Y_R(x^i)$
$[j_{a\,0}, j_{b\,0}] = F_{ab}{}^c j_{c\,0}$	$[D_a, D_b] = F_{ab}{}^c D_c$
$\sum_R e(R) \vert\phi_R\rangle$	$\sum_R e(R) Y_R(x^i) := e(x^i)$

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$[j_{a\,0}, j_{b\,0}] = F_{ab}{}^c j_{c\,0}$	$[D_a, D_b] = F_{ab}{}^c D_c$
$\sum_R e(R)\lvert\phi_R\rangle$	$\sum_R e(R) Y_R(x^i) := e(x^i)$

$$E_A{}^I = \begin{pmatrix} e_a{}^i & 0 \\ 0 & e_{\bar a}{}^{\bar i} \end{pmatrix} \qquad S_{AB} = \begin{pmatrix} \eta_{ab} & 0 \\ 0 & \eta_{\bar a \bar b} \end{pmatrix} \qquad \eta_{AB} = \begin{pmatrix} \eta_{ab} & 0 \\ 0 & -\eta_{\bar a \bar b} \end{pmatrix}$$

Weak constraint (level matching)

- level matched string field $(L_0 - \bar{L}_0)|\Psi\rangle = 0$ requires

$$(D_a D^a - D_{\bar{a}} D^{\bar{a}}) \cdot = 0 \quad \text{with} \quad \cdot \in \{\epsilon^{a\bar{b}}, e, \bar{e}, f^a, f^{\bar{b}}\}$$

- rewritten in terms of η^{AB} and $D_A = \begin{pmatrix} D_a & D_{\bar{a}} \end{pmatrix}$

$$\eta^{AB} D_A D_B \cdot = D_A D^A \cdot = 0 \qquad \text{compare with} \qquad \partial_M \partial^M \cdot = 0$$

Weak constraint (level matching)

- level matched string field $(L_0 - \bar{L}_0)|\Psi\rangle = 0$ requires

$$(D_a D^a - D_{\bar{a}} D^{\bar{a}}) \cdot = 0 \quad \text{with} \quad \cdot \in \{\epsilon^{a\bar{b}}, e, \bar{e}, f^a, f^{\bar{b}}\}$$

- rewritten in terms of η^{AB} and $D_A = \begin{pmatrix} D_a & D_{\bar{a}} \end{pmatrix}$

$$\eta^{AB} D_A D_B \cdot = D_A D^A \cdot = 0 \qquad \text{compare with} \qquad \partial_M \partial^M \cdot = 0$$

- change to curved indices using ${E_A}^M$

$$(\partial_M \partial^M - 2\partial_M d \partial^M) \cdot = 0 \qquad \text{with} \qquad d = \phi - \frac{1}{2} \log \sqrt{g}$$

- additional term which is absent in DFT $\rightarrow$ adsorb in cov. derivative

$$\boxed{\nabla_M \partial^M \cdot = 0} \quad \text{with} \quad \nabla_M V^N = \partial_M V^N + {\Gamma_{MK}}^N V^K \,, \quad {\Gamma_{MK}}^M = -2\partial_K d$$

Results (leading order k^{-1})

- calculate quadratic and cubic string functions
- integrate out auxiliary fields f^a and $f^{\bar{b}}$
- perform field redefinition

$$(2\kappa^2)S = \int d^{2D}X\sqrt{H}\,\Big[\frac{1}{4}\epsilon_{a\bar{b}}\Box\epsilon^{a\bar{b}} + \dots$$
$$-\frac{1}{4}\epsilon_{a\bar{b}}\big(F^{ac}{}_{d}\bar{D}^{\bar{e}}\epsilon^{d\bar{b}}\epsilon_{c\bar{e}} + F^{\bar{b}\bar{c}}{}_{\bar{d}}D^{e}\epsilon^{a\bar{d}}\epsilon_{e\bar{c}}\big)$$
$$-\frac{1}{12}F^{ace}F^{\bar{b}\bar{d}\bar{f}}\epsilon_{a\bar{b}}\epsilon_{c\bar{d}}\epsilon_{e\bar{f}} + \dots\Big]$$

- additional terms e.g. potential
- vanish in abelian limit $F_{abc} \to 0$ and $F_{\bar{a}\bar{b}\bar{c}} \to 0$

