Poisson-Lie Symmetry and Double Field Theory

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based on

1707.08624, 1611.07978

and

1502.02428 with Pascal du Bosque, Dieter Lüst and Ralph Blumenhagen

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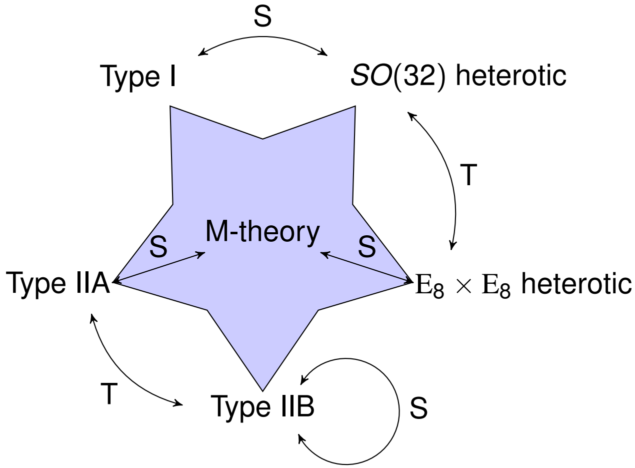
Holography, Strings and Exceptional Field Theory

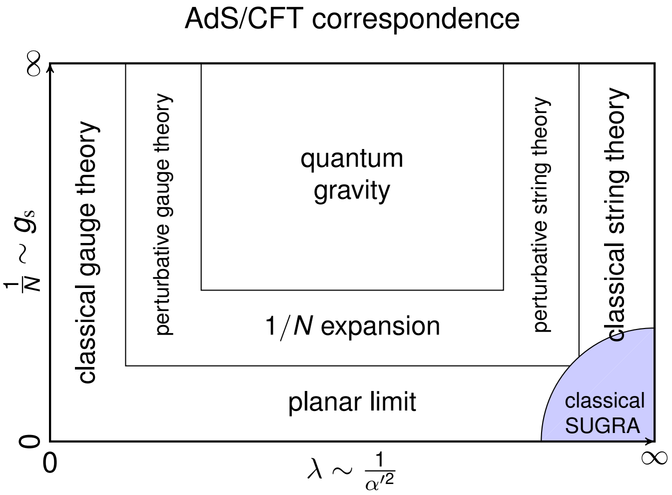
Canonical motivation for Exception/Double Field Theory



T = T-duality

S = S-duality

But there is also another interesting story…



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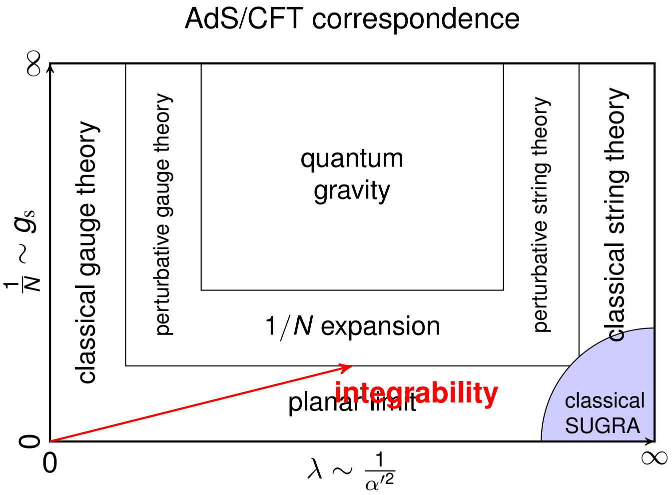
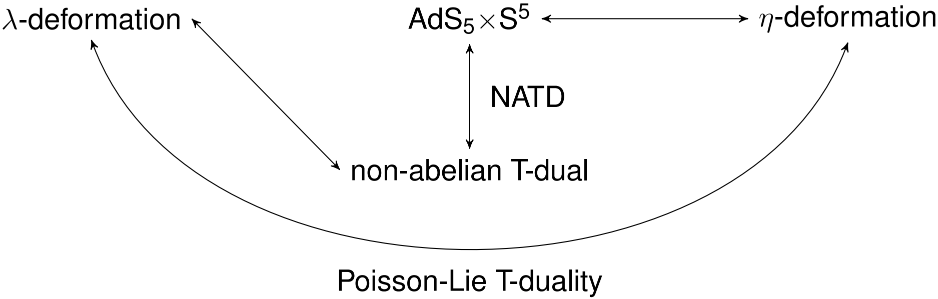
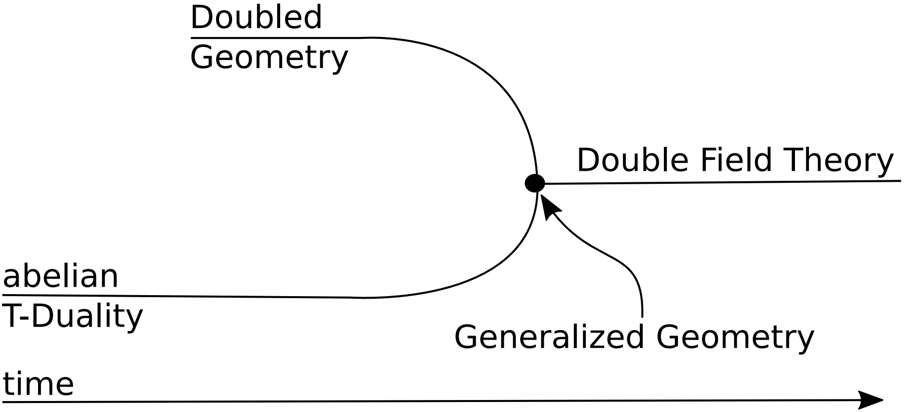


