

## 14. Grand Unified Theories

Remember, last time we packaged the first generation standard model particles into irreps of  $SU(3) \times SU(2) \times U(1)$

1) right-handed

$$(3,1)_{2/3} \oplus (3,1)_{-1/3} \oplus (1,1)_{-1} \oplus (\bar{3},2)_{-1/6} \oplus (1,2)_{1/2}$$

$u^+$  (up quark)     $d^+$  (down quark)     $e^+$  (electron)     $\bar{\Psi}^+ = \begin{pmatrix} \bar{d}^+ \\ \bar{u}^+ \end{pmatrix}$  } anti-quarks     $\bar{\ell}^+ = \begin{pmatrix} \bar{e}^+ \\ \bar{\nu}^+ \end{pmatrix}$  } anti-leptons

2) left-handed

in the complex conjugate representation of 1:

$$(\bar{3},1)_{-2/3} \oplus (\bar{3},1)_{1/3} \oplus (1,1)_1 \oplus (\bar{3},2)_{1/6} \oplus (1,2)_{-1/2}$$

anti-particles                           $\psi^+$  (quarks)                           $\ell^+$  (leptons)

Question: Can these particles originate from a bigger group  $G \supset SU(3) \times SU(2) \times U(1)$  which is spontaneously broken?

### 14.1. $SU(5)$

Constraint: rank  $G \geq \text{rank}(SU(3) \oplus SU(2) \oplus U(1)) = 4$

i.e.  $G = SU(5)$

branching rule from section 12.5:

$$5 \rightarrow (3,1)_2 \oplus (1,2)_{-3} \rightarrow \text{rescale } U(1)-\text{generator}$$

$$5 \rightarrow (3,1)_{-1/3} \oplus (1,2)_{1/2} \quad (\cdot - 1/6)$$

We can now also compute the branching:

$$\boxed{\phantom{0}} = 10 \rightarrow (\bar{3},1)_{-2/3} \oplus (1,1)_1 \oplus (\bar{3},2)_{1/6}$$

$\rightsquigarrow$  1) right-handed & 2) left-handed

$$5 \oplus \overline{10} \qquad \qquad \overline{5} \oplus 10$$

Breaking the  $SU(5)$  to  $SU(3) \times SU(2) \times U(1)$  by the Higgs-mechanism requires:

1) Higgs field in the adjoint.

$$24 \rightarrow (1,1)_0 \oplus (1,3)_0 \oplus (8,1)_0 \oplus (3,2)_{-5/6} \oplus (\bar{3},2)_{5/6}$$

U(1)-ads      su(2)-adj      su(3)-adj

↑ give vacuum expectation value

2) Masses for fermions through Yukawa-coupling

coupling  $\bar{\Psi} \psi \phi \leftarrow$  Higgs field  
particle    ↑  
anti-particle

positron  $\bar{e}$  and d-quark d in the 5

electron e and anti-d-quark  $\bar{d}$  in the  $\bar{10}$

$$\hookrightarrow 5 \otimes \bar{10} = \bar{5} \oplus \bar{45}$$

branchings:

$$\begin{aligned} \bar{5} &\rightarrow (\bar{3}, 1)_{1/2} \oplus (1, 2)_{-1/2} \\ \bar{45} &\rightarrow (1, 2)_{-1/2} \oplus \dots \end{aligned}$$

← correct irreps  
for standard-  
model Higgs-  
field

right-handed u- and anti-u-quark are both in the  $\bar{10}$ ,

therefore:  $\bar{10} \otimes \bar{10} = 5 + 45 + \textcircled{50}$

no  $(1, 2)$  contribution for Higgs

Problem:  $\bar{10}$  contains:  
• electron  $(1, 1)_{-1}$   
• anti-quarks  $(\bar{3}, 2)_{-1/6}$   
• quarks  $(3, 1)_{2/3}$

→ some  $Su(5)$  interactions do not conserve baryon number

$$B = \frac{1}{3} (n_q - n_{\bar{q}})$$

$n_q$  number of quarks  
 $n_{\bar{q}}$  number of anti-quarks

→ proton decay?  
Yet, not observed in nature.

## 14.2. SO(10)

So far only consider particles we know from the SM. But,  
there could be more, very massive ones.

i.e. take the spinor irrep, 16, of  $SO(10)$  and branch to  $SU(5)$

$$16 \rightarrow 5 + \bar{10} + \textcircled{1} \quad \text{and additional particle, should get large mass}$$

$$\bar{16} \rightarrow \bar{5} + 10 + 1$$

Interesting: all right-/left-handed particles of one gen. in one irrep.

$\hookrightarrow$   $SU(5)$  is not a maximal regular subgroup

$$D_5: \begin{array}{c} \bullet \\ \diagup \quad \diagdown \\ \bullet - \bullet \end{array} \rightarrow \begin{array}{c} \bullet \\ \diagup \quad \diagdown \\ \bullet \end{array} \oplus \bullet \oplus \bullet$$

$$= su(4) \oplus su(2) \oplus su(2)$$

with branchings:

$$16 \rightarrow (4, 1, 2) \oplus (\bar{4}, 2, 1)$$

$$\bar{16} \rightarrow (\bar{4}, 1, 2) \oplus (4, 2, 1)$$

contains color  
 $su(3)$

mirror image of  
weak interaction

This mirror image has to be broken to restore CP-violation of the weak interaction observed in nature.

breaking from  $SO(10)$  to  $SU(5)$

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$$Yukawa couplings: 16 \otimes \bar{16} = 10 \oplus 120 \oplus 126$$

only the branching of

$$126 \rightarrow \textcircled{1} \oplus 5 \oplus \bar{10} \oplus 15 \oplus \bar{45} \oplus 50$$

contains singlet

choose a Higgs field that couples to this singlet does not generate mass terms for the 5 or  $\bar{10}$ , just for the  $1 \rightleftharpoons$  right-handed neutrino

adjoint representation:

$$45 \rightarrow 24 \oplus \bar{10} \oplus 10 \oplus 1$$

only VEV for 45 gives rise to an unwanted  
 $SU(3) \times SU(2) \times U(1)^{\textcircled{2}}$

broken by additional VEV of 16 or 126,

breaking  $SO(10)$  to  $SU(4) \times SU(2) \times SU(2)$

done by VEV of 54 and VEV of 16 or 126

break further to  $SU(3) \times SU(2) \times U(1)$