Tutorial for String Theory I, WiSe2013/14
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2. Point particle action and reparameterization

To be discussed on Thursday, October 31, 2013 in the tutorial.

Exercise 2.1: Massive relativistic point particle

Consider the action of a massive relativistic point particle

$$S = -mc \int_{\tau_0}^{\tau_1} d\tau \sqrt{-\dot{x}^\mu \dot{x}_\mu}\,.$$

a) Using $x^0 = ct$, show the equivalence of S to the action

$$\hat{S} = \int_{t_0}^{t_1} dt L(t) = -mc^2 \int_{t_0}^{t_1} dt \sqrt{1 - \frac{\vec{v}^2}{c^2}}\,,$$

where $\vec{v} = \vec{v}(t)$ denotes the ordinary velocity of the particle with respect to the physical time t.

b) Verify that for small velocities, $|\vec{v}| \ll c$, $L(t)$ reduces to the standard form of the Lagrange function, i.e., kinetic minus potential energy. What plays the rôle of the potential energy in this case.

Exercise 2.2: Reparameterization invariance of the point particle action

The advantage of the action S over the action $\hat{S}$ is that it treats time x^0 and the space coordinate on an equal footing, making Poincaré invariance manifest. This comes at the expense of a new, unphysical parameter τ. Verify that the covariant action S is indeed invariant under changes of this unphysical parameter, i.e., under reparameterizations

$$\tau \to \tilde{\tau}(\tau)\,.$$

Exercise 2.3: Point particle action without square root

Consider now the action

$$S' = \frac{1}{2} \int_{\tau_0}^{\tau_1} d\tau \left(e^{-1} \dot{x}^\mu \dot{x}_\mu - e m^2 c^2 \right)\,.$$

a) How does e have to transform under the reparameterization $\tau \to \tilde{\tau}(\tau)$ in order to ensure the reparameterization invariance of S'?

b) Find the equation of motion for e by varying S'. Insert the resulting equation into S' and verify that S' is classically equivalent to the action S of problem 2.1.