

# 10. Spontaneous Symmetry Breaking

remember:

symmetries in (Q)FT

global symmetries

local = gauge symmetries

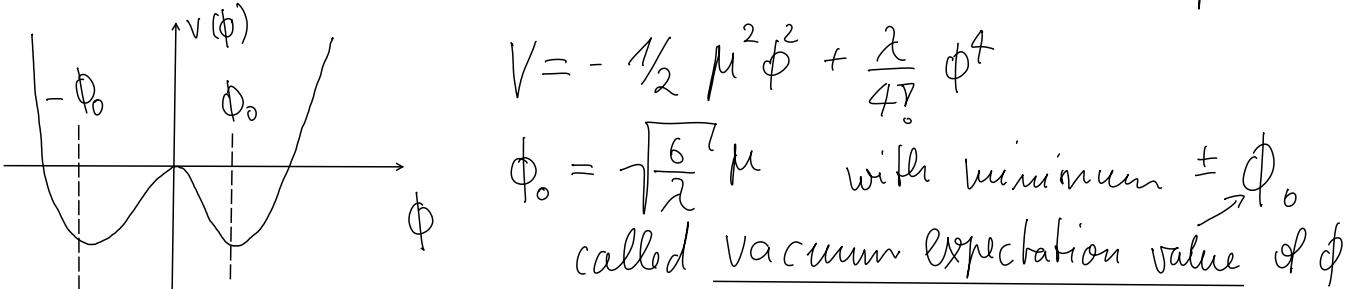
Today, we see why and how they can be broken.

## 10.1. Global Symmetries & Goldstone's Theorem

Example:  $\mathcal{L} = \frac{1}{2} (\partial_\mu \phi)^2 + \frac{1}{2} \mu^2 \phi^2 - \frac{\lambda}{4!} \phi^4 = T - V$

introduce  $m = i\mu$  in  $\phi^4$ -theory

Theory has a  $\mathbb{Z}_2$ -symmetry  $\phi \rightarrow -\phi$  and the potential



expanding  $\phi(x) = \phi_0 + \delta(x)$  results in

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \delta)^2 - \underbrace{\frac{1}{2} (2\mu^2) \delta^2}_{\text{mass term for } m = \sqrt{2}\mu} - \underbrace{\frac{\lambda}{6} M \delta^3}_{\text{breaks } \mathbb{Z}_2\text{-Sym.}} - \frac{\lambda}{4!} \delta^4$$

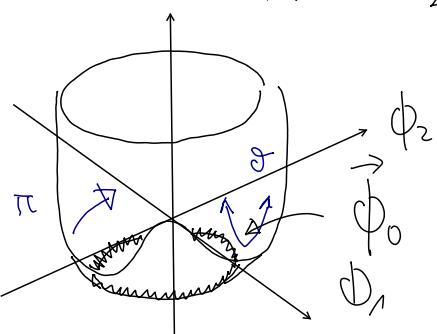
mass term for  $m = \sqrt{2}\mu$  brakes  $\mathbb{Z}_2$ -Sym.  $\phi \rightarrow -\phi$

More interesting are continuous symmetries:  $i=1, \dots, N$

like  $\mathcal{L} = \frac{1}{2} (\partial_\mu \phi^i)^2 + \frac{1}{2} \mu^2 (\phi^i)^2 - \frac{\lambda}{4} [(\phi^i)^2]^2 = T - V$

which has a  $\phi^i \rightarrow R^i_j \phi^j$ ,  $R^T R = 1$ ,  $O(N)$ -symmetry

with  $V(\phi^i) = -\frac{1}{2} \mu^2 [\phi^i]^2 + \frac{\lambda}{4} [(\phi^i)^2]^2$  and minima at



$$(\phi_0^i)^2 = \frac{\mu^2}{\lambda}$$

$\stackrel{1}{=} \text{vector } \vec{\phi}_0 \text{ with fixed length}$

choose (hyper)-spherical coordinates such that

$$\phi(x) = \left( \frac{\mu}{\sqrt{\lambda}} + \theta(x), \pi^k(x) \right) \quad \text{and we hit the minima } \vec{\phi} \text{ for } \theta(x) = 0$$

radius      angles

results in:

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \pi^k)^2 + \frac{1}{2} (\partial_\mu \theta)^2 - \frac{1}{2} (2\mu^2) \theta^2 - \sqrt{2} \mu \theta^3 - \sqrt{\lambda} \mu (\pi^k)^2 \theta - \frac{\lambda}{4} \theta^4 - \frac{\lambda}{2} (\pi^k)^2 \theta^2 - \frac{\lambda}{4} [(\pi^k)^2]^2$$

- $\theta$  is massive ( $m_\theta = \sqrt{2}\mu$ ), like before
- $\pi^k$ ,  $k=1, \dots, N-1$ , massless fields with  $O(N-1)$ -sym.

