Poisson-Lie T-duality in Double Field Theory

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based on

1707.08624, 1611.07978

and

1502.02428 with Pascal du Bosque, Dieter Lüst and Ralph Blumenhagen

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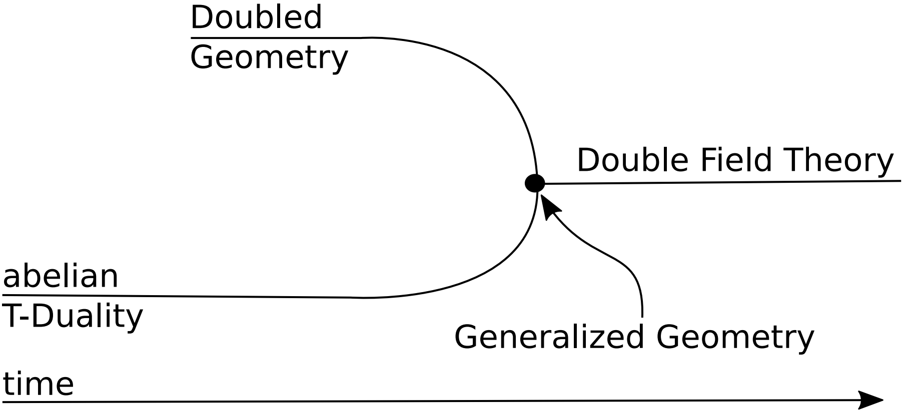
Motivation

Chris Hull
Barton Zwiebach
Olaf Hohm



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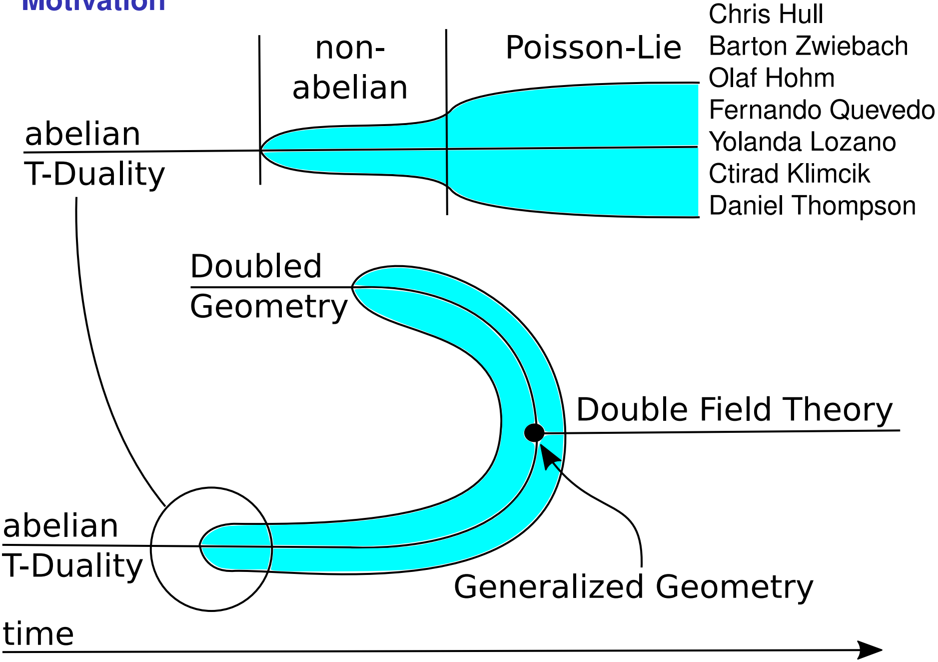
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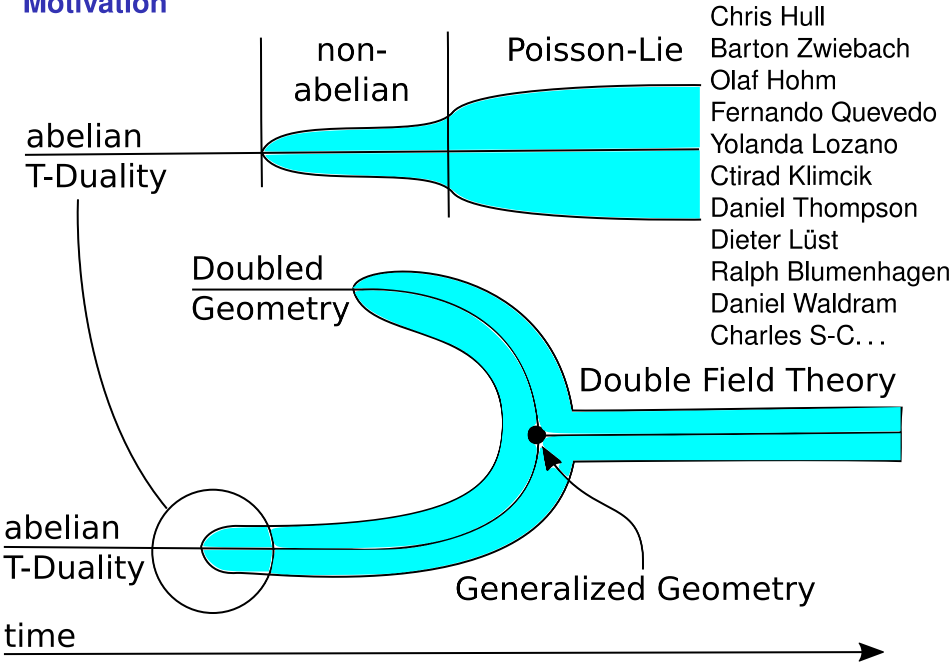
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Outline