Gauge transformations

- tree level gauge transformation in CSFT [Zwiebach, 1993]

$$\delta_\Lambda|\Psi\rangle = Q|\Lambda\rangle + [\Lambda, \Psi]_0 + \cdots$$

- string field for gauge parameter [Hull and Zwiebach, 2009]

$$|\Lambda\rangle = \sum_R \Big[\frac{1}{2}\lambda^a(R) j_{a\,-1} c_1 - \frac{1}{2}\lambda^{\bar{b}}(R) j_{\bar{b}\,-1} \bar{c}_1 + \mu(R) c_0^+\Big]|\phi_R\rangle$$

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- after field redefinition and μ gauge fixing

$$\begin{aligned}\delta_\lambda \epsilon_{a\bar b} =& D_{\bar b}\lambda_a + \frac{1}{2}\big[D_a\lambda^c\epsilon_{c\bar b} - D^c\lambda_a\epsilon_{c\bar b} + \lambda_c D^c\epsilon_{a\bar b} + F_{ac}{}^d\lambda^c\epsilon_{d\bar b}\big]\\ & D_a\lambda_{\bar b} + \frac{1}{2}\big[D_{\bar b}\lambda^{\bar c}\epsilon_{a\bar c} - D^{\bar c}\lambda_{\bar b}\epsilon_{a\bar c} + \lambda_{\bar c} D^{\bar c}\epsilon_{a\bar b} + F_{\bar b\bar c}{}^{\bar d}\lambda^{\bar c}\epsilon_{a\bar d}\big]\\ \delta_\lambda d =& -\frac{1}{4}D_a\lambda^a + \frac{1}{2}\lambda_a D^a d - \frac{1}{4}D_{\bar a}\lambda^{\bar a} + \frac{1}{2}\lambda_{\bar a} D^{\bar a} d\end{aligned}$$

Generalized Lie derivative

- "doubled" version of fluctuations $\epsilon^{a\bar{b}}$

$$\epsilon^{AB} = \begin{pmatrix} 0 & -\epsilon^{a\bar{b}} \\ -\epsilon^{\bar{a}b} & 0 \end{pmatrix} \quad \text{with} \quad \epsilon^{a\bar{b}} = (\epsilon^T)^{\bar{b}a}$$

- generate generalized metric

$$\mathcal{H}^{AB} = S^{AB} + \epsilon^{AB} + \frac{1}{2}\epsilon^{AC} S_{CD} \epsilon^{DB} + \dots = \exp(\epsilon^{AB})$$

with the defining property $\mathcal{H}^{AC}\eta_{CD}\mathcal{H}^{DB} = \eta^{AB}$

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$$\mathcal{L}_\lambda \mathcal{H}^{AB} = \lambda^C D_C \mathcal{H}^{AB} + (D^A\lambda_C - D_C\lambda^A)\epsilon^{CB} + (D^B\lambda_C - D_C\lambda^B)\mathcal{H}^{AC} + F^A{}_{CD}\lambda^C\mathcal{H}^{DB} + F^B{}_{CD}\lambda^C\mathcal{H}^{AD}$$

- setting $\delta_\lambda S^{AB} := 0$ and using

$$\delta_\lambda \epsilon^{AB} = \frac{1}{2}\left(\mathcal{L}_\lambda S^{AB} + \mathcal{L}_\lambda \epsilon^{AB} + \mathcal{L}_\lambda S^{(A}{}_C S^{B)}{}_D \epsilon^{CD}\right) .$$

results in $$\boxed{\delta_\lambda \mathcal{H}^{AB} = \frac{1}{2}\mathcal{L}_\lambda \mathcal{H}^{AB} + \mathcal{O}(\epsilon^2)}$$

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$$\mathcal{L}_\lambda V^A = \lambda^C \nabla_C V^A + (\nabla^A \lambda_C - \nabla_C \lambda^A) V^C$$

Gauge algebra

- CSFT to cubic order fulfills

$$\delta_{\Lambda_1}\delta_{\Lambda_2} - \delta_{\Lambda_2}\delta_{\Lambda_1} = \delta_{\Lambda_{12}} \quad \text{with} \quad \Lambda_{12} = [\Lambda_2, \Lambda_1]_0$$

- after field redefinition and μ fixing $\lambda^A_{12} = \frac{1}{2}[\lambda_2, \lambda_1]^A_{\mathrm{C}}$ with

$$[\lambda_1, \lambda_2]^A_C = \lambda_1^B \nabla_B \lambda_2^A - \frac{1}{2}\lambda_1^B \nabla^A \lambda_{2\,B} - (1 \leftrightarrow 2)$$

