Generalized Geometry of α'-corrections

Falk Hassler

Based on 2409.00176, 2412.17893 and 2412.17900 with

Daniel Butter, Achilles Gitsis,
Ondřej Hulík and David Osten

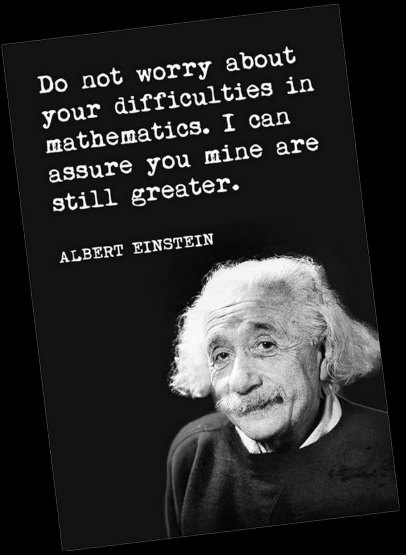
Uniwersytet Wrocławski

Narodowe Centrum Nauki

The Problem

- Einstein-Hilbert action is not renormalizable in d>2 → only EFT

$$S = \int \mathrm{d}x^d \sqrt{-g} \left(R + a_1 R^2 + a_2 R_{ij} R^{ij} + \ldots \right)$$



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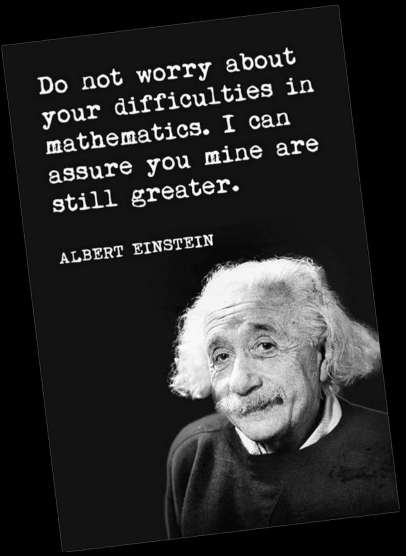
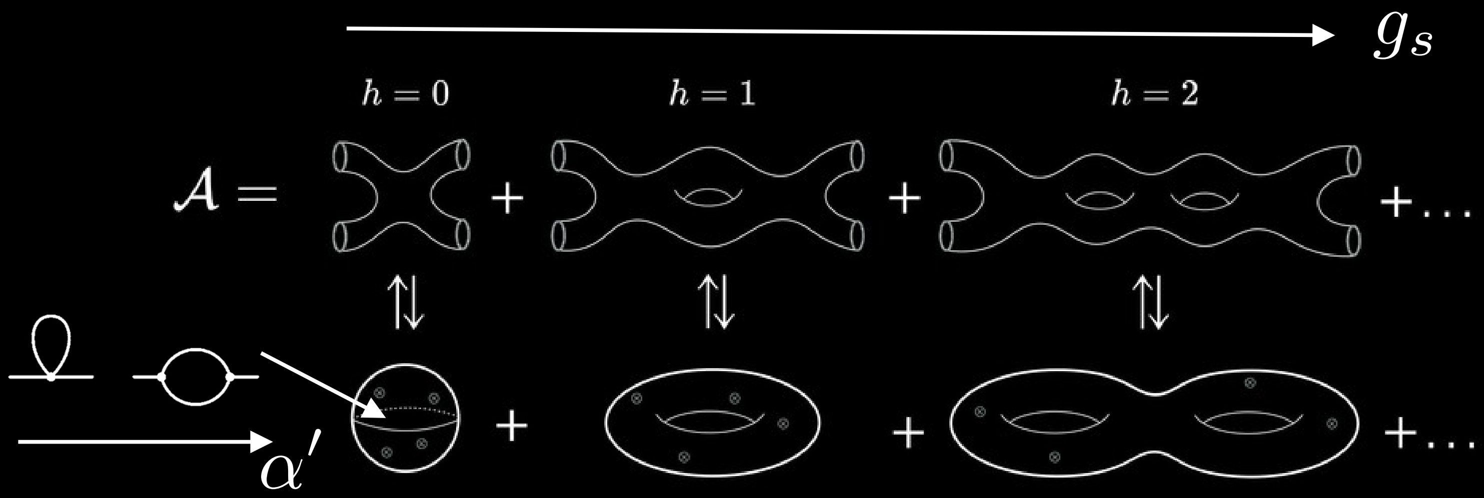
Question: How do we obtain all the coefficients ?