1. Motivation
2. Poisson-Lie T-duality
3. Double Field Theory on Drinfeld doubles
4. Application: Dilaton transformation
5. Summary

Drinfeld double [Drinfeld, 1988]

Definition: A **Drinfeld double** is a $2D$-dimensional Lie group $\mathcal{D}$, whose Lie-algebra $\mathfrak{d}$

1. has an ad-invariant bilinear for $\langle \cdot\,,\,\cdot \rangle$ with signature (D, D)
2. admits the decomposition into two maximal isotropic subalgebras $\mathfrak{g}$ and $\tilde{\mathfrak{g}}$

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- $\begin{pmatrix} t^a & t_a \end{pmatrix} = t_A \in \mathfrak{d}\,, \quad t_a \in \mathfrak{g} \quad \text{and} \quad t^a \in \tilde{\mathfrak{g}}$
- $\langle t_A, t_B\rangle = \eta_{AB} = \begin{pmatrix} 0 & \delta_b^a \\ \delta_a^b & 0 \end{pmatrix}$

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- $\langle t_A, t_B \rangle = \eta_{AB} = \begin{pmatrix} 0 & \delta^a_b \\ \delta_a^b & 0 \end{pmatrix}$
- $[t_A, t_B] = F_{AB}{}^C t_C$ with non-vanishing commutators

$$[t_a, t_b] = f_{ab}{}^c t_c \qquad [t_a, t^b] = \tilde{f}^{bc}{}_a t_c - f_{ac}{}^b t^c$$
$$[t^a, t^b] = \tilde{f}^{ab}{}_c t^c$$

- ad-invariance of $\langle \cdot\,, \cdot \rangle$ implies $F_{ABC} = F_{[ABC]}$

Poisson-Lie T-duality: 1. Definition [Klimcik and Severa, 1995]

- 2D σ-model on target space M with action

  $S(E, M) = \int dzd\bar{z}\, E_{ij}\partial x^i \bar{\partial} x^j$
- $E_{ij} = g_{ij} + B_{ij}$ captures metric and two-from field on M
- inverse of E_{ij} is denoted as E^{ij}

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- *left* invariant vector field $v_a{}^i$ on G is the inverse transposed of *right* invariant Maurer-Cartan form $t_a v^a{}_i dx^i = dg\, g^{-1}$
- adjoint action of $g \in G$ on $t_A \in \mathfrak{d}$: $\mathrm{Ad}_g\, t_A = g t_A g^{-1} = M_A{}^B t_B$
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Definition: $S(E, \mathcal{D}/\tilde{G})$ and $S(\tilde{E}, \mathcal{D}/G)$ are **Poisson-Lie T-dual** if

$$E^{ij} = {v_c}^i {M_a}^c (M^{ae} {M^b}_e + E_0^{ab}) {M_b}^d {v_d}^j$$
$$\tilde{E}^{ij} = \tilde{v}^{ci} {\tilde{M}^a}_c (\tilde{M}_{ae} {\tilde{M}_b}^e + E_{0\,ab}) {\tilde{M}^b}_d \tilde{v}^{dj}$$

holds, where E_0^{ab} is constant and invertible with the inverse $E_{0\,ab}$.

