

## 13. Application: Particle Physics

remember original motivation: describe symmetries

more precise: action  $S = \int d^4x \mathcal{L}(\phi_i)$

Lagrangian

degrees of freedom, i.e. fields

Symmetry: transformation  $\phi_i \rightarrow \lambda_i^j(x) \phi_j$

that does not change the action

check: 1) infinitesimal transformation

$$S_\lambda \phi_i = \lambda_i^j(x) \phi_j$$

2) what happens for two such transformations?

$$S_{\lambda_2} S_{\lambda_1} \phi - S_{\lambda_1} S_{\lambda_2} \phi = S_{\lambda_3} \phi \text{ (closure)}$$

$$\lambda_3 := [\lambda_1, \lambda_2] \quad \leftarrow \text{Lie bracket}$$

(symmetries)  $\leftrightarrow$  Lie algebra

global symmetry

- $\lambda_i^j$  does not depend on the coordinates  $x^\mu$

local (gauge) symmetry

- $\lambda_i^j(x)$  depends on the coordinates
- introduces new fields, gauge potential, whose excitations are force carriers

Examples:

• Lorentz symmetry

$$SO(3,1; \mathbb{R}) \hookrightarrow SO(3,1; \mathbb{C})$$

$$\stackrel{1}{=} SU(2) \oplus SU(2)$$

$(1,1)$ : scalar

$(2,1)$ : left-handed Weyl-spinor

• Standard model

$$\underbrace{SU(3) \times SU(2) \times U(1)}_{\text{organises the particle content}}$$

i.e.  $u$ : up quark

$d$ : down -

$(1, 2)$ : right-handed -u-  
 $(2, 2)$ : 4-vector  
 $\dots$

e: electron  
 $\nu$ : neutrino

### 13. 1. Glashow - Salam - Weinberg theory

$SU(2) \times U(1)$  Lie algebra describes the electro weak  
gen. by  $R_a^{\alpha}$  and  $S$   
 $\alpha = 1, 2, 3$   
 $\Rightarrow$  electro magnetic force /  
 $\Rightarrow$  weak nuclear force

take now the creation operators:

$\underbrace{u^+, d^+, e^+}_{\text{right-handed}}, \underbrace{\bar{u}^+, \bar{d}^+, \bar{e}^+, \bar{\nu}^+}_{\text{left-handed}}$

$$l_r^+ = \begin{pmatrix} l_1^+ \\ l_2^+ \end{pmatrix} = \begin{pmatrix} \bar{e}^+ \\ \bar{\nu}^+ \end{pmatrix}$$

$$\Psi_r^+ = \begin{pmatrix} \Psi_1^+ \\ \Psi_2^+ \end{pmatrix} = \begin{pmatrix} \bar{d}^+ \\ \bar{u}^+ \end{pmatrix}$$

split into two  $SU(2)$  doublets

symmetry acts as:

1) on right-handed particles:

$$\begin{aligned} [R_a, u^+] &= 0 \\ [R_a, d^+] &= 0 \\ [R_a, e^+] &= 0 \end{aligned} \quad \left. \begin{array}{l} \text{trivial} \\ \text{under} \\ SU(2) \end{array} \right.$$

$$\begin{aligned} [S, u^+] &= \frac{2}{3} u^+ \\ [S, d^+] &= -\frac{1}{3} d^+ \\ [S, e^+] &= -e^+ \end{aligned}$$

2) on left-handed particles:

$$\begin{aligned} [R_a, \Psi_r^+] &= \frac{1}{2} \Psi_r^+ \partial_{sr}^a \\ [R_a, l_r^+] &= \frac{1}{2} l_r^+ \partial_{sr}^a \end{aligned} \quad \left. \begin{array}{l} \text{both} \\ \text{SU(2)} \\ \text{doublet} \end{array} \right.$$

$$\begin{aligned} [S, \Psi_r^+] &= -\frac{1}{6} \Psi_r^+ \\ [S, l_r^+] &= \frac{1}{2} l_r^+ \end{aligned}$$

Pauli-matrices, generate the 2 of  $SU(2)$

electrical charge:  $Q = R_3 + S$  with

$$[Q, u^+] = \frac{2}{3} u^+$$

$$[Q, e^+] = -e^+$$

$$[Q, d^+] = -\frac{1}{3} d^+$$

$$[Q, \bar{\nu}] = 0,$$

- we found the charges of the first gen. fermions ?
- $R_A$  and  $S$  represent gauge bosons that mediate the interaction

$$W^\pm = R_1 \pm i R_2 \quad Z = R_3 - S \quad \left. \begin{array}{l} \text{weak-} \\ \text{interaction} \end{array} \right\} \quad P = R_3 + S \quad \left. \begin{array}{l} \text{electro-} \\ \text{magnetic-} \\ \text{interaction} \end{array} \right\}$$

