Quanten Field Theory Summer 2022

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# 12. Renormalisation group flow

To be discussed on Tuesday, 7th June, 2022 in the seminar.

In addition to computing the $\beta$-functions of massless Yukawa theory, which we started last exercise and will finish together in teams of two, we further practice how to obtain the renormalisation group flow for the Gross-Neveu model in two dimensions. It was originally introduced by David Gross (Nobel prize 2004) and André Neveu as a toy model for quantum chromodynamics.

### Exercise 12.1: The Gross-Neveu model

We want to compute the one-loop $\beta$-function for the coupling $g$ of the Lagrangian

$$\mathcal{L} = \sum_{i=1}^{N} \left[ i\overline{\psi}_i \partial\!\!\!/ \psi_i + \frac{1}{2} g^2 (\overline{\psi}_i \psi_i)^2 \right] .$$

It describes $N$ Dirac fermions in two dimensions interacting through a four fermion term.

a) Derive the Feynman rules for this model. *Hint: We have done this step already very often and the final result will be not given here. If you are in double you should rather try to have a look at the literature. This is in general a good idea when you start to work with a new model.*

b) Compute the superficial degree of divergence and identify the divergent diagrams. *Hint: You should find two.*

c) Obtain the counter terms for the divergent diagrams and write down their Feynman rules.

d) Fix the divergent contribution to the counter terms for the propagator at one-loop in dimensional regularisation.

e) Repeat this step for the vertex diagram. *Hint: It has four external legs. Therefore, you already know that there has to be an s-, t- and u-channel. However, you cannot compute them in one shot, like we did for scalars because here the Feynman rules have a more complicated index structure which you have to take into account.*

f) Compute the $\beta$-function for the coupling $g$. *Hint: For this computation you need the* $\log M^2$ *divergent term from the counter terms fixed above. But, we just computed the divergent part yet. Use dimensional analysis to see how the* $\log M^2$ *contribution can be directly obtained from the* $\epsilon^{-1}$ *part we already have.*