Poisson-Lie T-duality: 2. Properties

- captures
$$\begin{cases} \text{abelian T-d.} & G \ \text{abelian} & \text{and} \ \tilde{G} \ \text{abelian} \\ \text{non-abelian T-d.} & G \ \text{non-abelian} & \text{and} \ \tilde{G} \ \text{abelian} \end{cases}$$
[Ossa and Quevedo, 1993;Giveon and Rocek, 1994; Alvarez, Alvarez-Gaume, and Lozano, 1994;. . .]

Poisson-Lie T-duality: 2. Properties

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  - non-abelian T-d. G non-abelian and $\tilde{G}$ abelian

  [Ossa and Quevedo, 1993;Giveon and Rocek, 1994; Alvarez, Alvarez-Gaume, and Lozano, 1994;. . .]
- dual σ-models related by canonical transformation

  [Klimcik and Severa, 1995;Klimcik and Severa, 1996;Sfetsos, 1998]

$\rightarrow$ equivalent at the classical level

- preserves conformal invariance at one-loop

  [Alekseev, Klimcik, and Tseytlin, 1996;Sfetsos, 1998;. . . ;Jurco and Vysoky, 2017]

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  [Alekseev, Klimcik, and Tseytlin, 1996;Sfetsos, 1998;...;Jurco and Vysoky, 2017]
- dilaton transformation [Jurco and Vysoky, 2017]

$$\phi = -\frac{1}{2}\log\left|\det\left(1 + \tilde{g}_0^{-1}(\tilde{B}_0 + \Pi)\right)\right|$$
$$\tilde{\phi} = -\frac{1}{2}\log\left|\det\left(1 + g_0^{-1}(B_0 + \tilde{\Pi})\right)\right| \qquad \text{details later}$$

Additional structure on the Drinfeld double

[Blumenhagen, Hassler, and Lüst, 2015, Blumenhagen, Bosque, Hassler, and Lüst, 2015]

- *right* invariant vector $E_A{}^I$ field on $\mathcal{D}$ is the inverse transposed of *left* invariant Maurer-Cartan form $t_A E^A{}_I dX^I = g^{-1} dg$

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- two η-compatible, covariant derivatives[1]
  1. flat derivative

     $$D_A V^B = E_A{}^I \partial_I V^B - w F_A V^B, \qquad F_A = D_A \log \left| \det(E^B{}_I) \right|$$

  2. convenient derivative

     $$\nabla_A V^B = D_A V^B + \frac{1}{3} F_{AC}{}^B V^C$$

[1] definitions here just for quantities with flat indices

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- generalized metric $\mathcal{H}_{AB}$ ($w = 0$)

$$\mathcal{H}_{AB} = \mathcal{H}_{(AB)}, \qquad \mathcal{H}_{AC} \eta^{CD} H_{DB} = \eta_{AB}$$

- generalized dilaton d with e^{-2d} scalar density of weight $w = 1$

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- generalized dilaton d with e^{-2d} scalar density of weight $w = 1$
- triple $(\mathcal{D}, \mathcal{H}_{AB}, d)$ captures the doubled space of DFT

[1] definitions here just for quantities with flat indices

Double Field Theory for $(\mathcal{D}, \mathcal{H}_{AB}, d)$ [Blumenhagen, Bosque, Hassler, and Lüst, 2015]

see also [Vaisman, 2012; Hull and Reid-Edwards, 2009;Geissbuhler, Marques, Nunez, and Penas, 2013; Cederwall, 2014; ...]

- action ($\nabla_A d = -\frac{1}{2} e^{2d} \nabla_A e^{-2d}$)

$$S_{\mathrm{NS}} = \int_{\mathcal{D}} d^{2D} X e^{-2d} \Big(\frac{1}{8} \mathcal{H}^{CD} \nabla_C \mathcal{H}_{AB} \nabla_D \mathcal{H}^{AB} - \frac{1}{2} \mathcal{H}^{AB} \nabla_B \mathcal{H}^{CD} \nabla_D \mathcal{H}_{AC} - 2 \nabla_A d \nabla_B \mathcal{H}^{AB} + 4 \mathcal{H}^{AB} \nabla_A d \nabla_B d + \frac{1}{6} F_{ACD} F_B{}^{CD} \mathcal{H}^{AB} \Big)$$