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- algebra closes up to a trivial gauge transformation if
  1. fluctuations and parameter fulfill strong constraint $D_A D^A$.
  2. background fulfills closure constraint (CC)

$$F_{E[AB} F^E{}_{C]D} = 0$$

- no strong constraint required for background

Covariant derivative

- non-vanishing torsion and Riemann curvature

$$[\nabla_A, \nabla_B] V_C = R_{ABC}{}^D V_D - T^D{}_{AB} \nabla_D V_C \quad \text{with}$$

$$T^A{}_{BC} = -\frac{1}{3} F^A{}_{BC} \quad \text{and} \quad R_{ABC}{}^D = \frac{2}{9} F_{AB}{}^E F_{EC}{}^D$$

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Comparison DFT and DFT_{WZW}

	DFT	DFT_{WZW}
background	torus	group manifold

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$[\lambda_1, \lambda_2]^I_{\mathrm{C}} =$	$\lambda^J_{[1} \partial_J \lambda^I_{2]} - \frac{1}{2} \lambda^J_{[1} \partial^I \lambda_{2]\,J}$	$\lambda^J_{[1} \nabla_J \lambda^I_{2]} - \frac{1}{2} \lambda^J_{[1} \nabla^I \lambda_{2]\,J}$

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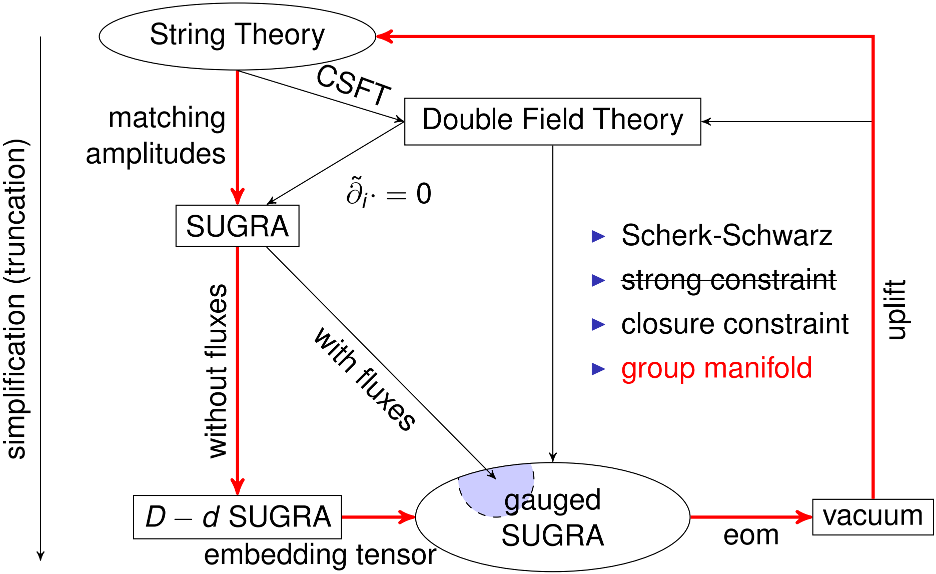
abelian limit

Reminder: Generalized Scherk-Schwarz compactification



- Scherk-Schwarz
- ~~strong constraint~~
- closure constraint
- group manifold

Embedding tensor

ID	$M_{mn}/\cos\alpha$	$\tilde{M}^{mn}/\sin\alpha$	range of α	gauging
1	diag(1, 1, 1, 1)	diag(1, 1, 1, 1)	$-\frac{\pi}{4} < \alpha \leq \frac{\pi}{4}$	$\begin{cases} \mathrm{SO}(4)\,, & \alpha \neq \frac{\pi}{4}\,, \\ \mathrm{SO}(3)\,, & \alpha = \frac{\pi}{4}\,. \end{cases}$
2	diag(1, 1, 1, −1)	diag(1, 1, 1, −1)	$-\frac{\pi}{4} < \alpha \leq \frac{\pi}{4}$	SO(3, 1)
[illegible]	…(1, 1…	diag(1… 1)	… $\frac{\pi}{4}$	SO(2,2) , α … $\frac{\pi}{4}$, …

[Dibitetto, Fernandez-Melgarejo, Marques, and Roest, 2012]

- fluxes for embedding one

$$F_{abc} = \sqrt{2}\epsilon_{abc}(\cos\alpha + \sin\alpha) \quad \text{and} \quad F_{\bar{a}\bar{b}\bar{c}} = \sqrt{2}\epsilon_{abc}(\cos\alpha - \sin\alpha)$$