String Theory





NS/NS-sector @ leading order in α'

$$\widetilde{H}_{ijk} = H_{ijk} - \tfrac{3}{2} a \Omega^{(-)}_{ijk} + \tfrac{3}{2} b \Omega^{(+)}_{ijk}$$

$$S = \int \mathrm{d}x^d \sqrt{-g} e^{-2\phi} \Big(R + 4(\partial\phi)^2 - \tfrac{1}{12} \widetilde{H}^2 + \tfrac{a}{8} R^{(-)\,b}_{ija} R^{(-)ij}{}_b{}^a + \tfrac{b}{8} R^{(+)\,b}_{ija} R^{(+)ij}{}_b{}^a + \dots \Big)$$

$a = -\alpha, b = 0$	heterotic
$a = b = -\alpha'$	bosonic
$a = b = 0$	type II

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- 3 coefficients for terms with 2
- 8 coefficients for terms with 4
- 60 coefficients for terms with 6

} derivatives

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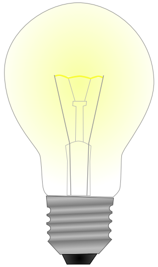
Too many terms. Nothing is known about > 8 derivatives.



A better approach:

Leverage symmetry to decrease number of possible terms.

Like diffeomorphisms, gauge-transformations and:

- SUSY
- Extended Generalized Lorentz Symmetry (today)
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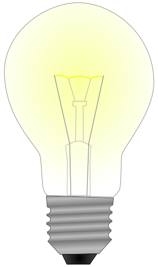

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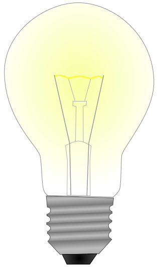
generalized frame

$$\mathrm{E}_A{}^I = \begin{pmatrix} e_a{}^i & e_a{}^j B_{ji} \\ 0 & e^a{}_i \end{pmatrix}$$



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$$\mathrm{E}_A{}^I = \begin{pmatrix} e_a{}^i & e_a{}^j B_{ji} \\ 0 & e^a{}_i \end{pmatrix}$$

invariant under O(d)xO(d) $\subset$ O(d,d)

$$\eta_{AB} = \begin{pmatrix} 0 & \delta_\alpha^\beta \\ \delta_\beta^\alpha & 0 \end{pmatrix} \quad \mathrm{H}_{AB} = \begin{pmatrix} \delta_{ab} & 0 \\ 0 & \delta^{ab} \end{pmatrix}$$

Leading Symmetries and Action

$$\delta E^A{}_M = \mathbb{L}_\xi E^A{}_M + \Lambda^A{}_B E^B{}_M \,, \qquad \Lambda^A{}_B \in \mathrm{O}(d) \times \mathrm{O}(d)$$

generalized Lie derivative

1) diffeomorphisms (gravity)
2) gauge tranformation

generalized Lorentz transformation

transformation of fermions

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generalized flux $F_{ABC} = 3D_{[A} E_B{}^I E_{C]I}$ with $\mathrm{D}_A = E_A{}^i \partial_i$

$$F_A = D_A d - \partial_i E_A{}^i \qquad d = -\tfrac{1}{2}\log(-g) + \phi$$

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$$S = \int dx^d \, e^{-2d} \, \mathcal{R}$$

one unique invariant

$$\mathcal{R}(F_{ABC}, F_A, D_A, H_{AB})$$

We need a Factory for Invariants...