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- $2D$-diffeomorphisms

$$L_\xi V^A = \xi^B D_B V^A + w D_B \xi^B V^A$$

- global O(D,D) transformations

$$V^A \to T^A{}_B V^B \quad \text{with} \quad T^A{}_C T^B{}_D \eta^{CD} = \eta^{AB}$$

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- generalized diffeomorphisms

$$\mathcal{L}_\xi V^A = \xi^B \nabla_B V^A + \left(\nabla^A \xi_B - \nabla_B \xi^A \right) V^B + w \nabla_B \xi^B V^A$$

- section condition (SC)

$$\eta^{AB} D_A \cdot D_B \cdot = 0$$

Symmetries of the action

- S_{NS} invariant for $X^I \rightarrow X^I + \xi^A E_A{}^I$ and
  1. $\mathcal{H}^{AB} \rightarrow \mathcal{H}^{AB} + \mathcal{L}_\xi \mathcal{H}^{AB}$ and $e^{-2d} \rightarrow e^{-2d} + \mathcal{L}_\xi e^{-2d}$
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object	gen.-diffeomorphisms	$2D$-diffeomorphisms	global O(D,D)
$\mathcal{H}_{AB}$	tensor	scalar	tensor
$\nabla_A d$	not covariant	scalar	1-form
e^{-2d}	scalar density (w=1)	scalar density (w=1)	invariant
η_{AB}	invariant	invariant	invariant
$F_{AB}{}^C$	invariant	invariant	tensor
$E_A{}^I$	invariant	vector	1-form
S_{NS}	invariant	invariant	invariant
SC	invariant	invariant	invariant
D_A	not covariant	covariant	covariant
∇_A	not covariant	covariant	covariant

(last two columns: manifest)

Poisson-Lie T-duality: 1. Solve SC [Hassler, 2016]

- fix D physical coordinates x^i from $X^I = \begin{pmatrix} x^i & x^{\tilde{i}} \end{pmatrix}$ on $\mathcal{D}$

  such that $\eta^{IJ} = E_A{}^I \eta^{AB} E_B{}^J = \begin{pmatrix} 0 & \cdots \\ \cdots & \cdots \end{pmatrix} \to$ SC is solved
- fields and gauge parameter depend just on x^i

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- only *two* SC solutions, relate them by symmetries of DFT

$$d(X^I) = g(x^i)\tilde{g}(x^{\tilde{i}}) \qquad t_A = \begin{pmatrix} t^a & t_a \end{pmatrix}$$

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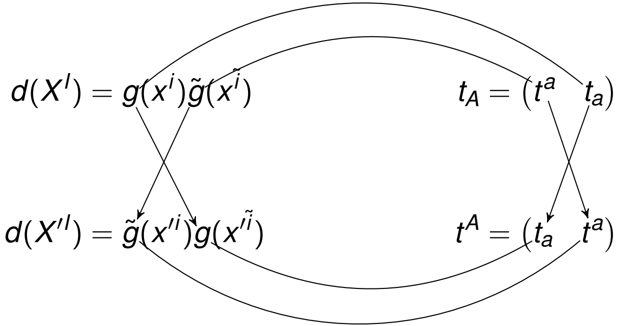

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- duality target spaces arise as different solutions of the SC

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- 2D-diffeomorphisms $X^I \to X'^I(X^1, \dots X^{2D})$ with $d(X^I) = d(X'^I)$
- global O(D,D) transformation $t_A \to \eta^{AB} t_B$

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Poisson-Lie T-duality is a manifest symmetry of DFT

Equivalence to supergravity: 1. Generalized parallelizable spaces

[Lee, Strickland-Constable, and Waldram, 2014]

- generalized tangent space element $V^{\hat{I}} = \begin{pmatrix} V^i & V_i \end{pmatrix}$
- generalized Lie derivative

$$\widehat{\mathcal{L}}_\xi V^{\hat{I}} = \xi^{\hat{J}} \partial_{\hat{J}} V^{\hat{I}} + \left(\partial^{\hat{I}} \xi_{\hat{J}} - \partial_{\hat{J}} \xi^{\hat{I}} \right) V^{\hat{J}} \qquad \text{with} \qquad \partial_{\hat{I}} = \begin{pmatrix} 0 & \partial_i \end{pmatrix}$$