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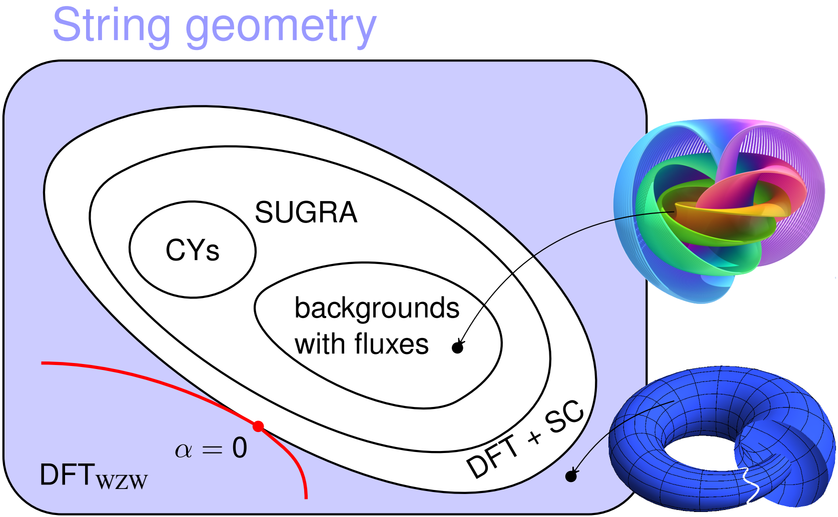
- DFT strong constraint holds only for

$$F_{ABC}F^{ABC} = 24\sin(2\alpha) = 0 \quad \rightarrow \alpha = \frac{\pi}{2}n \quad n \in \mathbb{Z}$$

- closure constraint holds always

The landscape again



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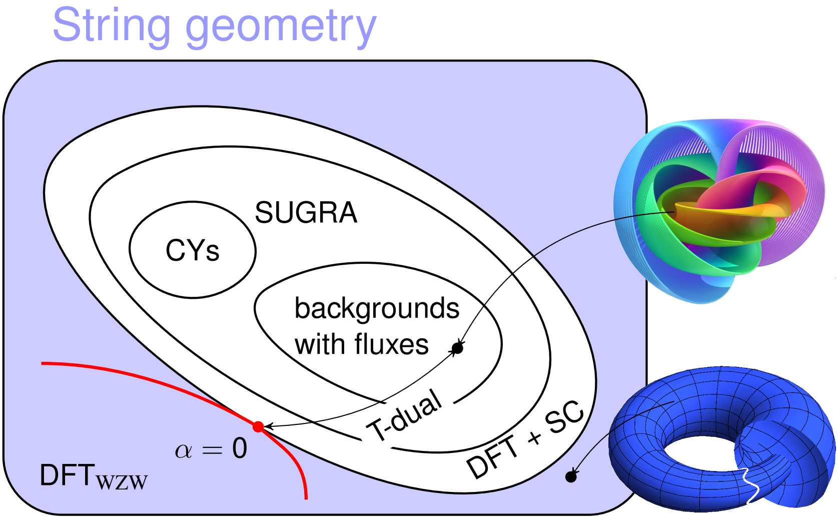

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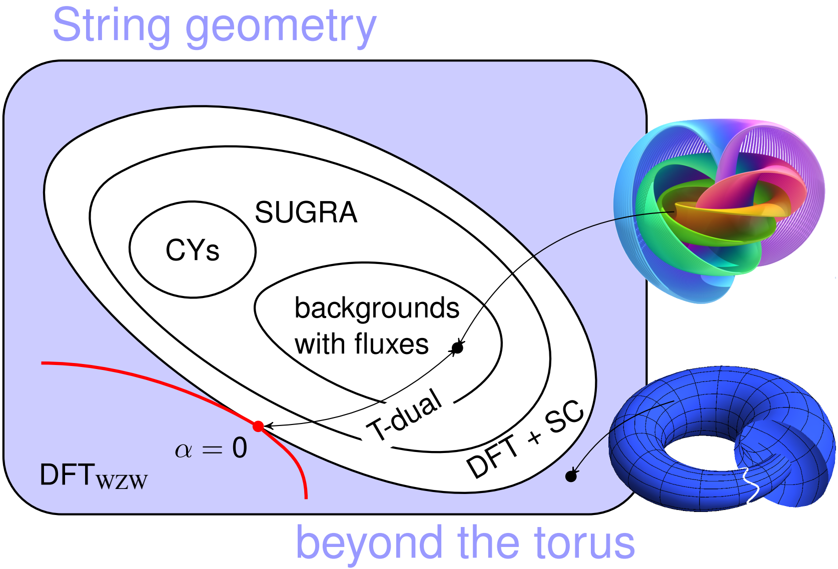

Summary

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- phenomenology of non-geometric backgrounds [Hassler, Lust, and Massai, 2014]

Thank you for your attention. Are there any questions?