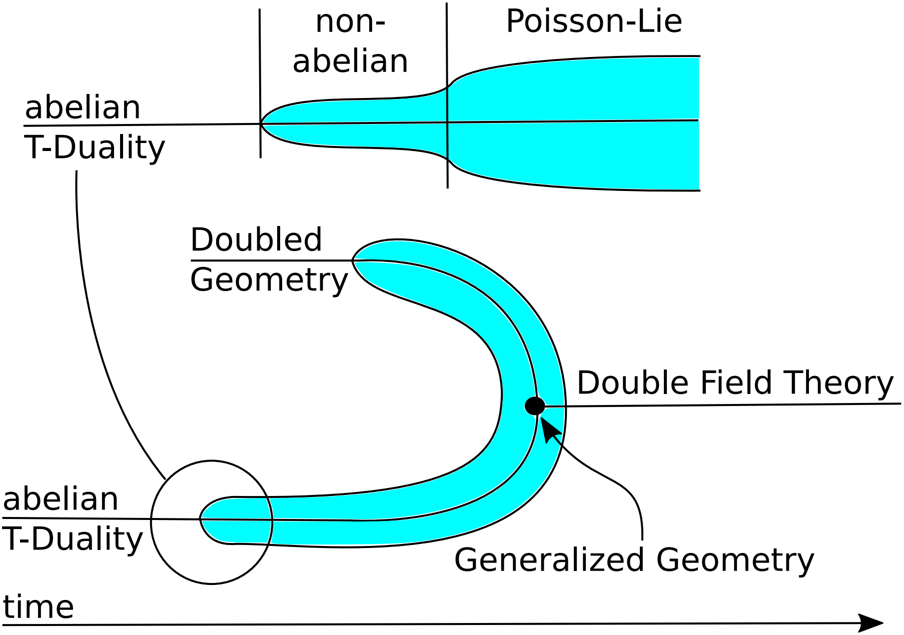
Integrable deformations



many more: bi-Yang-Baxter, with or without WZW term ...

ALL POISSON-LIE SYMMETRY


Doubled
Geometry
Double Field Theory
abelian
T-Duality
Generalized Geometry
time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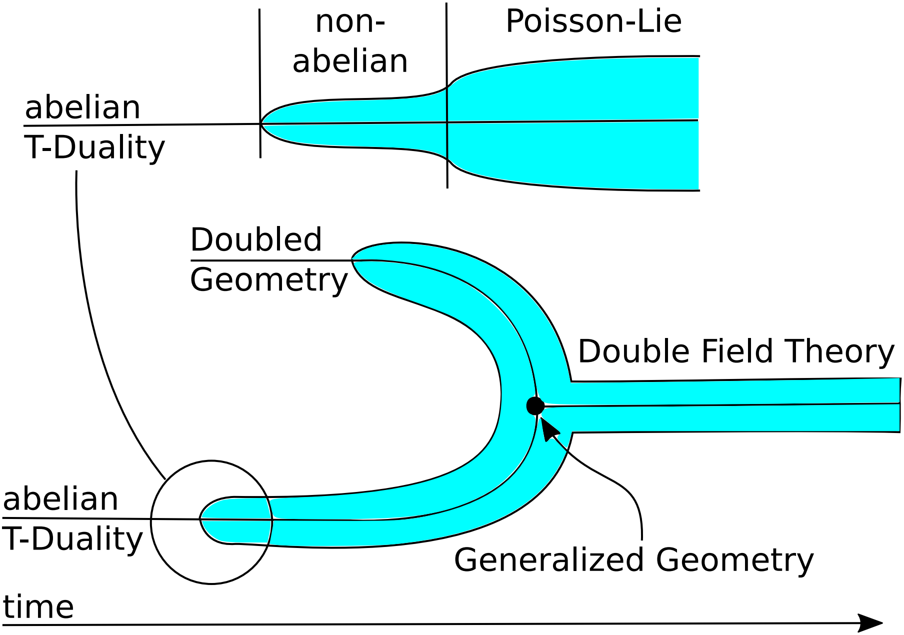
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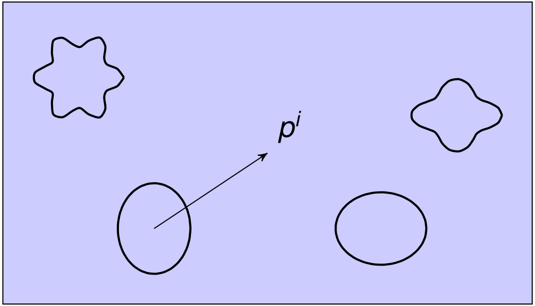
Outline