[Polacek, Siegel 13;
Butter 21;
Butter, FH, Pope, Zhang 23]

Symmetries

- gen. diff
- gen. Lorentz

Invariants

$\mathcal{R}, \ldots$

Generalized Cartan Geometry*

*) also known as Poláček-Siegel construction, or for mathematicians: symplectic reduction of Courant algebroids

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Covariant derivative:

gen. spin and affine connection, related by

$$\nabla_A E_B{}^M = 0$$

$$\nabla_A E_B{}^M = E_A{}^N \partial_N E_B{}^M + \Omega_{AB}{}^C E_C{}^M - E_A{}^N \Gamma_{NL}{}^M E_B{}^L$$

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Curvature and torsion???:

$$[\nabla_A, \nabla_B] V^C = R_{ABD}{}^C V^D + T_{AB}{}^D \nabla_D V^C$$

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$R_{ABC}{}^D \,\&\, T_{AB}{}^C$ are not covariant

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Solution: Poláček-Siegel constr.

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2 connections are required: $\Omega_A^{\alpha}\,,\,\rho^{\alpha\beta}$ ← adjoint index of the gen. Lorentz group G_{S}

$\Omega_A^{\alpha}\,,\,\rho^{\alpha\beta}\,,\,E_A{}^I$ parameterize a mega-frame $\mathcal{E}_{\mathcal{A}}{}^{\mathcal{I}} \in G_{\mathrm{PS}}$

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$$\mathrm{O}(d+n,d+n) \to \mathrm{O}(d,d) \times G_{\mathrm{S}}\,, n = \dim(G_{\mathrm{S}})$$

$$\cup$$

$$G_{\mathrm{PS}} \supset G_{\mathrm{S}}$$

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<u>Cartan connection</u> $\theta(x) : T_x P \to \mathfrak{g}$

$$\theta^{\hat{a}}{}_{\hat{i}} = \begin{pmatrix} \delta^{\alpha}_{\mu} & \omega^{\alpha}{}_{i} \\ 0 & e^{a}{}_{i} \end{pmatrix}$$

$$\theta = t_{\hat{a}} \theta^{\hat{a}}{}_{\hat{i}} \mathrm{d}x^{\hat{i}}$$

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Cartan curvature

$$\Theta = -\mathrm{d}\theta + \frac{1}{2}[\theta, \theta]$$

$$T = -\mathrm{d}e + [\omega, e]$$

$$R = -\mathrm{d}\omega + \frac{1}{2}[\omega, \omega]$$

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Generalized Cartan curvature

$$\Theta_{\hat{A}\hat{B}} = -[\theta_{\hat{A}}, \theta_{\hat{B}}]_{\mathrm{D},\mathfrak{d}}$$

$\mathfrak{d}$-twisted Dorfman-bracket

Choosing G_{S} and G_{PS}

Objective:

1) fix all connections by
   1) gauge fixing
   2) torsion constraints

   in terms of the generalized frame (and its derivatives)
2) as few invariants as possible

We do the same in General Relativity.

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$$G_{\mathrm{S}} = \mathrm{O}(d+p) \times \mathrm{O}(d+q)$$
$$G_{\mathrm{PS}} = \mathrm{O}(d+p, d+q)$$

G_{PS} in more detail

- we split the generators into $\mathrm{O}(d,d) \subset G_{\mathrm{PS}}$

$$\mathcal{K}_{\mathcal{AB}} = \begin{pmatrix} K_{AB} & -\frac{1}{2} R_A^{\beta} \\ \frac{1}{2} R_B^{\alpha} & -\frac{1}{2} R^{\alpha\beta} \end{pmatrix}$$

which are governed by

$$[\mathcal{K}_{\mathcal{AB}}, \mathcal{K}_{\mathcal{CD}}] = 2\eta_{[\mathcal{A}|[\mathcal{C}}\mathcal{K}_{\mathcal{D}]|\mathcal{B}]} \quad \text{with} \quad \eta_{\mathcal{AB}} = \begin{pmatrix} \eta_{AB} & 0 \\ 0 & \kappa^{\alpha\beta} \end{pmatrix}$$