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Definition: A manifold M which admits a globally defined generalized frame field $\widehat{E}_A{}^{\hat{I}}(x^i)$ satisfying

1. $\widehat{\mathcal{L}}_{\widehat{E}_A} \widehat{E}_B{}^{\hat{I}} = F_{AB}{}^C \widehat{E}_C{}^{\hat{I}}$

   where $F_{AB}{}^C$ are the structure constants of a Lie algebra $\mathfrak{h}$

2. $\widehat{E}_A{}^{\hat{I}} \eta^{AB} \widehat{E}_B{}^{\hat{J}} = \eta^{\hat{I}\hat{J}} = \begin{pmatrix} 0 & \delta_i^j \\ \delta^i_j & 0 \end{pmatrix}$

is a **generalized parallelizable space** $(M, \mathfrak{h}, \widehat{E}_A{}^{\hat{I}})$.

Equivalence to supergravity: 2. Generalized metric and dilaton

[Klimcik and Severa, 1995; Hull and Reid-Edwards, 2009; du Bosque, Hassler, Lüst, 2017]

- Drinfeld double $\mathcal{D} \to$ two generalized parallelizable spaces:

$$(D/\tilde{G}, \mathfrak{d}, \widehat{E}_A{}^{\widehat{I}})$$

$$\widehat{E}_A{}^{\widehat{I}} = M_A{}^B \begin{pmatrix} v^b{}_i & 0 \\ 0 & v_b{}^i \end{pmatrix}_B{}^{\widehat{I}}$$

and

$$(D/G, \mathfrak{d}, \widetilde{\widehat{E}}_A{}^{\widehat{I}})$$

$$\widetilde{\widehat{E}}_A{}^{\widehat{I}} = \tilde{M}_{AB} \begin{pmatrix} \tilde{v}_{bi} & 0 \\ 0 & \tilde{v}^{bi} \end{pmatrix}^{B\widehat{I}}$$

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- Drinfeld double $\mathcal{D} \to$ two generalized parallelizable spaces:

$$(D/\tilde{G}, \mathfrak{d}, \widehat{E}_A{}^{\widehat{I}}) \quad \text{and} \quad (D/G, \mathfrak{d}, \widetilde{\widehat{E}}_A{}^{\widehat{I}})$$

$$\widehat{E}_A{}^{\widehat{I}} = M_A{}^B \begin{pmatrix} v^b{}_i & 0 \\ 0 & v_b{}^i \end{pmatrix}_B{}^{\widehat{I}} \qquad \widetilde{\widehat{E}}_A{}^{\widehat{I}} = \tilde{M}_{AB} \begin{pmatrix} \tilde{v}_{bi} & 0 \\ 0 & \tilde{v}^{bi} \end{pmatrix}^{B\widehat{I}}$$

- express $\mathcal{H}^{AB}$ in terms of the generalized $\widehat{\mathcal{H}}^{\hat{I}\hat{J}}$ on $TD/\tilde{G} \oplus T^*D/\tilde{G}$

$$\mathcal{H}^{AB} = \widehat{E}^A{}_{\hat{I}} \widehat{\mathcal{H}}^{\hat{I}\hat{J}} \widehat{E}^B{}_{\hat{J}} \qquad \text{with} \qquad \widehat{\mathcal{H}}^{\hat{I}\hat{J}} = \begin{pmatrix} g_{ij} - B_{ik} g^{kl} B_{lk} & -B_{ik} g^{kl} \\ g^{ik} B_{kj} & g^{ij} \end{pmatrix}$$

- express d in terms of the standard generalized dilaton $\widehat{d}$

$$d = \widehat{d} - \frac{1}{2} \log |\det \tilde{v}_{ai}|$$

$$\widehat{d} = \phi - 1/4 \log \left|\det g_{ij}\right|$$

Equivalence to supergravity: 2. Generalized metric and dilaton

[Klimcik and Severa, 1995; Hull and Reid-Edwards, 2009; du Bosque, Hassler, Lüst, 2017]

