Quanten Field Theory Summer 2022

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7. Quantum Electrodynamics

To be discussed on Tuesday, 26th April, 2022 in the seminar.

We have completed our discussion of the path integral in quantum field theory. Therefore it is time to put it to some use. We do not know yet how to deal with divergent loop diagrams but we can already compute some simple scattering processes. This is exactly what we will do.

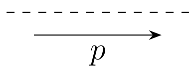
Exercise 7.1: QED Coupled To A Scalar Field

This problems considers quantum electrodynamics coupled to a complex scalar field ϕ of mass m. The Lagrangian is

$$\mathcal{L} = \mathcal{L}_{\text{QED}} + (D_\mu \phi)^* D^\mu \phi - m^2 \phi$$

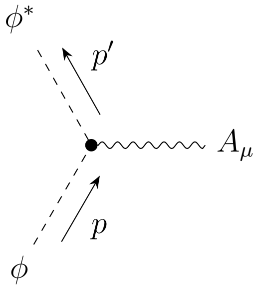
where $D_\mu = \partial_m u + ieA_\mu$ is the usual gauge-covariant derivative.

a) Show that the propagator of the complex scalar field is the same as that of a real field, namely

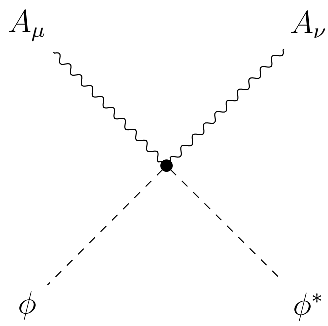


$$= \frac{i}{p^2 - m^2 + i\epsilon} \,.$$

b) Derive the Feynman rules for the interactions between photons and scalar particles. *Hint: You should find*

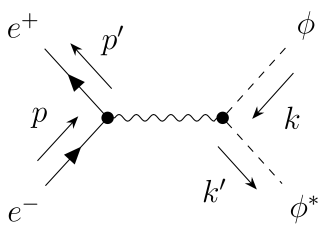


$= -ie(p + p')^\mu$ *and*



$$= 2ie^2 g^{\mu\nu} \quad (1)$$

c) We want to compute the process $\mathcal{M}e^+e^- \to \phi\phi^*$, given by the diagram



$$= i\mathcal{M}(e^+e^- \to \phi\phi^*) \,.$$

First, write down the Feynman rule for the annihilation of an electron and a positron (the first part of the diagram). Then combine it with (1) to show

$$\mathcal{M}(e^+e^- \to \phi\phi^*) = e^2 \frac{\overline{v}(p')(\not{k}' - \not{k})u(p)}{(p + p')^2} \,.$$

d) Average the result over the different spin of the electrons/positrons. *Hint: You should find*

$$\overline{\mathcal{M}} = \frac{1}{4}\sum_{\text{spin}} |\mathcal{M}(e^+e^- \to \phi\phi^*)|^2 = \frac{1}{4}\left(\frac{e^2}{(p+p')^2}\right)^2 \text{Tr}\left[(\not{p}' - m_e)(m_e - \not{k})(\not{p} + m_e)(m_e - \not{k}')\right] . \tag{2}$$

You might also want to checkout Casimir's Trick, named after Dutch physicist Hendrik Casimir.

e) Evaluate (2) in the center of mass frame ($\vec{p} = -\vec{p}'$ and $\vec{k} = -\vec{k}'$). Assume that the total energy is small compared to the mass of the electron m_e. *Hint: You should get*

$$\overline{\mathcal{M}} = 8\pi^2\alpha^2\beta^2 \sin^2\theta \,,$$

where $\beta = \sqrt{1 - m^2/E^2}$, $\alpha = 4\pi e^2$ *and* θ *the angle between* $\vec{p}$ *and* $\vec{k}$.

f) Compute the cross section for this process. *Hint: You can use the formula*

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{2E^2}\frac{|\vec{k}|}{16\pi^2 E}\overline{\mathcal{M}} \,.$$

We did not discuss the transition from correlation functions to cross sections in the lecture. If your are not familiar with it from a previous course you may consult a textbook.

g) Using the Feynman rules (1), write down the two one-loop diagrams that contribute to the self-energy of the photon (this is the same effect as we have discussed in exercise 5 for the ϕ^4 theory). Write down the corresponding loop integrals. You do not have to solve them, because we just learn in the next lecture how to regularise such integrals.