1. Motivation
2. SUGRA & DFT in a nutshell
3. Poisson-Lie symmetry
4. Double Field Theory on Drinfeld doubles
5. Summary

SUGRA

- closed strings in D-dim. flat space
- truncate all massive excitations
- match scattering amplitudes of strings with EFT

$$S_{\mathrm{NS}} = \int \mathrm{d}^D x \sqrt{g} e^{-2\phi} \left(\mathcal{R} + 4\partial_i\phi\partial^i\phi - \frac{1}{12} H_{ijk} H^{ijk} \right)$$



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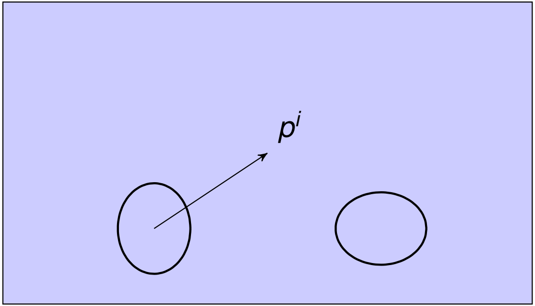

SUGRA

- closed strings in *D*-dim. flat space
- truncate all massive excitations
- match scattering amplitudes of strings with EFT

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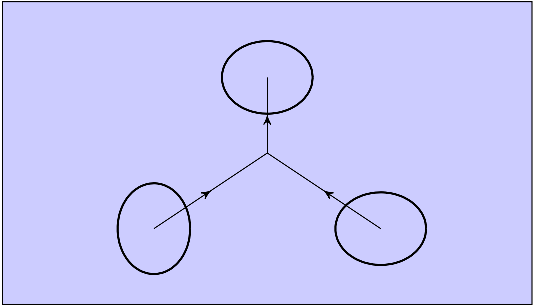
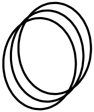
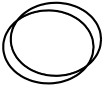




$g_{ij}(p^k)$



$\phi(p^i)$

Manifest & hidden symmetries

- S_{NS} = action for NS/NS sector of Type IIA and Type IIB
- manifest invariant under

  diffeomorphisms $\quad g_{ij} = \mathcal{L}_\xi g_{ij}$

  gauge transformations $\quad B_{ij} = \mathcal{L}_\xi B_{ij} + \partial_i \alpha_j - \partial_j \alpha_i$

- compactification on circle $\to U(1)$ isometry
- Buscher rules implement T-duality [Buscher, 1987]

$$\tilde{g}_{\theta\theta} = \frac{1}{g_{\theta\theta}}\,, \quad \tilde{g}_{\theta i} = \frac{1}{g_{\theta\theta}} B_{\theta i}\,, \quad \tilde{g}_{ij} = g_{ij} + \frac{1}{g_{\theta\theta}}(g_{\theta i} g_{\theta j} - B_{\theta i} B_{\theta j})\,, \ \ldots$$

  from NS/NS sector of IIA to IIB

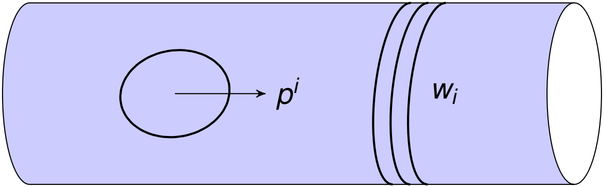
- T-duality is a hidden symmetry

DFT (Double Field Theory) [Siegel, 1993,Hull and Zwiebach, 2009,Hohm, Hull, and Zwiebach, 2010]

- closed strings on a flat torus
- combine conjugated variables x_i and $\tilde{x}^i$ into $X^M = \begin{pmatrix} \tilde{x}_i & x^i \end{pmatrix}$
- repeat steps from SUGRA derivation

$$S_{\mathrm{DFT}} = \int \mathrm{d}^{2D} X \, e^{-2d} \mathcal{R}(\mathcal{H}_{MN}, d)$$

- fields are constrained by strong constraint $\boxed{\partial_M \partial^M \cdot = 0}$



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$$d = \phi - \frac{1}{2} \log \sqrt{g}$$

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$$S_{\mathrm{DFT}} = \int \mathrm{d}^{2D}X\, e^{-2d}\mathcal{R}(\mathcal{H}_{MN}, d)$$

$$\mathcal{R} = 4\mathcal{H}^{MN}\partial_M\partial_N d - \partial_M\partial_N\mathcal{H}^{MN} - 4\mathcal{H}^{MN}\partial_M d\partial_N d + 4\partial_M\mathcal{H}^{MN}\partial_N d + \frac{1}{8}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\partial_N\mathcal{H}_{KL} - \frac{1}{2}\mathcal{H}^{MN}\partial_N\mathcal{H}^{KL}\partial_L\mathcal{H}_{MK}$$

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$$\mathcal{H}^{MN} = \begin{pmatrix} g_{ij} - B_{ik}g^{kl}B_{lj} & -B_{ik}g^{kj} \\ g^{ik}B_{kj} & g^{ij} \end{pmatrix} \in O(D,D) \to \text{T-duality}$$