- we also need

$$R_{\alpha} = \frac{1}{2} f_{\alpha\beta}{}^{\gamma} \kappa^{\beta\delta} R_{\gamma\delta}$$

structure coefficients of G_{S}

Recursive embedding of G_{S}

- G_{PS} is generated by $K_{AB}, {R_\alpha}^A, R_{\alpha\beta}$
- and G_{S} by $\tau_\alpha = \begin{pmatrix} \tau_{\overline{\alpha}} & \tau_{\underline{\alpha}} \end{pmatrix}$

How to relate them ???

left and right factors of G_{S}

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- exponential growth of generators
- can be truncated at every order

A game of splitting indices

$$[\tau_{\underline{\alpha}}, \tau_{\underline{\beta}}] = -f_{\underline{\alpha}\underline{\beta}}{}^{\underline{\gamma}}\tau_{\underline{\gamma}}$$

$$[\tau_{\underline{\alpha}_I\underline{\beta}_J}, \tau_{\underline{\gamma}_K\underline{\delta}_L}] = 2g_-\eta_{[\underline{\alpha}_I|[\underline{\gamma}_K}\tau_{\underline{\delta}_L]|\underline{\beta}_J]}$$

$$\frac{1}{g_-^2}\langle\!\langle\tau_{\underline{\alpha}_I\underline{\beta}_J}, \tau_{\underline{\gamma}_K\underline{\delta}_L}\rangle\!\rangle = \eta_{[\underline{\alpha}_I|[\underline{\gamma}_K}\eta_{\underline{\delta}_L]|\underline{\beta}_J]} = \kappa_{\underline{\alpha}_I\underline{\beta}_J\underline{\gamma}_K\underline{\delta}_L}$$

Example: $\kappa_{\underline{\alpha}_1\underline{\beta}_1} = \kappa_{\underline{a}_1\underline{a}_2\underline{b}_1\underline{b}_2} = \eta_{[\underline{a}_1|[\underline{b}_1}\eta_{\underline{b}_2]|\underline{a}_2]}$

Torsion contraints and gauge fixing

- Poláček-Siegel construction results one quantity (product):

The generalized Cartan curvature $\Theta_{\mathcal{ABC}}$ ← fundamental index of G_{PS}

- Remember, it contains all curvatures and torsions of the gen. connections

$$\mathcal{A}_{\mathcal{A}}{}^{\beta} = \begin{pmatrix} \Omega_A^{\beta} & \rho^{\beta\alpha} \end{pmatrix} .$$

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- To fix them completely, we impose:

$$\Theta_{\overline{\mathcal{A}}\underline{\mathcal{BC}}} = \Theta_{\underline{\mathcal{A}}\overline{\mathcal{BC}}} = 0$$

Torsion contraint

$$\Omega_{\overline{a}}^{\overline{\alpha}} = \Omega_{\underline{a}}^{\underline{\alpha}} = \rho^{\overline{\alpha}\overline{\beta}} = \rho^{\underline{\alpha}\underline{\beta}} = 0$$

