Tutorial for String Theory I, WiSe2013/14
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1. Light-cone coordinates and compact dimensions

To be discussed on Thursday, October 24, 2013 in the tutorial.

Exercise 1.1: Lorentz transformations in light-cone coordinates

Consider coordinates $x^\mu = (x^0\,, x^1\,, x^2\,, x^3)$ and the associated light-cone coordinates $(x^+\,, x^-\,, x^2\,, x^3)$. Write the following Lorentz transformations in terms of the light-cone coordinates:

a) A boost with velocity parameter β in the x^1 direction.

b) A rotation with angle θ in the x^1, x^2 plane.

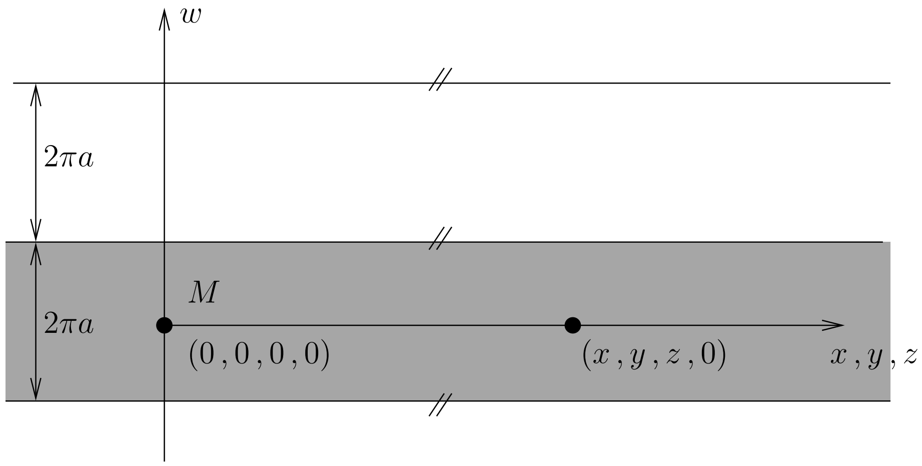
c) A boost with velocity parameter β in the x^3 direction.

Exercise 1.2: Gravitational field of a point mass in a compactified five-dimensional world.

Consider a five-dimensional spacetime with space coordinates $(x\,, y\,, z\,, w)$ *not* yet compactified. A point mass M is located at the orgin $(x\,, y\,, z\,, w) = (0\,, 0\,, 0\,, 0)$.

a) Find the gravitation potential $V_g^{(5)}(r)$. Write your answer in terms of M, $G^{(5)}$, and $r = \sqrt{x^2 + y^2 + z^2 + w^2}$. *Hint: Use* $\nabla^2 V_g^{(5)} = 4\pi G^{(5)} \rho_m$ *and the divergence theorem.*

Now let w become a circle with radius a while keeping the mass fixes.



b) Write an exact expression for the gravitational potential $V_g^{(5)}(x, y, z, 0)$. This potential is a function of $R = \sqrt{x^2 + y^2 + z^2}$ and can be written as an infinite sum.

c) Show that for $R \gg a$ the gravitational potential takes the form of a four dimensional graviational potential, with Newton's constant $G^{(4)}$ given in term of $G^{(5)}$ as

$$G^{(4)} = \frac{G^{(5)}}{2\pi a}\,. \tag{1}$$

Hint: Turn the infinite sum into an integral.

These results confirm both, the relations between the four- and five-dimensional Newton constants in a compactification and the emergence of a four-dimensional potential at distances large compared to the size of the compact dimensions.