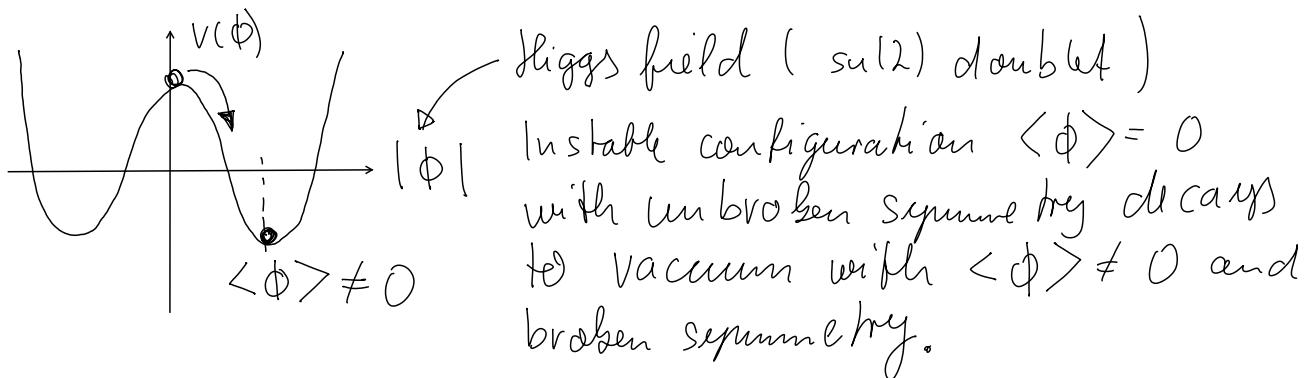
Photon

$W$  &  $Z$  bosons

weak-interaction is much different from em.  
i.e., short-ranged vs. long ranged

→ This symmetry is spontaneously broken in nature by the Higgs mechanism.

Idea: Vacuum of theory  $|0\rangle$  is just invariant under  $R_3 + S$  but not the other 3 generators.



### 13.2. Adding color

full gauge group  $SU(3) \times SU(2) \times U(1)$  with creation

op.  $a_{y,r}^+$  in  
new color index

irrep	$(D, d)_S$
$[T_i, a_{x,r}^+]$	$= a_{y,r}^+ (T_i^D)_{yx}$
$[R_a, a_{x,r}^+]$	$= a_{x,s}^+ (R_a^d)_{sr}$
$[S, a_{x,r}^+]$	$= S a_{x,r}^+$

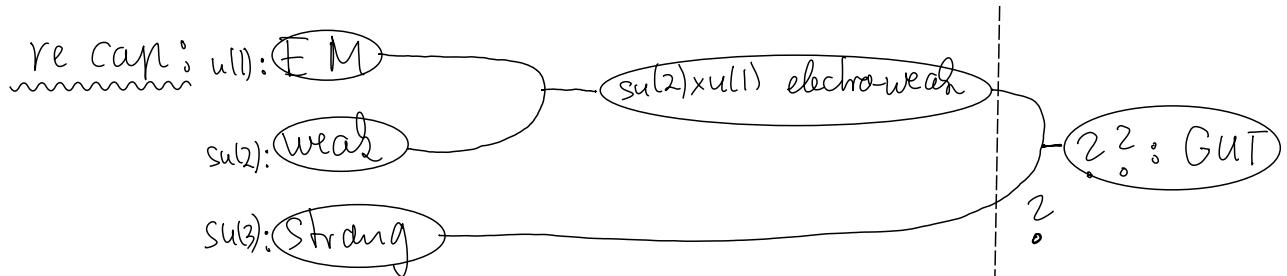
and we find:

right handed particle

$$u^+ : (3, 1)_{2/3}, \quad d^+ : (3, 1)_{-1/3}, \quad e^+ : (1, 1)_{-1}$$

left handed particle

$$\psi^+ : (\bar{3}, 2)_{-1/6}, \quad \ell^+ : (1, 2)_{1/2}$$



today  $\vdash$  fact

energy  
new weak,  
not confirmed