- Drinfeld double $\mathcal{D} \rightarrow$ two generalized parallelizable spaces:

$$(D/\tilde{G}, \mathfrak{d}, \widehat{E}_A{}^{\widehat{I}}) \qquad \text{and} \qquad (D/G, \mathfrak{d}, \widetilde{\widehat{E}}_A{}^{\widehat{I}})$$

$$\widehat{E}_A{}^{\widehat{I}} = M_A{}^B \begin{pmatrix} v^b{}_i & 0 \\ 0 & v_b{}^i \end{pmatrix}_B{}^{\widehat{I}} \qquad \widetilde{\widehat{E}}_A{}^{\widehat{I}} = \tilde{M}_{AB} \begin{pmatrix} \tilde{v}_{bi} & 0 \\ 0 & \tilde{v}^{bi} \end{pmatrix}^{B\widehat{I}}$$

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- plug into the DFT action S_{NS}

Equivalence to supergravity: 3. IIA/B bosonic sector action

- if G and $\tilde{G}$ are unimodular

$$S_{\mathrm{NS}} = V_{\tilde{G}} \int d^D x \, e^{-2\widehat{d}} \Big(\frac{1}{8} \widehat{\mathcal{H}}^{\hat{K}\hat{L}} \partial_{\hat{K}} \widehat{\mathcal{H}}_{\hat{I}\hat{J}} \partial_{\hat{L}} \widehat{\mathcal{H}}^{\hat{I}\hat{J}} - 2 \partial_{\hat{I}} \widehat{d} \partial_{\hat{J}} \widehat{\mathcal{H}}^{\hat{I}\hat{J}} - \frac{1}{2} \widehat{\mathcal{H}}^{\hat{I}\hat{J}} \partial_{\hat{J}} \widehat{\mathcal{H}}^{\hat{K}\hat{L}} \partial_{\hat{L}} \widehat{\mathcal{H}}_{\hat{I}\hat{K}} + 4 \widehat{\mathcal{H}}^{\hat{I}\hat{J}} \partial_{\hat{I}} \widehat{d} \partial_{\hat{J}} \widehat{d} \Big)$$

- $V_{\tilde{G}} = \int_{\tilde{G}} d\tilde{x}^D \det \tilde{v}_{ai}$ volume of group $\tilde{G}$

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$$S_{\mathrm{NS}} = V_{\tilde{G}} \int \mathrm{d}^D x \sqrt{\det(g_{ij})} e^{-2\phi} \left(\mathcal{R} + 4 \partial_i \phi \partial^i \phi - \frac{1}{12} H_{ijk} H^{ijk} \right)$$

- holds for all $\mathcal{H}_{AB}(x^i)$ / $\widehat{\mathcal{H}}^{\hat{I}\hat{J}}(x^i)$
- only D-diffeomorphisms and B-field gauge trans. as symmetries

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- similar story for R/R sector

Restrictions on $\mathcal{H}_{AB}$ and d to admit Poisson-Lie T-duality

- in general $\mathcal{H}_{AB}(x^i) \xrightarrow{\text{Poisson-Lie T-duality } (2D\text{-diff.})} \mathcal{H}_{AB}(x'^i, x'^{\tilde{i}})$
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A doubled space $(\mathcal{D}, \mathcal{H}_{AB}, d)$ admits Poisson-Lie T-dual supergravity descriptions iff

1. $L_\xi \mathcal{H}_{AB} = 0 \quad \forall \xi \quad \rightarrow \quad D_A \mathcal{H}_{AB} = 0$
2. $L_\xi d = 0 \quad \forall \xi \quad \rightarrow \quad D_A e^{-2d} = 0$

Application: Dilaton transformation

- $D_A e^{-2d} = 0 \quad \rightarrow \quad \partial_I \underbrace{(2d + \log|\det v| + \log|\det \tilde{v}|)}_{= 2\phi_0 = \text{const.}} = 0$

Application: Dilaton transformation

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- $d = \phi - 1/4 \log|\det g| - \frac{1}{2}\log|\det \tilde{v}|$
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- $g = v^T e^T e v \quad \text{with} \quad \begin{cases} (\tilde{B}_0 + \tilde{g}_0)^{ab} = E^{0\,ab} \\ \Pi^{ab} = M^{ac} M^b{}_c \\ e^{-1} e^{-T} = \tilde{g}_0 - (\tilde{B}_0 + \Pi) \tilde{g}_0^{-1} (\tilde{B}_0 + \Pi) \\ \tilde{e}_0^T \tilde{e}_0 = \tilde{g}_0 \\ e^{-T} = \tilde{e}_0 + \tilde{e}_0^{-T} (\tilde{B}_0 + \Pi) \end{cases}$