DFT (Double Field Theory) [Siegel, 1993,Hull and Zwiebach, 2009,Hohm, Hull, and Zwiebach, 2010]

- lower/raise indices with $\eta_{MN} = \begin{pmatrix} 0 & \delta_i^j \\ \delta_j^i & 0 \end{pmatrix}$ and $\eta^{MN} = \begin{pmatrix} 0 & \delta^i_j \\ \delta^j_i & 0 \end{pmatrix}$

$$X^M = \begin{pmatrix} \tilde{x}_i & x^i \end{pmatrix}$$

$$d = \phi - \tfrac{1}{2} \log \sqrt{g}$$

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$$\mathcal{R} = 4\mathcal{H}^{MN}\partial_M\partial_N d - \partial_M\partial_N\mathcal{H}^{MN} - 4\mathcal{H}^{MN}\partial_M d\partial_N d + 4\partial_M\mathcal{H}^{MN}\partial_N d + \frac{1}{8}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\partial_N\mathcal{H}_{KL} - \frac{1}{2}\mathcal{H}^{MN}\partial_N\mathcal{H}^{KL}\partial_L\mathcal{H}_{MK}$$

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Gauge transformations [Hull and Zwiebach, 2009]

- generalized Lie derivative combines

  1. diffeomorphisms
  2. *B*-field gauge transformations
  3. β-field gauge transformations

  (1. and 2.: available in SUGRA)

$$\mathcal{L}_\lambda \mathcal{H}^{MN} = \lambda^P \partial_P \mathcal{H}^{MN} + (\partial^M \lambda_P - \partial_P \lambda^M)\mathcal{H}^{PN} + (\partial^N \lambda_P - \partial_P \lambda^N)\mathcal{H}^{MP}$$

$$\mathcal{L}_\lambda d = \lambda^M \partial_M d + \frac{1}{2}\partial_M \lambda^M$$

- closure of algebra

$$\mathcal{L}_{\lambda_1}\mathcal{L}_{\lambda_2} - \mathcal{L}_{\lambda_2}\mathcal{L}_{\lambda_1} = \mathcal{L}_{\lambda_{12}} \quad \text{with} \quad \lambda_{12} = [\lambda_1, \lambda_2]_C$$

- only if strong constraint holds

What about PL T-duality?

- DFT makes abelian T-duality manifest
- but does not work for full PL-Td
- what do we have to change?

Ingredient 1: Drinfeld double [Drinfeld, 1988]

Definition: A **Drinfeld double** is a $2D$-dimensional Lie group $\mathcal{D}$, whose Lie-algebra $\mathfrak{d}$

1. has an ad-invariant bilinear for $\langle\cdot\,,\,\cdot\rangle$ with signature (D, D)
2. admits the decomposition into two maximal isotropic subalgebras $\mathfrak{g}$ and $\tilde{\mathfrak{g}}$

- $\begin{pmatrix} t^a & t_a \end{pmatrix} = t_A \in \mathfrak{d}\,,\quad t_a \in \mathfrak{g}$ and $t^a \in \tilde{\mathfrak{g}}$
- $\langle t_A, t_B\rangle = \eta_{AB} = \begin{pmatrix} 0 & \delta^a_b \\ \delta_a^b & 0 \end{pmatrix}$
- $[t_A, t_B] = F_{AB}{}^C t_C$ with non-vanishing commutators

$$[t_a, t_b] = f_{ab}{}^c t_c \qquad [t_a, t^b] = \tilde{f}^{bc}{}_a t_c - f_{ac}{}^b t^c$$

$$[t^a, t^b] = \tilde{f}^{ab}{}_c t^c$$

- ad-invariance of $\langle\cdot\,,\,\cdot\rangle$ implies $F_{ABC} = F_{[ABC]}$

Ingredient 2: Poisson-Lie Symmetry [Klimcik and Severa, 1995]

- 2D σ-model on target space M with action

  $S(E, M) = \int dz d\bar{z}\, E_{ij} \partial x^i \bar{\partial} x^j$
- $E_{ij} = g_{ij} + B_{ij}$ captures metric and two-from field on M
- inverse of E_{ij} is denoted as E^{ij}
- *left* invariant vector field $v_a{}^i$ on G is the inverse transposed of *right* invariant Maurer-Cartan form $t_a v^a{}_i dx^i = dg\, g^{-1}$
- adjoint action of $g \in G$ on $t_A \in \mathfrak{d}$: $\mathrm{Ad}_g\, t_A = g t_A g^{-1} = M_A{}^B t_B$
- analog for $\tilde{G}$

Definition: $S(E, \mathcal{D}/\tilde{G})$ has **Poisson-Lie Symmetry** if

$$E^{ij} = v_c{}^i M_a{}^c (M^{ae} M^b{}_e + E_0^{ab}) M_b{}^d v_d{}^j$$

holds, where E_0^{ab} is constant and invertible with the inverse $E_{0\,ab}$.