Gauge fixing of chiral/anti-chiral sector

Torsion contraints

$$\mathcal{A}^{(l)\beta}_{\mathcal{A}} \tau_\beta \cong -\boldsymbol{\mathcal{F}}^{(l)}_{\mathcal{A}} = \mathcal{A}^{(l)\beta}_{\mathcal{A}} R_\beta - \boldsymbol{\mathcal{F}}^{(l)}_{\mathcal{A}}[A^{(<l)}]$$

$$\boldsymbol{\mathcal{F}}_{\mathcal{A}} = \prec_{\mathcal{A}}|\mathcal{A}|^{B}\succ\left(D_B \mathcal{A}\mathcal{A}^{-1} + \mathcal{A}\boldsymbol{F}_B\mathcal{A}^{-1}\right) - \prec_{\mathcal{A}}|\mathcal{A}|_\beta\succ \mathcal{A}\,\mathrm{R}^\beta \mathcal{A}^{-1} + \prec_{\mathcal{A}}|D_B\mathcal{A}\mathcal{A}^{-1}\mathcal{Z}\mathcal{A}|^{B}\succ$$

$$\mathcal{A} = \exp\left(A + c_3 A^3 + c_5 A^5 + \dots\right)$$

$$\boldsymbol{F}_A = F_{ABC} K^{BC}$$

fixed by torsion constraints

Collapsing towers

The real identification is $$\mathcal{A}^{(l)\beta}_{\mathcal{A}} t_\beta \cong -\boldsymbol{\mathcal{F}}^{(l)}_{\mathcal{A}}[A^{(<l)}]$$

$$\downarrow$$

$$\tau_\alpha = t_\alpha + R_\alpha$$

similarity transformation required $$t_{\underline{\alpha}} = S_{\underline{\alpha}}{}^{\underline{\beta}}\tau_{\underline{\beta}}$$

and we have to compute $$\widetilde{\kappa}_{\alpha\beta} := (S^{-1})_{\underline{\alpha}}{}^{\underline{\gamma}}\,\kappa_{\underline{\gamma}\underline{\delta}}\,(S^{-1})_{\underline{\beta}}{}^{\underline{\delta}}$$

$$\widetilde{f}_{\underline{\alpha}\underline{\beta\gamma}} := (S^{-1})_{\underline{\alpha}}{}^{\underline{\delta}}(S^{-1})_{\underline{\beta}}{}^{\underline{\epsilon}}(S^{-1})_{\underline{\gamma}}{}^{\underline{\rho}}f_{\underline{\delta\epsilon}\underline{\rho}}$$

Solutions for torsion constraints

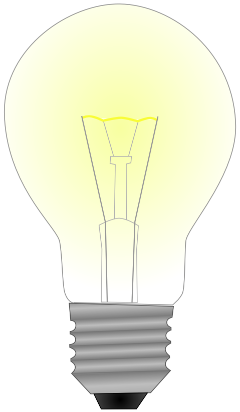
for convenience we define $\widetilde{A}^{(l)}_{\mathcal{A}}{}^{\beta}\boldsymbol{\tau}_{\beta} = A^{(l)}_{\mathcal{A}}{}^{\beta} t_{\beta}$ to get

$$\widetilde{A}^{(1)}_{\overline{a}\underline{\beta}_1} = \widetilde{A}^{(1)}_{\overline{a}}{}^{\underline{b}_1\underline{b}_2} = -\frac{1}{g_-}F_{\overline{a}}{}^{\underline{b}_1\underline{b}_2}$$

$$\widetilde{A}^{(2)}_{\overline{a}}{}^{\underline{b}}{}_{\underline{\beta}} = \frac{1}{g_-}\left(F_{\overline{a}}{}^{\underline{b}\overline{c}}A^{(1)}_{\overline{c}\underline{\beta}} + D^{\underline{b}}A^{(1)}_{\overline{a}\underline{\beta}}\right)$$

$$\widetilde{A}^{(2)}_{\overline{\alpha}}{}^{\underline{b}_1\underline{b}_2} = -\frac{1}{g_-}\Big(2D^{[\underline{b}_1}A^{(1)\underline{b}_2]}{}_{\overline{\alpha}} - A^{(1)\underline{a}}{}_{\overline{\alpha}}F_{\underline{a}}{}^{\underline{b}_1\underline{b}_2} - A^{(1)\underline{b}_1\overline{\beta}}A^{(1)\underline{b}_2\overline{\gamma}}f_{\overline{\alpha}\overline{\beta}\overline{\gamma}}\Big)$$