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- $\phi = \phi_0 + \frac{1}{2}\log|\det e| = \phi_0 - \frac{1}{2}\log|\det \tilde{e}_0| - \frac{1}{2}\log\left|\det\left(1 + \tilde{g}_0^{-1}(\tilde{B}_0 + \Pi)\right)\right|$
- reproduces [Jurco and Vysoky, 2017]

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  - Drinfeld doubles $\rightarrow$ quantum groups $\rightarrow$ rich mathematical structure
  - new way to organized α' corrections?
  - new way to construct non-geometric backgrounds?
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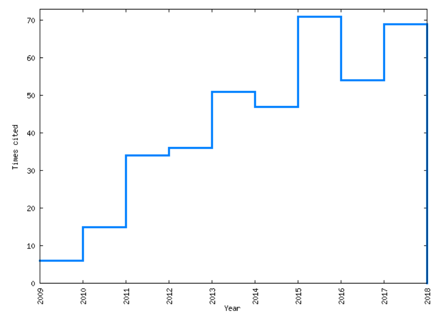
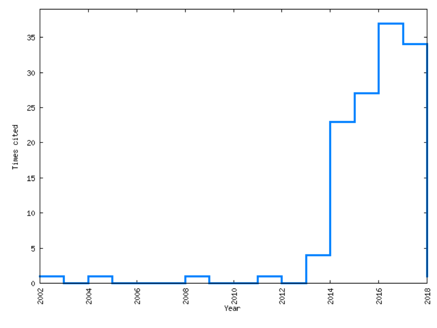
- facilitates new applications
  - integrable deformations of 2D σ-models (see Daniel's talk)
  - solution generating technique
  - explore underlying structure of AdS/CFT (see Yolanda's talk)

Summary

- DFT, Poisson-Lie T-duality and Drinfeld doubles fit together naturally
- interpretation of doubled space does not require winding modes



Hull and Zwiebach, 2009



Klimcik, 2002

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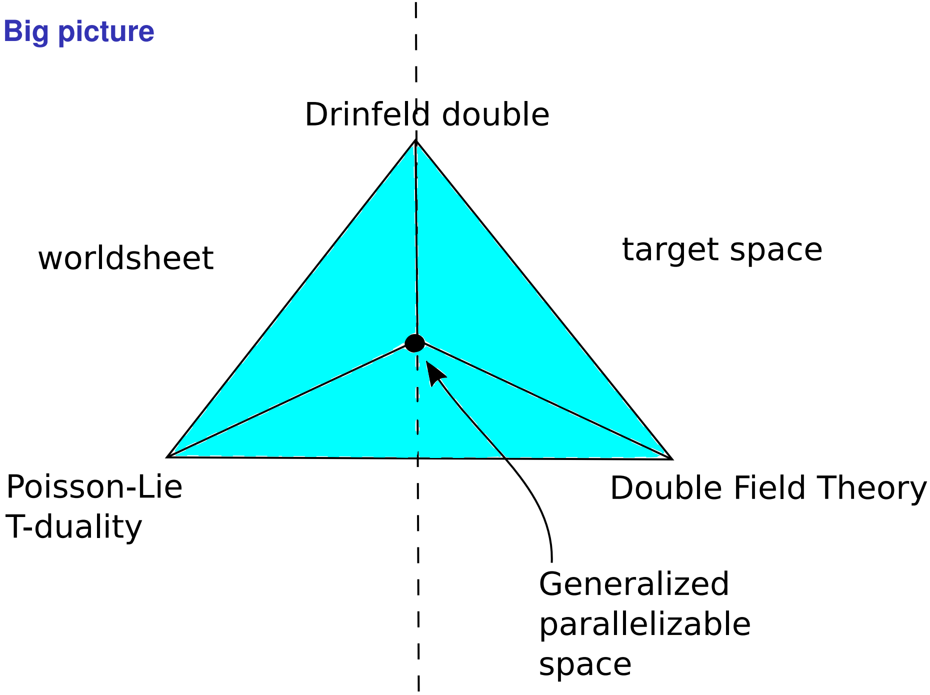
Big picture


Drinfeld double
worldsheet
target space
Poisson-Lie
T-duality
Double Field Theory
Generalized
parallelizable
space