Immediate consequence: Poisson-Lie T-duality

- exchanging G and $\tilde{G}$ results in dual σ-model with

$$\tilde{E}^{ij} = \tilde{v}^{ci} \tilde{M}^a{}_c (\tilde{M}_{ae} \tilde{M}_b{}^e + E_{0\,ab}) \tilde{M}^b{}_d \tilde{v}^{dj}$$

- captures

abelian T-d.	G	abelian	and	$\tilde{G}$	abelian
non-abelian T-d.	G	non-abelian	and	$\tilde{G}$	abelian

[Ossa and Quevedo, 1993;Giveon and Rocek, 1994; Alvarez, Alvarez-Gaume, and Lozano, 1994;. . .]

- dual σ-models related by canonical transformation
[Klimcik and Severa, 1995;Klimcik and Severa, 1996;Sfetsos, 1998]

$\rightarrow$ equivalent at the classical level

- preserves conformal invariance at one-loop
[Alekseev, Klimcik, and Tseytlin, 1996;Sfetsos, 1998;. . . ;Jurco and Vysoky, 2017]

- dilaton transformation [Jurco and Vysoky, 2017]

$$\phi = -\frac{1}{2} \log \left| \det \left(1 + \tilde{g}_0^{-1} (\tilde{B}_0 + \Pi) \right) \right|$$
$$\tilde{\phi} = -\frac{1}{2} \log \left| \det \left(1 + g_0^{-1} (B_0 + \tilde{\Pi}) \right) \right|$$

SUGRA

- DFT can make PL-Symmetry manifest
- consistent tructions are central
- get the dialton, R/R sector nearly for free

Additional structure on the Drinfeld double

[Blumenhagen, Hassler, and Lüst, 2015, Blumenhagen, Bosque, Hassler, and Lüst, 2015]

- *right* invariant vector $E_A{}^I$ field on $\mathcal{D}$ is the inverse transposed of *left* invariant Maurer-Cartan form $t_A E^A{}_I dX^I = g^{-1}dg$
- two η-compatible, covariant derivatives[1]
  1. flat derivative

     $$D_A V^B = E_A{}^I \partial_I V^B - w F_A V^B, \qquad F_A = D_A \log \left|\det(E^B{}_I)\right|$$

  2. convenient derivative

     $$\nabla_A V^B = D_A V^B + \tfrac{1}{3} F_{AC}{}^B V^C$$

- generalized metric $\mathcal{H}_{AB}$ ($w = 0$)

$$\mathcal{H}_{AB} = \mathcal{H}_{(AB)}, \qquad \mathcal{H}_{AC}\eta^{CD}H_{DB} = \eta_{AB}$$

- generalized dilaton d with e^{-2d} scalar density of weight $w = 1$
- triple $(\mathcal{D}, \mathcal{H}_{AB}, d)$ captures the doubled space of DFT

[1] definitions here just for quantities with flat indices

Double Field Theory for $(\mathcal{D}, \mathcal{H}_{AB}, d)$ [Blumenhagen, Bosque, Hassler, and Lüst, 2015]

see also [Vaisman, 2012; Hull and Reid-Edwards, 2009;Geissbuhler, Marques, Nunez, and Penas, 2013; Cederwall, 2014; . . .]

- action ($\nabla_A d = -\frac{1}{2} e^{2d} \nabla_A e^{-2d}$)

$$S_{\text{NS}} = \int_{\mathcal{D}} d^{2D} X e^{-2d} \Big(\frac{1}{8} \mathcal{H}^{CD} \nabla_C \mathcal{H}_{AB} \nabla_D \mathcal{H}^{AB} - \frac{1}{2} \mathcal{H}^{AB} \nabla_B \mathcal{H}^{CD} \nabla_D \mathcal{H}_{AC}$$
$$- 2 \nabla_A d \nabla_B \mathcal{H}^{AB} + 4 \mathcal{H}^{AB} \nabla_A d \nabla_B d + \frac{1}{6} F_{ACD} F_B{}^{CD} \mathcal{H}^{AB} \Big)$$

- generalized diffeomorphisms

$$\mathcal{L}_\xi V^A = \xi^B \nabla_B V^A + \left(\nabla^A \xi_B - \nabla_B \xi^A \right) V^B + w \nabla_B \xi^B V^A$$

- $2D$-diffeomorphisms

$$L_\xi V^A = \xi^B D_B V^A + w D_B \xi^B V^A$$

- global O(D,D) transformations

$$V^A \to T^A{}_B V^B \quad \text{with} \quad T^A{}_C T^B{}_D \eta^{CD} = \eta^{AB}$$

- section condition (SC)

$$\eta^{AB} D_A \cdot D_B \cdot = 0$$

Symmetries of the action

- $S_{\rm NS}$ invariant for $X^I \to X^I + \xi^A E_A{}^I$ and
  1. $\mathcal{H}^{AB} \to \mathcal{H}^{AB} + \mathcal{L}_\xi \mathcal{H}^{AB}$ and $e^{-2d} \to e^{-2d} + \mathcal{L}_\xi e^{-2d}$
  2. $\mathcal{H}^{AB} \to \mathcal{H}^{AB} + L_\xi \mathcal{H}^{AB}$ and $e^{-2d} \to e^{-2d} + L_\xi e^{-2d}$