Linus' and Stan's
"generalized Riemann tensor"



Gauge fixing

objective: preserve

$$\Omega^{\overline{\alpha}}{}_{\overline{a}} = \Omega^{\underline{\alpha}}{}_{\underline{a}} = \rho^{\overline{\alpha}\overline{\beta}} = \rho^{\underline{\alpha}\underline{\beta}} = 0$$

Gauge fixing of chiral/anti-chiral sector

under gauge transformations

$$\delta A^{(l)} + \delta E^{(l)} E^{-1} + \cdots = \xi^{(l)\alpha} t_{\alpha} + \ldots$$

depend on lower orders already fixed

allows to fix $\delta A^{(l)},\, \delta E^{(l)},\, \xi^{(l)}$

Universal gen. GS transformations

$$\delta E^{(2m)}_{\underline{a}\overline{b}} = A_{\underline{a}\overline{\alpha}} D_{\overline{b}} \xi^{\overline{\alpha}} - \text{c.c.}\Big|^{(2m)}$$

*) TODO:

- collapsing the towers
- inserting previous solutions from torsion contraints & gauge fixing

$$\begin{aligned}\delta E^{(4)}_{\underline{a}\overline{b}} =& \overline{\chi} \widetilde{A}^{(3)}_{\underline{a}\overline{\alpha}_1} D_{\overline{b}} \widetilde{\xi}^{(0)\overline{\alpha}_1} + \overline{\chi} \widetilde{A}^{(2)}_{\underline{a}\overline{\alpha}} D_{\overline{b}} \widetilde{\xi}^{(1)\overline{\alpha}} + \overline{\chi} \widetilde{A}^{(1)}_{\underline{a}\overline{\alpha}_1} D_{\overline{b}} \widetilde{\xi}^{(2)\overline{\alpha}_1} + \\ & \overline{\chi}_1 \widetilde{A}^{(3)}_{\underline{a}\overline{\alpha}} D_{\overline{b}} \widetilde{\xi}^{(0)\overline{\beta}_1} (\widetilde{S}^{-1})_{\overline{\beta}_1}{}^{\overline{\alpha}} + \overline{\chi} \widetilde{A}^{(1)\overline{\alpha}_1}_{\underline{a}} D_{\overline{b}} \widetilde{\xi}^{(2)}_{\overline{\beta}} (\widetilde{S}^{-1})_{\overline{\alpha}_1}{}^{\overline{\beta}} - \text{c.c.}\,.\end{aligned}$$