The observation from this example generalises:

For every spontaneously broken continuous symmetry, the theory must contain a massless particle

= Goldstone's theorem

Check:  $\dim(O(N)) = \frac{N(N-1)}{2}$ ,  $\dim(O(N-1)) = \frac{(N-1)(N-2)}{2}$

broken symmetry generators =  $\dim(O(N)) - \dim(O(N-1)) = N-1 = \#\pi^k$

Works not only classically, but to all orders in perturbation theory.

## 10.2. Gauge Symmetries & The Higgs Mechanism

Simplest example: Abelian gauge theory with

$$\mathcal{L} = -\frac{1}{4} (F_{\mu\nu})^2 + |\nabla_\mu \phi|^2 - V(\phi), \quad \text{remember } D_\mu = \partial_\mu + ieA_\mu,$$

and we find the local symmetry  $\phi(x) \rightarrow e^{i\alpha(x)} \phi(x)$  and  $A_\mu(x) \rightarrow A_\mu(x) - \frac{1}{e} \partial_\mu \alpha(x)$

motivated by the examples from the last section we choose:

$$V(\phi) = -\mu^2 \phi^* \phi + \frac{\lambda}{2} (\phi^* \phi)^2 \text{ with minima at}$$

$$\langle \phi \rangle = \phi_0 = \left( \frac{\mu^2}{\lambda} \right)^{1/2}$$

again we expand around the minimum:  $\phi(x) = \phi_0 + \frac{1}{2}(\phi_1(x) + i\phi_2(x))$

with  $V(\phi) = -\frac{1}{2\lambda}\mu^4 + \frac{1}{2}2\mu^2\phi_1^2 + \mathcal{O}(\phi_i^3)$

→  $\phi_1$  acquires mass  $m = \sqrt{2}\mu$

$\phi_2$  is the Goldstone boson, but something new happens  
for

Kinetic term:  $|D_\mu \phi|^2 = \frac{1}{2}(\partial_\mu \phi_1)^2 + \frac{1}{2}(\partial_\mu \phi_2)^2 + \sqrt{2}e\phi_0 A_\mu A^\mu \phi_2 +$   
 $\underbrace{e^2 \phi_0^2 A_\mu A^\mu}_{\text{new mass term for the photon}} + \dots$

new mass term for the photon =  $\frac{1}{2}m_A^2 A_\mu A^\mu$   
 with  $m_A = 2e\phi_0^2$

Note: In contrast to the examples before, we can remove the Goldstone boson  $\phi_2$  by gauge fixing (unitary gauge).

On the other hand the massive U(1)-gauge field allows a longitudinal mode (in contrast to the massless one → section 3.5).

 The Goldstone boson gets "eaten up" by this mode.

Non-abelian generalisation:

gauge transformation:  $\phi_i \rightarrow [S_i^j + i \stackrel{\circ}{\omega}{}^a(t_a)_i^j] \phi_j$   
 ↪ here real fields  $\stackrel{\circ}{\omega}$  purely imag.

with covariant derivative:  $D_\mu \phi = (\partial_\mu - ig A_\mu^a t_a) \phi$   $T_a = it_a$

Kinetic term:  $\frac{1}{2}(D_\mu \phi_i)^2 = \frac{1}{2}(\partial_\mu \phi_i)^2 + g A_\mu^a (\partial^\mu \phi^i (T_a)^j \phi_j)$   
 $+ \frac{1}{2}g^2 A_\mu^a A^{b\mu} (T^a \phi)^i (T^b \phi)_i$

expanding around VEV (vacuum expectation value)  $\langle \phi_i \rangle = (\phi_0)_i$ :

→ new mass term  $\Delta \mathcal{L} = \frac{1}{2}m_{ab}^2 A_\mu^a A^{b\mu}$  with

$$m_{ab}^2 = g^2 (T^a \phi_0)^i (T^b \phi_0)_i$$

The matrix  $m_{ab}^2$  is positive semi-definite  $\rightarrow$  diagonalisable with eigenvalues  $\lambda_a \geq 0$

If  $\lambda_a = 0$ ,  $(T_a)_i^j (\phi)_i^j = 0$  unbroken symmetry

$\rightarrow$  no Goldstone boson & mass term for gauge boson

SU(2) example:  $t_a = \frac{\sigma_a}{2} \leftarrow$  Pauli matrices

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \text{ results in the kinetic term}$$

$$|D_\mu \phi|^2 = \frac{1}{2} g^2 (0 \ v) t_a t_b \begin{pmatrix} 0 \\ v \end{pmatrix} A_\mu^a A_\nu^b + \dots$$

$$\Rightarrow \Delta L = \frac{g^2 v^2}{8} A_\mu^a A_a^\mu \quad \Rightarrow \text{all three gauge bosons acquire the same mass}$$

$$m_A = \frac{g v}{2}$$