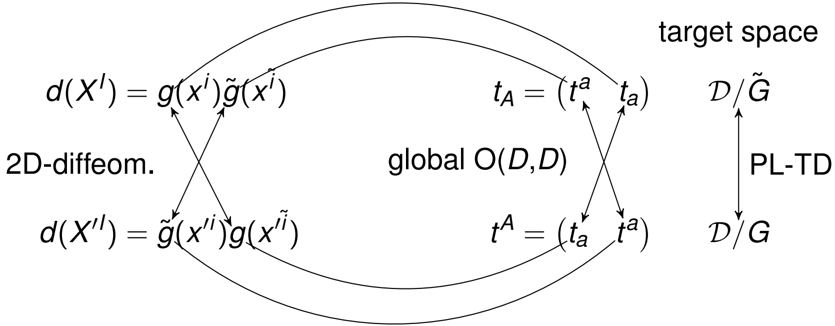
object	gen.-diffeomorphisms	$2D$-diffeomorphisms	global O(D,D)
$\mathcal{H}_{AB}$	tensor	scalar	tensor
$\nabla_A d$	not covariant	scalar	1-form
e^{-2d}	scalar density (w=1)	scalar density (w=1)	invariant
η_{AB}	invariant	invariant	invariant
$F_{AB}{}^C$	invariant	invariant	tensor
$E_A{}^I$	invariant	vector	1-form
$S_{\rm NS}$	invariant	invariant	invariant
SC	invariant	invariant	invariant
D_A	not covariant	covariant	covariant
∇_A	not covariant	covariant	covariant

manifest (last two columns)

Poisson-Lie T-duality: 1. Solve SC [Hassler, 2016]

- fix D physical coordinates x^i from $X^I = \begin{pmatrix} x^i & x^{\tilde{i}} \end{pmatrix}$ on $\mathcal{D}$

  such that $\eta^{IJ} = E_A{}^I \eta^{AB} E_B{}^J = \begin{pmatrix} 0 & \cdots \\ \cdots & \cdots \end{pmatrix} \to$ SC is solved
- fields and gauge parameter depend just on x^i
- different SC solutions, relate them by symmetries of DFT



Poisson-Lie T-duality: 2. As manifest symmetry of DFT

- same structure as in the original paper [Klimcik and Severa, 1995]
- duality target spaces arise as different solutions of the SC

Poisson-Lie T-duality:

- 2D-diffeomorphisms $X^I \to X'^I(X^1, \dots X^{2D})$ with $d(X^I) = d(X'^I)$
- global O(D,D) transformation $t_A \to \eta^{AB} t_B$

$\underbrace{\hspace{10em}}$ manifest symmtries of DFT

- for abelian T-duality $X^I \to X'^I = X^I$

$\rightarrow$ no 2D-diffeomorphisms needed, only global O(D,D) transformation

Poisson-Lie Symmetry is a manifest symmetry of DFT

Equivalence to supergravity: 1. Generalized parallelizable spaces

[Lee, Strickland-Constable, and Waldram, 2014]

- generalized tangent space element $V^{\hat{I}} = \begin{pmatrix} V^i & V_i \end{pmatrix}$
- generalized Lie derivative

$$\widehat{\mathcal{L}}_\xi V^{\hat{I}} = \xi^{\hat{J}} \partial_{\hat{J}} V^{\hat{I}} + (\partial^{\hat{I}} \xi_{\hat{J}} - \partial_{\hat{J}} \xi^{\hat{I}}) V^{\hat{J}} \qquad \text{with} \qquad \partial_{\hat{I}} = \begin{pmatrix} 0 & \partial_i \end{pmatrix}$$

Definition: A manifold M which admits a globally defined generalized frame field $\widehat{E}_A{}^{\hat{I}}(x^i)$ satisfying

1. $\widehat{\mathcal{L}}_{\widehat{E}_A} \widehat{E}_B{}^{\hat{I}} = F_{AB}{}^C \widehat{E}_C{}^{\hat{I}}$

   where $F_{AB}{}^C$ are the structure constants of a Lie algebra $\mathfrak{h}$
2. $\widehat{E}_A{}^{\hat{I}} \eta^{AB} \widehat{E}_B{}^{\hat{J}} = \eta^{\hat{I}\hat{J}} = \begin{pmatrix} 0 & \delta_i^j \\ \delta_j^i & 0 \end{pmatrix}$

is a **generalized parallelizable space** $(M, \mathfrak{h}, \widehat{E}_A{}^{\hat{I}})$.