…and again

$$
\begin{aligned}
\delta E^{(4)}_{\underline{a}\bar{b}} = -\frac{a^2}{2}\Big[& D_{\underline{a}} D_{\underline{c}} \Lambda_{\underline{d}\underline{e}} \Big(F^{\underline{c}}{}_{\bar{f}\bar{b}} F^{\bar{f}\underline{d}\underline{e}} + D^{\underline{c}} F_{\bar{b}}{}^{\underline{d}\underline{e}} \Big) - F_{\bar{b}\underline{f}}{}^{\underline{g}} F^{\bar{c}}{}_{\underline{d}\underline{g}} \big(F_{\bar{c}}{}^{\underline{e}\underline{d}} D_{\underline{a}} \Lambda_{\underline{e}}{}^{\underline{f}} - F_{\bar{c}}{}^{\underline{e}\underline{f}} D_{\underline{a}} \Lambda_{\underline{e}}{}^{\underline{d}} \big) \\
& + D_{\underline{a}} \Lambda_{\underline{e}\underline{f}} F^{\bar{c}\underline{e}}{}_{\underline{d}} \Big(F_{\bar{b}\bar{c}\bar{g}} F^{\bar{g}\underline{f}\underline{d}} - D_{\bar{b}} F_{\bar{c}}{}^{\underline{f}\underline{d}} + 2 D_{\bar{c}} F_{\bar{b}}{}^{\underline{f}\underline{d}} \Big) + F_{\bar{b}}{}^{\underline{e}\underline{d}} D_{\underline{a}} \Big(D^{\bar{c}} \Lambda_{\underline{e}}{}^{\underline{f}} F_{\bar{c}\underline{f}\underline{d}} \Big) \Big] \\
-\frac{ab}{4}\Big[& D_{\underline{a}} \Lambda^{\underline{c}\underline{d}} \Big(F_{\bar{b}\underline{c}\underline{g}} F^{\underline{g}\bar{e}\bar{f}} F_{\underline{d}\bar{e}\bar{f}} - D_{\bar{b}} F_{\underline{c}}{}^{\bar{e}\bar{f}} F_{\underline{d}\bar{e}\bar{f}} \Big) + F_{\bar{b}\underline{c}\underline{d}} D_{\underline{a}} \Big(D^{\underline{c}} \Lambda_{\bar{e}\bar{f}} F^{\underline{d}\bar{e}\bar{f}} \Big) \\
& - D_{\bar{b}} \Lambda^{\bar{c}\bar{d}} \Big(F_{\underline{a}\bar{c}\bar{g}} F^{\bar{g}\underline{e}\underline{f}} F_{\bar{d}\underline{e}\underline{f}} - D_{\underline{a}} F_{\bar{c}}{}^{\underline{e}\underline{f}} F_{\bar{d}\underline{e}\underline{f}} \Big) - F_{\underline{a}\bar{c}\bar{d}} D_{\bar{b}} \Big(D^{\bar{c}} \Lambda_{\underline{e}\underline{f}} F^{\bar{d}\underline{e}\underline{f}} \Big) \Big] \\
+\frac{b^2}{2}\Big[& D_{\bar{b}} D_{\bar{c}} \Lambda_{\bar{d}\bar{e}} \Big(F^{\bar{c}}{}_{\underline{f}\underline{a}} F^{\underline{f}\bar{d}\bar{e}} + D^{\bar{c}} F_{\underline{a}}{}^{\bar{d}\bar{e}} \Big) - F_{\underline{a}\bar{f}}{}^{\bar{g}} F^{\underline{c}}{}_{\bar{d}\bar{g}} \Big(F_{\underline{c}}{}^{\bar{e}\bar{d}} D_{\bar{b}} \Lambda_{\bar{e}}{}^{\bar{f}} - F_{\underline{c}}{}^{\bar{e}\bar{f}} D_{\bar{b}} \Lambda_{\bar{e}}{}^{\bar{d}} \Big) \\
& + D_{\bar{b}} \Lambda_{\bar{e}\bar{f}} F^{\underline{c}\bar{e}}{}_{\bar{d}} \Big(F_{\underline{a}\underline{c}\underline{g}} F^{\underline{g}\bar{f}\bar{d}} - D_{\underline{a}} F_{\underline{c}}{}^{\bar{f}\bar{d}} + 2 D_{\underline{c}} F_{\underline{a}}{}^{\bar{f}\bar{d}} \Big) + F_{\underline{a}}{}^{\bar{e}\bar{d}} D_{\bar{b}} \Big(D^{\underline{c}} \Lambda_{\bar{e}}{}^{\bar{f}} F_{\underline{c}\bar{f}\bar{d}} \Big) \Big]
\end{aligned}
$$

matches [Baron, Marques 20]

Remarks

- requires a particular prescription of “collapsing the towers”
- for other “regularizations” residual transformations will not close
- invariant action follows from the mega-space (=standard two derivative action there)

Results

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$$G_{\mathrm{S}} \to \widehat{\mathrm{O}}(d) \times \widehat{\mathrm{O}}(d)$$

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There is a hidden symmetry in string theory which controls higher-derivative(α’)-corrections. How far can we push it?