Equivalence to supergravity: 2. Generalized metric and dilaton

[Klimcik and Severa, 1995; Hull and Reid-Edwards, 2009; du Bosque, Hassler, Lüst, 2017]

- Drinfeld double $\mathcal{D} \to$ two generalized parallelizable spaces:

$$(D/\tilde{G}, \mathfrak{d}, \widehat{E}_A{}^{\hat{I}}) \qquad \text{and} \qquad (D/G, \mathfrak{d}, \widetilde{\widehat{E}}_A{}^{\hat{I}})$$

$$\widehat{E}_A{}^{\hat{I}} = M_A{}^B \begin{pmatrix} v^b{}_i & 0 \\ 0 & v_b{}^i \end{pmatrix}_B{}^{\hat{I}} \qquad \widetilde{\widehat{E}}_A{}^{\hat{I}} = \tilde{M}_{AB} \begin{pmatrix} \tilde{v}_{bi} & 0 \\ 0 & \tilde{v}^{bi} \end{pmatrix}^{B\hat{I}}$$

- express $\mathcal{H}^{AB}$ in terms of the generalized $\widehat{\mathcal{H}}^{\hat{I}\hat{J}}$ on $TD/\tilde{G} \oplus T^*D/\tilde{G}$

$$\mathcal{H}^{AB} = \widehat{E}^A{}_{\hat{I}} \widehat{\mathcal{H}}^{\hat{I}\hat{J}} \widehat{E}^B{}_{\hat{J}} \qquad \text{with} \qquad \widehat{\mathcal{H}}^{\hat{I}\hat{J}} = \begin{pmatrix} g_{ij} - B_{ik} g^{kl} B_{lk} & -B_{ik} g^{kl} \\ g^{ik} B_{kj} & g^{ij} \end{pmatrix}$$

- express d in terms of the standard generalized dilaton $\widehat{d}$

$$d = \widehat{d} - \frac{1}{2} \log |\det \tilde{v}_{ai}|$$

$$\widehat{d} = \phi - 1/4 \log \left|\det g_{ij}\right|$$

- plug into the DFT action S_{NS}

Equivalence to supergravity: 3. IIA/B bosonic sector action

- if G and $\tilde{G}$ are unimodular

$$S_{\mathrm{NS}} = V_{\tilde{G}} \int d^D x \, e^{-2\widehat{d}} \Big(\frac{1}{8} \widehat{\mathcal{H}}^{\hat{K}\hat{L}} \partial_{\hat{K}} \widehat{\mathcal{H}}_{\hat{I}\hat{J}} \partial_{\hat{L}} \widehat{\mathcal{H}}^{\hat{I}\hat{J}} - 2 \partial_{\hat{I}} \widehat{d} \partial_{\hat{J}} \widehat{\mathcal{H}}^{\hat{I}\hat{J}} - \frac{1}{2} \widehat{\mathcal{H}}^{\hat{I}\hat{J}} \partial_{\hat{J}} \widehat{\mathcal{H}}^{\hat{K}\hat{L}} \partial_{\hat{L}} \widehat{\mathcal{H}}_{\hat{I}\hat{K}} + 4 \widehat{\mathcal{H}}^{\hat{I}\hat{J}} \partial_{\hat{I}} \widehat{d} \partial_{\hat{J}} \widehat{d} \Big)$$

- $V_{\tilde{G}} = \int_{\tilde{G}} d\tilde{x}^D \det \tilde{v}_{ai}$ volume of group $\tilde{G}$.
- equivalent to IIA/B NS/NS sector action
  [Hohm, Hull, and Zwiebach, 2010; Hohm, Hull, and Zwiebach, 2010]

$$S_{\mathrm{NS}} = V_{\tilde{G}} \int \mathrm{d}^D x \sqrt{\det(g_{ij})} e^{-2\phi} \big(\mathcal{R} + 4 \partial_i \phi \partial^i \phi - \frac{1}{12} H_{ijk} H^{ijk} \big)$$

- holds for all $\mathcal{H}_{AB}(x^i)$ / $\widehat{\mathcal{H}}^{\hat{I}\hat{J}}$
- only D-diffeomorphisms and B-field gauge trans. as symmetries

- similar story for R/R sector

Restrictions on $\mathcal{H}_{AB}$ and d to admit Poisson-Lie Symmetry

- in general $\mathcal{H}_{AB}(x^i) \xrightarrow{\text{Poisson-Lie T-duality } (2D\text{-diff.})} \mathcal{H}_{AB}(x'^i, x'^{\tilde{i}})$
- $x'^{\tilde{i}}$ part not compatible with ansatz for SC solutions $\rightarrow$ avoid it

A doubled space $(\mathcal{D}, \mathcal{H}_{AB}, d)$ admits Poisson-Lie T-dual supergravity descriptions iff

1. $L_\xi \mathcal{H}_{AB} = 0 \quad \forall \xi \quad \rightarrow \quad D_C \mathcal{H}_{AB} = 0$
2. $L_\xi d = 0 \quad \forall \xi \quad \rightarrow \quad D_A e^{-2d} = 0$

Application: Dilaton profile

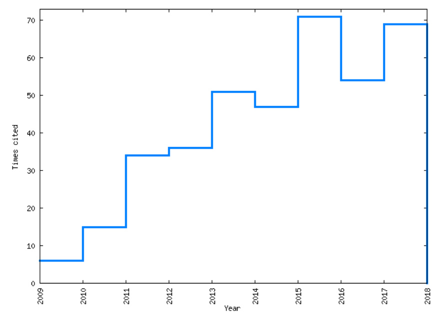
- $D_A e^{-2d} = 0 \quad \rightarrow \quad \partial_I \underbrace{(2d + \log|\det v| + \log|\det \tilde{v}|)}_{=2\phi_0 = \text{const.}} = 0$
- $d = \phi - 1/4 \log|\det g| - \frac{1}{2}\log|\det \tilde{v}| \quad \rightarrow \quad \phi = \phi_0 + \frac{1}{4}\log|\det g| - \frac{1}{2}\log|\det v|$
- $g = v^T e^T e v \quad \text{with} \quad \begin{cases} (\tilde{B}_0 + \tilde{g}_0)^{ab} = E^{0\,ab} \\ \Pi^{ab} = M^{ac} M^b{}_c \\ e^{-1} e^{-T} = \tilde{g}_0 - (\tilde{B}_0 + \Pi)\tilde{g}_0^{-1}(\tilde{B}_0 + \Pi) \\ \tilde{e}_0^T \tilde{e}_0 = \tilde{g}_0 \\ e^{-T} = \tilde{e}_0 + \tilde{e}_0^{-T}(\tilde{B}_0 + \Pi) \end{cases}$
- $\phi = \phi_0 + \frac{1}{2}\log|\det e| = \phi_0 - \frac{1}{2}\log|\det \tilde{e}_0| - \frac{1}{2}\log\left|\det\left(1 + \tilde{g}_0^{-1}(\tilde{B}_0 + \Pi)\right)\right|$
- reproduces [Jurco and Vysoky, 2017]

Summary

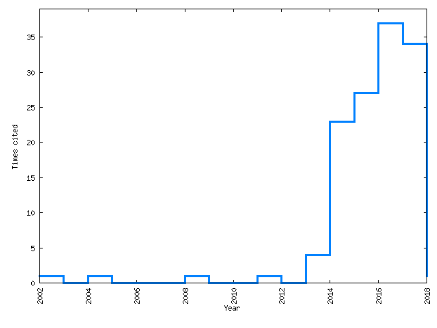
- DFT, Poisson-Lie T-duality and Drinfeld doubles fit together naturally
- interpretation of doubled space does not require winding modes anymore (phase space perspective instead)
- various new directions for research in DFT
  - connection to integrability in SUGRA
  - Drinfeld doubles $\to$ quantum groups $\to$ rich mathematical structure
  - new way to organized α' corrections?
  - implication for consistent truncation
  - branes in curved space [Klimcik, and Severa, 1996 (D-branes)]?
- facilitates new applications
  - integrable deformations of 2D σ-models
  - solution generating technique
  - explore underlying structure of AdS/CFT

Summary

- DFT, Poisson-Lie T-duality and Drinfeld doubles fit together naturally
- interpretation of doubled space does not require winding modes



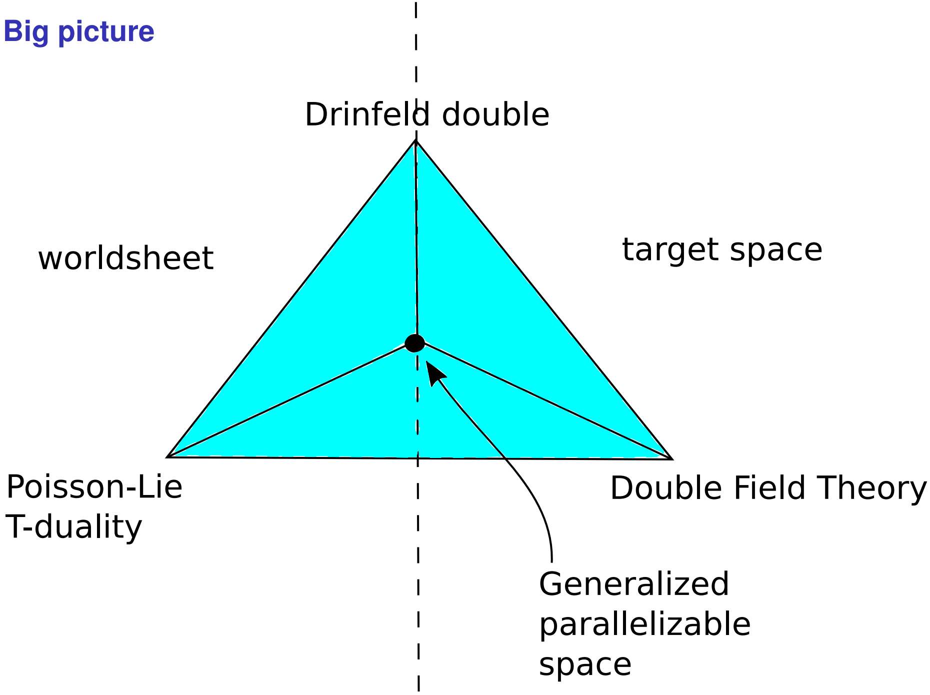
Hull and Zwiebach, 2009



Klimcik, 2002

  - solution generating technique
  - explore underlying structure of AdS/CFT

Big picture



Drinfeld double
worldsheet
target space
Poisson-Lie
T-duality
Double Field Theory
Generalized
parallelizable
space